

Flavor-Space Analog to the Aharonov-Bohm Effect for a Constant Scalar Matter Potential in Neutrino Flavor Interferometry

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The Aharonov-Bohm effect is one of the most surprising wonders of the quantum world. Its interpretation is debated between a physical significance of the potential versus nonlocality of quantum physics, ambiguity which cannot be resolved from all performed spatial interference experiments. We put forward the idea of replacing its study by flavor interferometry as observed in neutrino oscillations. The neutrino propagation through the crust of the Earth is affected by the constant scalar matter potential, with a phase shift between flavors and no force field anywhere. Here we show how to signal and experimentally disentangle this phase shift by its symmetry properties. The energy dependence of the neutrino-antineutrino asymmetry in the golden transition $\nu_\mu \rightarrow \nu_e$ allows for a clear separation of this matter component against the genuine asymmetry. Thus we find how to observe in a single experiment, like DUNE, the physical significance of the potential, the neutrino mass hierarchy, and the genuine matter-antimatter asymmetry in the lepton sector.

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The Ehrenberg-Siday-Aharonov-Bohm effect [1,2], in which a charged particle is affected by an electromagnetic potential ($V, \mathbf{A}$) in a region with neither a magnetic field $\mathbf{B}$ nor an electric field $\mathbf{E}$, is one of the Seven Wonders of the Quantum World [3]. The underlying mechanism is in the phase of the wave function, so the Aharonov-Bohm effect can be observed by interference experiments. The most commonly described case, the Aharonov-Bohm solenoid effect, takes place when the wave function of a charged particle passing around a long solenoid experiences a phase shift as a result of the enclosed magnetic field. This is so despite the magnetic field being absent in the region through which the particle travels. This phase shift has been observed [4,5]. An electric Aharonov-Bohm phenomenon, in which a charged particle is affected by regions with different scalar potentials and zero electric field, was also predicted [6], but this has no experimental confirmation

yet. Another experiment involving a ring geometry interrupted by tunnel barriers, with a bias voltage V relating the potentials of the two halves of the ring, leads to an Aharonov-Bohm phase shift, and it has been observed [7]. Recently the split of a cloud of cold atoms into two atomic wave packets, one of them subjected to gravitational interaction with a large mass, has been reported with an observed phase shift due to a gravitational Aharonov-Bohm effect [8].

The prevailing interpretation of the effect is the physical significance of the potentials in quantum mechanics governing the phase of the probability amplitudes given by the action. However, in the solenoid effect, for example, the result can be written in terms of the enclosed magnetic field, leading to a school of thought giving the weight of the argument to nonlocality of quantum mechanics. The debate is provoked by the formalism itself via the Stoke's theorem because the phase shift—the induced relative phase between the two interfering amplitudes—is given by

$$\frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{q}{\hbar} \Phi_B, \quad (1)$$

where q is the particle charge and Φ_B is the enclosed magnetic flux. The alternative is thus centered between a local effect of the potential—the left side of Eq. (1)—against a nonlocal effect of the force field—the right side of Eq. (1).

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Interesting phenomena also exhibiting quantum interference effects can be found in particle physics. Gell-Mann and Pais [9] were the first to point out that the weak interaction induced $K^0 - \bar{K}^0$ transition implies the need to consider the K -mesons as superposition of neutral particles having a different combined parity. Soon afterward, Pontecorvo raised the question of the possible existence of other “mixed” neutral particles besides the K -mesons [10–12]. Eventually, it was realized that the assumption of neutrino flavor nonconservation would lead to a decrease in the number of detectable solar neutrinos at earth due to the $\nu_e - \nu_\mu$ oscillations similar to the K -meson system [13], thus potentially providing an explanation for early experimental indications of a solar neutrino deficit [14]. However, it was not until a proper understanding of matter effects on neutrino propagation was formulated in Refs. [15,16] and a direct search for matter effects in the Earth [17] resulted in a 2.7σ indication that the electron flavor content of solar neutrinos during nighttime, where Earth matter effects are present, exceeds that of daytime, that all these led to neutrino oscillations cementing as the resolution of the solar neutrino problem. The Mikheyev-Smirnov-Wolfenstein effect is, in this context, often compared to an index of refraction acting on electron-neutrinos by the presence of their interaction with the ambient matter. In what follows we will simplify the discussion by considering the coherent forward elastic scattering of neutrinos in matter as the sole source of matter effects on the neutrinos, neglecting inelastic scattering effects.

