

Squeezing-Enhanced Sensing at an Exceptional Point

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Pushing the boundaries of measurement precision is central for sensing and metrology, pursued by nonclassical resources such as squeezing, as well as non-Hermitian degeneracies with distinct spectral response. Their convergence, however, remains challenging. We find extraordinary enhancement of sensitivity by unifying both effects in a general framework for quantum sensing in open systems. At the parametric oscillation threshold and an exceptional point, the sensing precision exhibits a unique quartic scaling with the perturbation strength. The result generalizes to multimode squeezed-state sensors with higher-order exceptional points catered to various quantum sensing platforms.

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Introduction—Quantum sensing—leveraging quantum phenomena such as entanglement and coherence to probe physical quantities—has rapidly emerged as a prominent and fast-growing field [1,2]. A long-lasting objective for quantum sensing is to boost the achievable precision in the presence of classical and quantum noise via improving measurement sensitivity and signal-to-noise ratio (SNR). Squeezing, with reduced uncertainty in one quadrature of the electromagnetic field at the expense of increasing the uncertainty in the orthogonal quadrature, constitutes a powerful resource to surpass the standard quantum limit (SQL) [3–5]. Recent years have witnessed remarkable advances in leveraging squeezing to enhance weak-signal detection across diverse quantum sensing platforms, such as LIGO [6] and Virgo [7] gravitational wave detectors, nonlinear integrated photonic circuits [8], lithium-niobate optical resonators [9], mechanical oscillators [10], optomechanical systems [11], and superconducting cavities [12]. In particular, operating squeezing-enhanced sensors near the parametric oscillation (PO) threshold, a phase transition point, leads to divergent behavior of susceptibility through critical quantum sensing (CQS) protocols [13–18].

As a parallel pursuit toward ultraprecise measurements, non-Hermitian physics [19–22] presents new opportunities by exploiting non-Hermitian degeneracies known as exceptional points (EPs) [23,24]. EPs are singularities in the

parameter space, where eigenvalues and their associated eigenstates of a non-Hermitian Hamiltonian or other non-Hermitian operators become degenerate [24–29]. Near an n th-order EP, a weak perturbation θ induces eigenspectral splitting scaling as θ^n [30,31], yielding amplified responses as $\theta \rightarrow 0$. Hitherto, EP-enhanced sensing schemes have been explored in a plethora of platforms, including photonic resonators [32–34], laser gyroscopes [35,36], plasmonics [37], electronic circuits [38], mechanical resonators [39], etc.

Despite these encouraging advances in EP-enhanced sensing, recent studies have sparked debates about the ability of EPs to break the fundamental precision limit imposed by quantum noise [40–45]. Notably, the SNR of gain-assisted EP sensors is strongly limited by the Petermann factor-enhanced laser noise arising from non-orthogonal eigenstates [46]. Nevertheless, a quantum-noise analysis has shown EP-enhanced SNR through a quadratic scaling between sensing precision and perturbation strength for a sensor operating at the lasing threshold with a second-order EP [47]. Furthermore, several studies revealed intriguing interplays between squeezing and non-Hermiticity, including EP-enhanced squeezing in anti-parity-time symmetric systems [48] and EP-assisted squeezing control in superconducting resonators [49]. Yet, key questions remain: (1) How do EPs modify the ultimate sensitivity and precision limit for quantum sensing? (2) What is the physical mechanism behind EP enhancement? and (3) How do EPs and squeezing jointly influence the sensing limit?

To address these questions, we develop a unified quantum-noise framework to clarify the role of EPs in bosonic-mode quantum sensors that concurrently generate squeezing. We evaluate the precision bound for non-Hermitian sensors composed of single or coupled squeezed bosonic modes near the PO threshold, by deriving the

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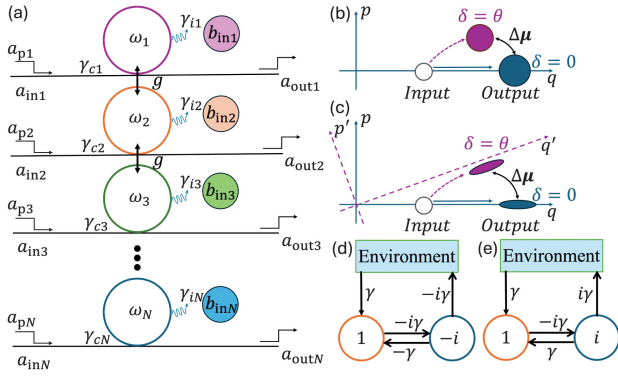