Nowadays, long baseline neutrino experiments (LBNE) are on the verge to measure fundamental properties of the neutrinos such as the CP phase and the mass ordering. In this Letter we wish to underscore certain interesting aspects—some of them unnoticed—related to the quantum nature of these particle oscillations. First of all and as is well known, the propagation of neutrinos takes place as a superposition of eigenstates of definite mass—those in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [10,11,18]. In an experiment measuring a neutrino beam, the detector determines the flavor state of a given incident neutrino by—in a manner of speaking—collapsing the sum of amplitudes in the superposition to the projected flavor detection. We recall that in quantum physics the interference is between probability amplitudes for different untagged alternatives, no matter whether the observables refer to either spatial or internal properties like the neutrino flavor charges. The flavor mixing and nondegenerate neutrino masses of neutrino oscillations are thus the ingredients of flavor interferometry. An analogy between spatial interference and two-family neutrino oscillation in vacuum has been invoked regularly in the literature; see, for example, Refs. [19–22] for illustration. Here we would like to add more to this concept. Second, LBNE with neutrino propagation in the crust of the Earth can be considered to occur, to a very good approximation, in a medium of

constant matter density. While the Earth crust is not strictly such a medium, it has been shown in Ref. [23] that effects on the oscillation probabilities by small departures from a constant matter density are not detectable in experiments such as DUNE. Henceforth we shall assume propagation in a constant matter potential. Third and last, consider, for example, a disappearing experiment with the mentioned conditions and with a muon neutrino source. During propagation and depending on the evolving masses content of their superposition, the neutrinos will trigger with a certain probability the interaction with the matter potential. This probability being of course that of the oscillation to an electron neutrino flavor state weighted by the coherent elastic forward scattering cross section per area of the beam. At the detector, a small fraction of the total initial neutrinos from the source will “disappear” into electron neutrinos. The point is that, for our purposes, there are only two possible outcomes for each of the propagating neutrinos: either the interaction with the matter potential is triggered or it is not. In this respect, we can state that there are two different types of effective “paths” in neutrino flavor state space and in only one of these are the neutrinos subjected to the interaction with a nonzero potential. These paths—or arms of the flavor interferometer—are identifiable by the presence or lack of a quantum phase shift whenever the neutrinos traverse them. As it turns out, the relative phases in the coherent neutrino propagation induced by the matter effects are given in terms of the vacuum parameters plus a potential phase shift, which surprisingly is measurable in the same way that the CP neutrino phase is.

Despite the caveat that surges from neglecting inelastic scattering effects on the neutrinos interacting with the matter particles, the above key aspects bear a strong resemblance with corresponding essential features of the electric variation of the Aharonov-Bohm effect mentioned before. The claim is that it is in this sense that what we have just described constitutes a new, neutrino flavor space analog of the Aharonov-Bohm effect. Rephrasing our claim, an appropriate interpretation [24,25] of the matter interaction in neutrino propagation, with a potential difference in flavors, leads to an analog effect and a potential phase shift in flavor interferometry.

The ideal aim is to identify experimentally the relative phase shift induced by the matter potential. One has to disentangle it from the intrinsic phase difference associated with neutrino oscillations in vacuum due to the oscillation frequencies corresponding to different neutrino masses. This will be done here by appropriate observables exploiting the symmetry properties of the potential. Is it possible to separate the effect of the phase shift due to the matter potential in LBNE? There are indeed observables which are characteristic of the matter potential, not accessible by interference effects in vacuum: the CPT -odd component of discrete asymmetries in neutrino flavor oscillations [26,27].

Moreover, as we shall show next, our full analysis also allow us to provide a very elegant and original recipe to determine the neutrino mass hierarchy, and this constitutes one of the salient points of our results. Nevertheless, as the title suggests, we have chosen to highlight the connection between neutrino interferometry and the Aharonov-Bohm effect since it is our belief that such a new point of view might bring further clarity or at the very least shed some light onto the enigma of the flavor problem in elementary particle physics [28–30].