FIG. 1. Squeezing-enhanced non-Hermitian sensing. (a) Schematic diagram of a squeezing-enhanced non-Hermitian sensor consisting of N coupled bosonic modes with χ^2 nonlinearity. The j th mode with resonance frequency ω_j ($j = 1, 2, \dots, N$) is coupled to an input-output channel (intrinsic loss channel $b_{in,j}$) with coupling strength γ_{cj} (γ_{ij}). Single-mode squeezing is generated via parametric down-conversion triggered by a pump a_{pj} . Parameter perturbation is estimated via the scattering from the input $a_{in,j}$ to the output $a_{out,j}$. (b) Sensing near the lasing threshold. The weak perturbation θ on the resonance frequency induces a variation of the mean of output ($\Delta\mu$) and its noise distribution represented by circles. (c) Sensing near the parametric amplification threshold. The perturbation θ rotates the eigenbasis in the quadrature space, causing a large displacement in the output ($\Delta\mu$). (d),(e) Energy flow at an exceptional point. Two modes with amplification (orange circle) and dissipation (blue circle) each at rate γ are coupled with strength γ . (d) For the eigenstate $(1, -i)^T$, destructive interference between the modal coupling channel and the gain/loss in each mode yields steady energy. (e) For a noneigenstate such as $(1, i)^T$, the interference becomes constructive, producing energy growth during evolution.

quantum Fisher information (QFI). Our Letter reveals that operating at the PO threshold with a second-order EP yields a quartic scaling between the precision limit and perturbation strength, enabling exceptional sensitivity to weak signals. We further generalize the analysis to squeezing-enhanced sensors at higher-order EPs.

Non-Hermitian bosonic sensors with squeezing—We consider a canonical bosonic sensing system consisting of N coupled harmonic oscillators, each supporting a bosonic mode a_j at the resonance frequency ω_j ($j = 1, 2, \dots, N$) [Fig. 1(a)]. A weak external signal imparts a uniform perturbation θ , shifting the resonance frequencies as $\omega_j \rightarrow \omega_j + \theta$. We assume that each cavity supports degenerate parametric amplification that produces single-mode squeezing of amplitude ϵ_j through χ^2 nonlinearity driven by a coherent pump field at ω_{pj} . Furthermore, each mode is coupled to an intrinsic loss channel b_j at rate γ_{0j} , and to an external input/output channel $a_{in,j}$ (such as a waveguide) with coupling strength γ_{cj} . To incorporate phase-insensitive amplification, such as optical gain, we include coupling to an auxiliary scattering channel c_j with

strength κ_j . One can describe the system using a generic Hamiltonian under the rotating frame

$$H = \sum_j \hbar(\delta_j + \theta) a_j^\dagger a_j + \sum_{j,k} \hbar g(a_k^\dagger a_j + \text{H.c.}) + \sum_j \frac{i\hbar}{2} (\epsilon_j a_j^{\dagger 2} - \epsilon_j^* a_j^2), \quad (1)$$

where $\delta_j = \omega_j - \omega_{pj}/2$.

Given a coherent probe, the unknown parameter θ is estimated via quadrature measurement of the output field. We define the quadratures as $q_j = (a_j + a_j^\dagger)$ and $p_j = -i(a_j - a_j^\dagger)$. In the frequency domain, the system response is characterized by the Green's function $G_\theta[\omega]$ expressed in the quadrature basis

$$G_\theta[\omega] = -(\omega \mathbf{I} - \mathbf{M})^{-1} (\mathbf{I}_N \otimes \mathbf{\Omega}), \quad (2)$$

where $\mathbf{M}$ is the Hamiltonian in the quadrature basis, $\mathbf{I}_N$ is an N -by- N identity matrix, and $\mathbf{\Omega} = (0, 0, 1, 0; 0, 0, 0, -1; -1, 0, 0, 0; 0, 1, 0, 0)$.