Symmetries—At present all parameters describing neutrino oscillations are well established, except the genuine CPV phase δ in the unitary PMNS mixing matrix. Mixing angles and neutrino mass differences are known [31] up to the sign of the neutrino mass hierarchy. In the basis of the three neutrino flavors, the Hamiltonian matrix is

$$H = \frac{1}{2E} \left\{ U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} = \frac{1}{2E} \tilde{U} \tilde{M}^2 \tilde{U}^\dagger, \quad (2)$$

where $a = 2EV$, with E being the neutrino energy and V being the interaction potential having a different value for the electron-neutrino flavor. The final side of Eq. (2) is written in terms of the effective values in matter of the mixing matrix $\tilde{U}$ and masses $\tilde{M}^2$. In terms of them, neutrino oscillations in matter have the same flavor interference properties as those in vacuum, as emphasized above. The connection of $\tilde{U}$ and $\tilde{M}^2$ to the vacuum U and M^2 plus the potential V shows that the arms in the coherent neutrino propagation in matter are affected in a different way by V , leading to additional different phases between the arms. As a consequence, the transition probabilities are written in terms of the vacuum parameters plus a potential phase shift Δ_p . This demonstrates that the conceptual analogy with the Aharonov-Bohm effect is also apparent in the formalism. For antineutrinos, a changes sign and U changes to its conjugate. Because of the energy mismatch between the two terms of the Hamiltonian H , all effective values in matter become energy dependent. The effective values of neutrino masses in matter and those of the rephasing-invariant mixing in matter can be connected with the parameters in vacuum plus the matter potential present in the Hamiltonian of Eq. (2). Actual experiments, like DUNE [32] or Hyper-Kamiokande [33], cover neutrino energies in a region such that a hierarchy exists between a and the neutrino mass differences

$$\Delta m_{21}^2 < |a| < |\Delta m_{31}^2|. \quad (3)$$

The corresponding hierarchy for the induced phases $\Delta_{21} = (\Delta m_{21}^2 L / 4E)$, $\Delta_p = (aL / 4E)$, and $\Delta \equiv \Delta_{31} = (\Delta m_{31}^2 L / 4E)$ suggests a perturbative expansion of the

TABLE I. Symmetry behavior of the different terms in the expansion of transition probabilities.

	<i>CPT</i> -even	<i>CPT</i> -odd
<i>CP</i> -even	1, $\Delta_{21} \times \cos(\delta)$, Δ_p^2	$\Delta_p \times \Delta_{21} \times \sin(\delta)$
<i>CP</i> -odd	$\Delta_{21} \times \sin(\delta)$	Δ_p , $\Delta_p \times \Delta_{21} \times \cos(\delta)$

different transition probabilities with respect to the contribution generated by the Δ phase. Relative to it, we keep linear terms in Δ_{21} , needed for bringing the CPV phase δ of the mixing matrix into operation, accompanied by the potential phase Δ_p up to second order, i.e., Δ_p , Δ_{21} , Δ_p^2 , and $\Delta_p \times \Delta_{21}$. This expansion, neglecting Δ_p^3 , $\Delta_p^2 \times \Delta_{21}$ and higher order terms, allows for a clear understanding of their contributions to the interesting asymmetries to be selected. Once constructed, the analytical perturbative result will be compared with numerical exact results to prove their good agreement. Taking into account that the potential in matter is odd under *CP* and *CPT*, changing sign for neutrinos and antineutrinos, we show in Table I the definite symmetry properties of these terms with T-even behavior in the direct diagonal terms and T-odd behavior in the inverse diagonal terms. The reference term 1 in Table I corresponds to the contribution with the Δ phase in neutrino oscillations. Each of the terms, in the considered neutrino flavor transition, will be accompanied by the corresponding rephasing invariant mixing and a function $f(\Delta)$. As the T-even asymmetry has to be even in the baseline L whereas the T-odd asymmetry has to be odd in the baseline L , a corollary of Table I tells us the parity of $f(\Delta)$ under the change of sign of Δ : an odd function $f(\Delta)$ has a precious interest in order to fix the pending question of the hierarchy in the neutrino masses. We observe that the first order term, the phase Δ_p induced by the matter potential, contains an odd function of Δ , so its sign fixes the neutrino mass hierarchy. This conclusion reinforces the interest of the present study to find observables able to signal the term with the phase Δ_p of the matter potential. Notice that, due to the symmetry behavior, there is no Δ_p^2 correction term in the asymmetries, thus an excellent agreement with the exact numerical result is expected.