In absence of squeezing, the output is a coherent state with isotropic quadrature noise distribution in the (q, p) plane. Phase-insensitive amplification amplifies the signal and noise equally, leaving the SNR of the output unchanged [Fig. 1(b)]. Nevertheless, the estimate accuracy improves since the same perturbation induces a larger excursion in the output. Amplifying the perturbation-to-output transduction is the core mechanism behind sensitivity enhancement under threshold conditions [47].

On the other hand, squeezing reshapes the noise distribution anisotropically, enabling phase-sensitive amplification to surpass the SQL. Combining squeezing and the threshold condition leads to our first essential discovery, i.e., the sensitivity is prominently enhanced when a mode operates near the PO threshold, where the dissipation is balanced by the squeezing-induced amplification. It relies on the phenomenon that approaching the PO threshold generates a giant displacement of mean (μ) and amplification of noise along one quadrature (for example, q), whereas a constant mean and uncertainty are associated with the orthogonal quadrature (p) [Fig. 1(c)]. The tiny perturbation causes a rotation of the squeezing orientation, i.e., the eigenbasis, from (q, p) to (q', p') , leading to a new output state with the same coherent probe. The difference between the means of the two outputs ($\Delta\mu$) is amplified by the large displacement in the antisqueezing direction. Measuring $\Delta\mu$ via the squeezed quadratures with minimum noise yields enhanced SNR.

Precision bound of a single-mode sensor—We first analyze a single-mode bosonic sensor using the quantum noise model to elucidate the sensing enhancement at the threshold, as compared to conventional sensing schemes. There exists a smallest achievable standard deviation that quantifies the lower bound of the sensing precision, known as the Cramér-Rao bound

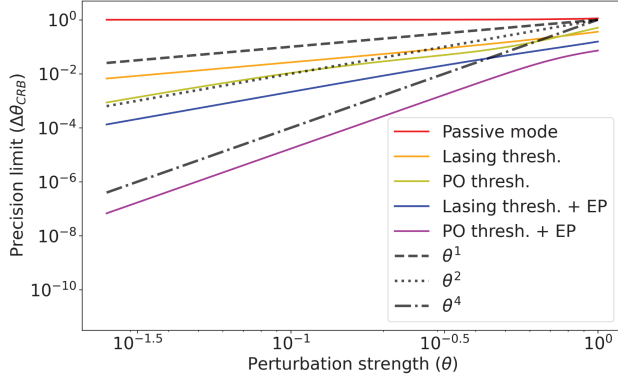


FIG. 2. Precision limit (Cramér-Rao bound $\delta\theta_{\text{CRB}}$) determined by QFI, as a function of the perturbation strength (θ) for different cases: a single passive mode, a single mode at the lasing threshold, a single mode at the PO threshold, two modes at an EP and the lasing threshold, and two modes at an EP and the PO threshold. The dashed, dotted, and dash-dotted lines represent the linear, quadratic, and quartic scaling, respectively.

$$\Delta\theta_{\text{CRB}} \geq \frac{1}{\sqrt{N_m} \sqrt{\mathcal{I}(\theta)}}, \quad (3)$$

where N_m is the number of measurement rounds, and $\mathcal{I}(\theta) = \mathcal{I}_\mu(\theta) + \mathcal{I}_V(\theta)$ is the QFI that characterizes the sensitivity of measurement, which generalizes the conventional scalar SNR. For sensing schemes measuring a Gaussian state with mean μ_θ and covariance V_θ , $\mathcal{I}_V(\theta)$ depends solely on V_θ (see Sec. II.C of Supplemental Material [50]) [47,55]. For sufficiently large μ_θ , QFI is dominated by $\mathcal{I}_\mu(\theta)$ given by

$$\mathcal{I}_\mu(\theta) = \left(\frac{d\mu_\theta}{d\theta} \right)^T V_\theta^{-1} \left(\frac{d\mu_\theta}{d\theta} \right). \quad (4)$$

For a passive single-mode sensor, the Green's function becomes θ independent for small perturbation. Consequently, the Cramér-Rao bound saturates as $\theta \rightarrow 0$ (red curve in Fig. 2). Nevertheless, one can surpass this precision limit through phase-insensitive amplification, since the diverging behavior of $G_\theta[\omega]$ boosts $d\mu/d\theta$. At the lasing threshold, where the total loss is balanced by the gain ($\gamma_0 + \gamma_c - \kappa = 0$), $G_\theta \propto \theta^{-1}$ as $\theta \rightarrow 0$. Hence QFI $\propto \theta^{-2}$ and $\delta\theta_{\text{CRB}} \propto \theta$ (orange curve in Fig. 2).

With squeezing, the Hamiltonian in the quadrature basis (denoted as M) has eigenvalues $\lambda_{1,2} = -i(\gamma/2) \pm \sqrt{\delta^2 - |\epsilon|^2}$, corresponding to an amplified and a deamplified eigenstate in the regime $|\delta| \leq |\epsilon|$, with the other pair $\lambda_{3,4} = -\lambda_{1,2}$. The PO threshold is reached when $\lambda_{1,3}$ become purely real, with the condition $\delta = \pm\sqrt{|\epsilon|^2 - (\gamma^2/4)}$. For $\delta = 0$ and weak perturbation $\theta \ll |\epsilon|$, we have

$$\lambda_{1,2} = -i\frac{\gamma}{2} \pm i|\epsilon| \left(1 - \frac{\theta^2}{2|\epsilon|^2} \right), \quad (5)$$

indicating a θ^2 scaling for $\lambda_{1,3}$ near the PO threshold ($|\epsilon| = (\gamma/2)$). Thus G_θ contains elements scaling as θ^{-2} , which ultimately yields a precision bound $\delta\theta_{\text{CRB}} \propto \theta^2$ (green curve in Fig. 2).

Precision bound of a coupled-mode sensor—We now clarify the role of EPs and their joint effects with squeezing using a coupled-mode model. In the previously-discussed cases, if one merely increases the number of bosonic modes without intermodal coupling, the Cramér-Rao bound retains the same scaling as in the single-mode case. In contrast, a coupled-mode sensor operating at an n th-order EP and the lasing threshold exhibits a θ^n scaling for the Cramér-Rao bound [47]. Yet the physical origin of the EP enhancement has not been fully clarified.

Using the quadrature-basis model, we identify the mechanism behind the EP-enhanced sensitivity near an amplification threshold. For phase-insensitive gain, the quadratures at ω and $-\omega$ are decoupled, enabling a simplified 4-by-4 Hamiltonian description. In particular, at a second-order EP and the lasing threshold, the effective Hamiltonian under perturbation is $M_\theta = \theta I_4 - I_2 \otimes M_{\text{EP}}$, where M_{EP} is a 2×2 Jordan block $(0, 1; 0, 0)$ [47]. The non-Hermitian degenerate eigenstate $(1, -i)^T$ for M_{EP} evolves with conserved energy since in each oscillator there is a destructive interference between the incoming wave via the modal coupling channel and the gain/loss through environment [Fig. 1(d)]. Under probe, the dominant order of the system response is determined by $M_\theta^{-1} = I_2 \otimes (\theta^{-1}, -\theta^{-2}; 0, \theta^{-1})$. Along the eigenstate, the response scales as θ^{-1} , similar to a single-mode sensor at the lasing threshold, yet a probe misaligned with the eigenstate produces a θ^{-2} response captured by the off-diagonal term. This higher-order scaling arises because exciting a noneigenstate breaks the gain-loss balance, causing net energy absorption from environment and amplification of the intracavity field. An example is shown in Fig. 1(e), where a constructive interference occurs for input that excites $(1, i)^T$, which is orthogonal to the eigenstate. Thus, an EP further enhances the response of a sensor at the lasing threshold. Such enhancement is absent in conventional EP sensing schemes that solely rely on perturbation-induced eigenspectral splitting, or that operate away from the lasing threshold, where the added noise scales as rapidly as the signal.