A priori, we may consider three possible observables with a μ -neutrino beam as reference. We give detailed expected results for a baseline of $L = 1300$ km, covering energies along the first and second oscillation peaks.

CPT-odd $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ —This choice has the beauty that a nonvanishing observation of the asymmetry is, by itself, a necessary and sufficient condition to demonstrate the effect of the Δ_p phase. Its experimental search would need a neutrino factory facility with a charge discrimination detector. We discuss the expected results for this asymmetry, linearly proportional to the potential, as well as its separation into the independent and dependent terms of the CPV δ phase in the mixing matrix.

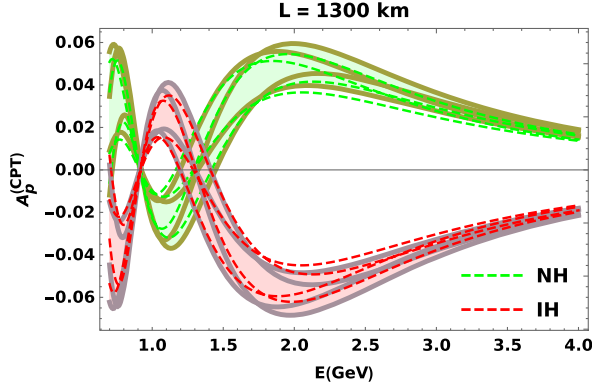


FIG. 1. $A_p^{(CPT)}(E)$ asymmetry for normal (green) and inverted (red) hierarchies. Solid (dashed) lines portray the approximate (numerical exact) equations. Colored bands depict the whole range of values of δ .

We calculate this asymmetry expanding cross-checking results [25,34,35] for the relevant transitions. We obtain

$$A_p^{(CPT)}(E) = A_1(E) + C_1(E) \cos(\delta) + S_1(E) \sin(\delta), \quad (4)$$

with the first two terms being T-even and CP -odd and the last term being CP -even and T-odd. The functions $A_1(E)$, $C_1(E)$, and $S_1(E)$ are given in the End Matter, showing the dominance of the $A_1(E)$ term. The resulting $A_p^{(CPT)}(E)$ is plotted in Fig. 1. This CPT -asymmetry, proportional to the phase shift Δ_p induced by the potential, has a peculiar energy dependence not given by the canonical oscillation peaks, and it leads to a magic energy [36] around the solution of $\tan(\Delta) = \Delta$, where the asymmetry vanishes independent of δ . For $L = 1300$ km the magic energy is 0.92 GeV, near the second oscillation peak of the transition probabilities. Needless to say, the proportionality of this asymmetry to Δ_p and the preeminence of the A_1 term make it ideal for the aim of separating out the effect of the potential. However, there is no facility at present able to measure it.

CPT - and CP -odd $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ —Again this alternative asymmetry, if observed, would represent a clear-cut confirmation of the effect of the potential in neutrino flavor interferometry. Although the effect of the potential in this disappearance experiment is expected to be small, we quantify here this statement, in view of being an indisputable effect of the potential for an observable in the DUNE experiment. We proceed to the discussed perturbative expansion of the survival probabilities [25,34,35], leading to an asymmetry generated by the matter potential:

$$A_p^{(CPT,CP)}(E) = A_2(E) + C_2(E) \cos(\delta), \quad (5)$$

with both the $A_2(E)$ and $C_2(E)$ terms being T-even. At the required perturbative order they are given in the End Matter. The result explains a small effect of $A_2(E)$ being due to a (2, 3) mixing around the maximum. An additional

consequence is that the dominance by the $C_2(E)$ term, which is an even function of Δ , implies that this observable is not sensitive to the neutrino mass hierarchy, in spite of its proportionality to the potential phase shift. Its actual value is fundamentally affected by the CPV phase δ of the mixing matrix. As expected, the same magic energy appears here. Although this asymmetry in the disappearance experiment is proportional to Δ_p , the small value of the $A_2(E)$ term in the entire energy region advises against its use for the purpose of this Letter.