To extend EP enhancement to squeezed-mode sensors, we bring two or multiple modes with squeezing to an EP, generating a configuration where all the amplified quadratures reach a non-Hermitian degeneracy through intricate balance of their coupling, dissipation, and amplification; so do all the deamplified quadratures. To estimate the sensing precision, we first consider a two-mode scheme described by the quantum Langevin equations

$$\begin{aligned} \frac{da_{1,2}}{dt} = & \left(-i\delta_{1,2} - \frac{\gamma_{1,2}}{2} \right) a_{1,2} - i g a_{2,1} + \epsilon_{1,2} a_{1,2}^\dagger \\ & + \sqrt{\gamma_{c1,c2}} a_{\text{in}1,\text{in}2} + \sqrt{\gamma_{01,02}} b_{1,2} + \sqrt{\kappa_{1,2}} c_{1,2}^\dagger, \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{da_{1,2}^\dagger}{dt} = & \left(i\delta_{1,2} - \frac{\gamma_{1,2}}{2} \right) a_{1,2}^\dagger + i g a_{2,1}^\dagger + \epsilon_{1,2}^* a_{1,2} \\ & + \sqrt{\gamma_{c1,c2}} a_{\text{in},\text{in}_2}^\dagger + \sqrt{\gamma_{01,02}} b_{1,2}^\dagger + \sqrt{\kappa_{1,2}} c_{1,2}. \end{aligned} \quad (7)$$

Single-mode squeezing in each mode couples a_j and $a_j^\dagger$ ($j = 1, 2$), thereby mixing the quadratures at ω and $-\omega$, in contrast to the case with phase-insensitive amplification where the two sidebands are decoupled. Transforming to the frequency domain, $a_j \rightarrow a_j[\omega]$, and defining the quadratures as $\hat{q}_j[\omega] = \hat{a}_j[\omega] + \{\hat{a}_j[\omega]\}^\dagger$, $\hat{p}_j[\omega] = -i(\hat{a}_j[\omega] - \{\hat{a}_j[\omega]\}^\dagger)$, one can derive an 8-by-8 non-Hermitian Hamiltonian matrix in the quadrature basis $\{\hat{q}_1[\omega], \hat{q}_1[-\omega], \hat{p}_1[\omega], \hat{p}_1[-\omega], \hat{q}_2[\omega], \hat{q}_2[-\omega], \hat{p}_2[\omega], \hat{p}_2[-\omega]\}^T$ and the corresponding Green's function $\mathbf{G}_\theta[\omega] = -(\omega\mathbf{I} - \mathbf{M}_\theta)^{-1}(\mathbf{I}_2 \otimes \mathbf{\Omega})$. Additionally, if phase-insensitive amplification is present, additional noise is introduced from the amplification channel with the scattering described by a secondary Green's function $\mathbf{G}'_\theta[\omega] = -(\omega\mathbf{I} - \mathbf{M}_\theta)^{-1}(\mathbf{I}_2 \otimes \mathbf{\Omega}')$, where $\mathbf{\Omega}' = (0, 0, 0, 1; 0, 0, -1, 0; 0, 1, 0, 0; -1, 0, 0, 0)$.

We model the input field vector with a mean amplitude μ_{in} and a covariance matrix $\mathbf{V}_{\text{in}}$, while the noise from the dissipation and amplification channels has zero means and covariance matrices $\mathbf{V}_{\text{in}}'$ and $\mathbf{V}_{\text{in}}''$, respectively. The scattering through the Gaussian channel yields an output field with the mean and covariance matrix given as

$$\mu_{\text{out}} = (\mathbf{I} - \mathbf{K}_{\text{ex}} \mathbf{G}_\theta \mathbf{K}_{\text{ex}}) \mu_{\text{in}}, \quad (8)$$

$$\begin{aligned} \mathbf{V}_{\text{out}} = & (\mathbf{I} - \mathbf{K}_{\text{ex}} \mathbf{G}_\theta \mathbf{K}_{\text{ex}}) \mathbf{V}_{\text{in}} (\mathbf{I} - \mathbf{K}_{\text{ex}} \mathbf{G}_\theta \mathbf{K}_{\text{ex}})^T \\ & + \mathbf{K}_{\text{ex}} (\mathbf{G}_\theta \mathbf{K}_i \mathbf{V}_{\text{in}}' \mathbf{K}_i^T \mathbf{G}_\theta^T + \mathbf{G}_\theta' \mathbf{K}_g \mathbf{V}_{\text{in}}'' \mathbf{K}_g^T \mathbf{G}_\theta^T) \mathbf{K}_{\text{ex}}^T, \end{aligned} \quad (9)$$

where $\mathbf{K}_{\text{ex}} = \text{diag}(\sqrt{\gamma_{c1}}, \sqrt{\gamma_{c2}}) \otimes \mathbf{I}_4$, $\mathbf{K}_i = \text{diag}(\sqrt{\gamma_{i1}}, \sqrt{\gamma_{i2}}) \otimes \mathbf{I}_4$, and $\mathbf{K}_g = \text{diag}(\sqrt{\kappa_1}, \sqrt{\kappa_2}) \otimes \mathbf{I}_4$.