CP -odd $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ —These probabilities are those of the golden transitions, available in the current experiments on neutrino appearance oscillations. Contrary to the previous asymmetries, this entire CP -odd asymmetry is not proportional to the phase shift Δ_p induced by the matter potential. It is in addition induced by a genuine CP -violation in free neutrino oscillations, the main focus in DUNE and HK experiments. In spite of this combined effect at a given energy E (and fixed baseline L), recent studies [26,27] have demonstrated a disentanglement theorem for the two components by means of their different behavior under the other discrete symmetries. The result for $A^{(CP)}(E) = A_p^{(CP,CPT)}(E) + A_g^{(CP,T)}(E)$ is a well defined unique separation with $A_p^{(CP,CPT)}(E)$ being a CPT -odd and T-even component, whereas $A_g^{(CP,T)}(E)$ is a T-odd and CPT -even component. The separation is not only a theoretical construct, it is experimentally accessible because—even for a fixed baseline L —the two components show different energy dependence. We exploit these findings for giving the analytic perturbative results for the two components,

$$A_p^{(CP,CPT)}(E) = A_3(E) + C_3(E) \cos(\delta), \quad (6)$$

$$A_g^{(CP,T)}(E) = S_3(E) \sin(\delta), \quad (7)$$

with the two CPT -odd and T-even terms,

$$A_3(E) = 16\Delta_p I [\sin(\Delta)/\Delta - \cos(\Delta)] \sin(\Delta), \quad (8)$$

$$C_3(E) = 16\Delta_p \Delta_{21} J_r [\sin(\Delta)/\Delta - \cos(\Delta)] \cos(\Delta), \quad (9)$$

and the result for $S_3(E)$ is identical, up to the required order, to the CPV-asymmetry for oscillation in vacuum:

$$S_3(E) = -16\Delta_{21} J_r \sin^2(\Delta). \quad (10)$$

The rephasing-invariant I and J_r couplings are given in the End Matter. In the left panel of Fig. 2 we give the separate results for $A_p^{(CP,CPT)}(E)$ for normal (green) and inverted (red) hierarchies, with the bands covering all values of δ , as well as $A_g^{(CP,T)}(E)$, with the blue band covering all values of δ . They correspond to a baseline $L = 1300$ km of the DUNE experiment, showing larger effects of the matter

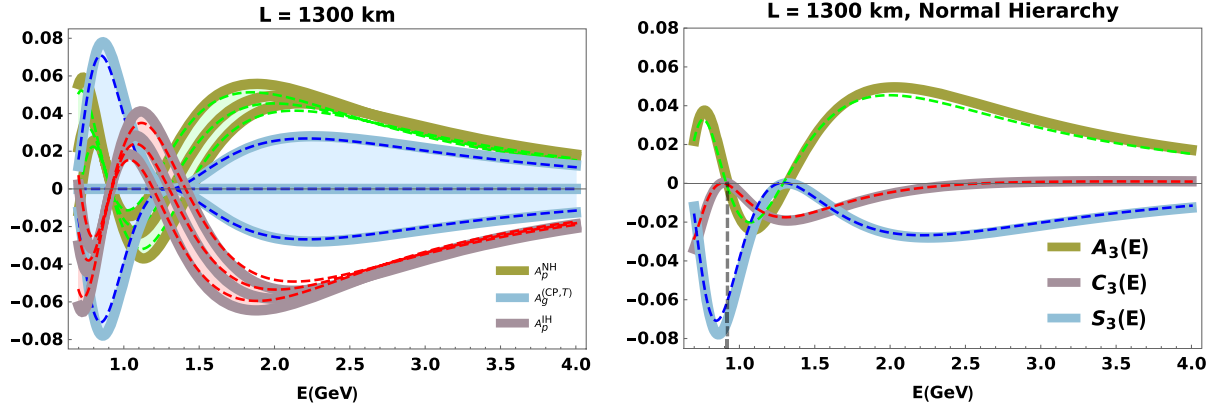


FIG. 2. Left panel: Two components $A_p^{(CP,CPT)}(E)$ for normal (green) and inverted (red) hierarchies, as well as $A_g^{(CP,T)}(E)$ (blue), of the CPV-asymmetry. Right panel: Separate $A_3(E)$, $C_3(E)$, and $S_3(E)$ terms. The black vertical dashed line depicts the location of the magic energy. Solid (dashed) lines portray the approximate (numerical exact) equations. Colored bands depict the whole range of values of δ .

potential. The dominance of $A_3(E)$ over the other components for energies above the first oscillation node is demonstrated in the right panel of Fig. 2 with a separate plot of $A_3(E)$, $C_3(E)$, and $S_3(E)$.