When operating the sensor at the PO threshold, one can reach a second-order EP by tailoring the parameters to engineer pairwise degenerate eigenstates (see Sec. IV.A of Supplemental Material [50] for specific conditions). There exists a $\mathbf{P}$ matrix that rotates $\mathbf{M}$ to the form $\tilde{\mathbf{M}} = \mathbf{P}^{-1} \mathbf{M} \mathbf{P} = \mathbf{M}_{\text{EP}} + \text{diag}\{a\theta^2, a\theta^2, -a\theta^2, -a\theta^2, b + a\theta^2, b + a\theta^2, -b - a\theta^2, -b - a\theta^2\}$, where a and b are constants and $\mathbf{M}_{\text{EP}} = \mathbf{I}_4 \otimes (0, 1; 0, 0)$ contains four Jordan blocks. The θ^2 scaling of the first four diagonal entries provides the sensitivity enhancement, as shown below.

For a general n th-order EP sensor at the PO threshold, $\mathbf{G}_\theta[\omega]$ can be evaluated using the Taylor expansion

$$\begin{aligned} \mathbf{G}_\theta[\omega] = & (\omega\mathbf{I} - \mathbf{P} \tilde{\mathbf{M}} \mathbf{P}^{-1})^{-1} (\mathbf{I}_n \otimes \mathbf{\Omega}) \\ \approx & \mathbf{P} \begin{pmatrix} -(-a)^{-n} \theta^{-2n} \mathbf{M}_{\text{EP},n}^{n-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -a^{-n} \theta^{-2n} \mathbf{M}_{\text{EP},n}^{n-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_n \end{pmatrix} \mathbf{P}^{-1} \\ & \times (\mathbf{I}_n \otimes \mathbf{\Omega}), \end{aligned} \quad (10)$$

where $\mathbf{F}_n = \text{diag}\{[(b + a\theta^2)\mathbf{I}_n + \mathbf{M}_{\text{EP},n}]^{-1}, [(-b - a\theta^2)\mathbf{I}_n + \mathbf{M}_{\text{EP},n}]^{-1}\}$. The higher-order series are terminated by $\mathbf{M}_{\text{EP}}^n = 0$ for the n th-order EP at $\omega = 0$. We thus obtain $\text{QFI} \sim \theta^{-4n}$ and $\delta\theta_{\text{CRB}} \sim \theta^{2n}$ for $\theta \rightarrow 0$, in contrast to the precision scaling $\delta\theta_{\text{CRB}} \sim \theta^n$ for EP sensors at the lasing threshold in the absence of squeezing [47].

This precision advantage originates from the nontrivial interplay between EPs and squeezing. If the two mechanisms acted independently, their contributions would be separable, yielding a θ^{n+2} scaling in the precision, which is analogous to sequentially passing a probe through an EP sensor and a squeezing-enhanced sensor. Instead, the θ^{2n} scaling indicates that the squeezing and EP effects are intrinsically coupled rather than additive, as confirmed by the third-order EP sensing case (see Sec. IV.B of Supplemental Material [50]).

For a second-order EP sensor at the PO threshold, we have $\delta\theta_{\text{CRB}} \sim \theta^4$ (purple curve in Fig. 2), which outperforms other conventional schemes, including EP sensors at the lasing threshold (blue curve in Fig. 2). Hence, the joint effects of squeezing and EPs provide a remarkable sensitivity advantage over conventional approaches.

Classical Fisher information for squeezing-enhanced EP sensing—The QFI-bounded precision limit can be approached through homodyne or heterodyne detection in realistic measurement. The precision lower bound is evaluated by the CFI, with the dominant term $I_\mu(\theta) = (d\mu_\theta/d\theta)^T \Sigma_\theta^{-1} (d\mu_\theta/d\theta)$ for a large input, where Σ_θ denotes the total noise covariance matrix (see Sec. V of Supplemental Material [50] for explicit forms). Figure 3 shows that heterodyne or homodyne detection yields a θ^4 precision scaling, consistent with the QFI-limited Cramér-Rao bound. Notably, the scaling is invariant to external attenuation in realistic setups such as transmission-line dissipation or detector inefficiency.

Effect of imperfect PO threshold—For EP sensors that operate slightly below the PO threshold, the scaling is cut off for small θ . Figure 3 displays the CFI and QFI with additional losses added to an EP sensor at the PO threshold. The precision limit no longer exhibits a quartic dependence on θ ; nevertheless, it still outperforms the conventional schemes if the imbalanced loss is much smaller than θ (see Sec. VI of Supplemental Material [50]).

Discussion—By developing a unified framework that integrates EPs and single-mode squeezing, we find that the precision limit scales as the $2n$ th power of the perturbation

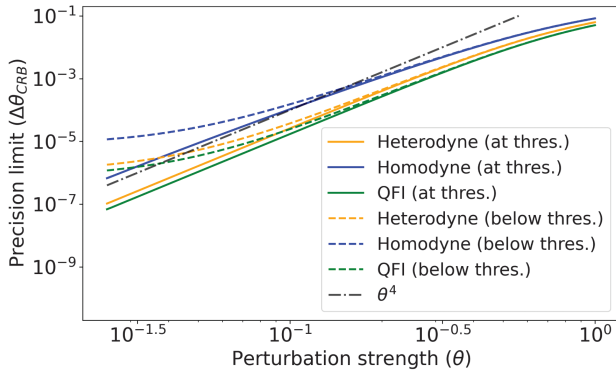


FIG. 3. Precision lower bound $\Delta\theta_{CRB}$ for an EP sensor at (solid curves) and slightly below (dashed curves) the PO threshold versus perturbation strength θ , derived from QFI, as well as from the classical quantum information (CFI) achieved by heterodyne and homodyne detection. Parameters: $g = 0.1$, $\epsilon_1 = 2i$, $\epsilon_2 = -2i$, $\gamma_{c1} = \gamma_{c2} = 1$. For the PO threshold: $\gamma_{01} = 2.8$, $\gamma_{02} = 3.2$. For below the PO threshold: $\gamma_{01} = 2.801$, $\gamma_{02} = 3.201$.

strength at the PO threshold for an n th-order EP. The results clarify how EPs enhance the fundamental detection limit of sensors combined with phase-sensitive amplification under noise constraints.

A relevant configuration operating near the PO threshold is the CQS scheme [56]. In the static CQS protocol, the system is adiabatically tuned to the critical point and measured immediately after a perturbation. In the dynamic CQS protocol, the system evolves from the vacuum state for a finite time before the measurement. Both yield QFI near a Heisenberg limit, scaling quadratically with the photon number ($\mathcal{I} \propto N^2$). In comparison, for an n th-order EP sensor at the PO threshold, QFI scales linearly with the photon number, since $G_\theta \propto \theta^{-2n}$ (see Sec. IV.C.a of Supplemental Material [50]). Yet the precision bounds exhibit a θ^{2n} scaling, with no analog in existing CQS protocols. Thus, integrating EPs and squeezing offers a superior precision for detecting ultraweak perturbations, and relies solely on a linear steady-state response to a coherent probe, without the time and procedures for state preparation or evolution.

The proposed scheme is compatible with a broad range of experimental platforms, notably nonlinear photonic devices (see Sec. VII of Supplemental Material [50] for details) and superconducting circuits. Optical squeezing is produced via parametric down-conversion [57] or degenerate four-wave mixing in microring resonators [58–60], while EPs can be realized by tuning resonance frequencies and intermodal coupling [25,32,61]. In circuit-QED systems, microwave squeezing arises from Josephson-mediated parametric amplification [62], and non-Hermitian dynamics can be engineered through pulse control, flux bias, and piezoelectric positioning [63]. Stabilization of EPs near the threshold can be facilitated by real-time feedback and automated parameter control.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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