We conclude that the dominance of $A_p^{(CP,CPT)}(E)$ over $A_g^{(CP,T)}(E)$ around the first oscillation peak, with the magnitude of A_3 much higher than C_3 , implies that the sign of the observed total CPV-asymmetry $A^{(CP)}$ dictates the neutrino mass hierarchy independent of δ . This conclusion holds for the expected value of the potential phase shift Δ_p present in $A_p^{(CP,CPT)}$ and absent in $A_g^{(CP,T)}$. Staying at energies around the first oscillation peak only, this conclusion is weakened without assuming the value of Δ_p : the inequalities $[A^{(CP)} - |S_3|] < A_p < [A^{(CP)} + |S_3|]$ keep the conclusion that the sign of $A^{(CP)}$ fixes the hierarchy if $|A^{(CP)}| > |S_3|$.

For the ideal aim of this Letter, an explicit measurement of the magnitude of the phase shift Δ_p , needing the experimental separation of A_p and A_g , can be made in the same experiment by energy dependence moving to energies around the second oscillation peak. The different energy dependence of the two components leads to the magic energy [26,27] where the component $A_p^{(CP,CPT)}$ due to the potential vanishes for all mixings and δ values. The observed total CPV-asymmetry $A^{(CP)}$ around the magic energy at $E = 0.92$ GeV measures $A_g^{(CP,T)}$ and thus δ . The magic energy in the CPV-asymmetry portrayed here is the same as that discussed for the other asymmetries. The zero of the magic energy is also there with the exact numerical calculation of the CPT -odd component of the asymmetry, not only to first order in Δ_p . The broken line in Fig. 2 demonstrates the excellence of the analytical approximation. The magic energy is 0.92 GeV in the approximation and 0.915 GeV in the numerical calculation. The experimental capability of separating the two components is illustrated for an energy bin width up to the feasible [37,38]

0.15 GeV: the mean value of A_p is below 2% of the mean value of A_g and this one is 85% of its peak value.

To summarize, this research demonstrates that a test of the flavor-space Aharonov-Bohm effect for a constant scalar potential can be performed with the appropriate interpretation of the CPV asymmetry in the golden transition for the DUNE experiment on neutrino oscillations. The phase shift due to the potential appears from the different constant potential of the e-flavor neutrinos with respect to the other flavors, not from a phase difference in space. The observable effect comes from flavor interferometry, not from spatial interference. Its measurement would prove without any loopholes the local effect of the potential in neutrino flavor interferometry. In this scenario, the neutrino potential is a physical property as it is the relative phase of flavor probability amplitudes.

We have discussed how to signal and identify the phase shift due to the potential by means of the CPT -odd component of the discrete asymmetries. The best case in existing experimental facilities corresponds to this component in the measurement of the CPV asymmetry in the $\nu_\mu \rightarrow \nu_e$ transition at the baseline of the DUNE experiment. The different energy dependence of the disentangled CPT -odd and CPT -even components of the CPV-asymmetry, by the comparison of the measurements in the first and second oscillation peaks, allows for obtaining in a single experiment an impressive observation of three fundamental open questions in physics: CP violation in the lepton sector, the physical significance of the potential in flavor interferometry, and the neutrino mass hierarchy.

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Data availability—The data that support the findings of this article are openly available [39].

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End Matter

$A_p^{(CPT)}(E)$ and $A_p^{(CPT,CP)}(E)$ asymmetries—The observable $A_p^{(CPT)}(E)$ asymmetry, plotted in Fig. 1, is written in terms of the three functions, $A_1(E)$, $C_1(E)$, and $S_1(E)$ in Eq. (4) of the text. They are calculated in the perturbative expansion discussed after Eq. (3), with the following result:

$$A_1(E) = 16\Delta_p I [\sin(\Delta)/\Delta - \cos(\Delta)] \sin(\Delta), \quad (\text{A1})$$

$$C_1(E) = 16\Delta_p \Delta_{21} J_r [\sin(\Delta)/\Delta - \cos(\Delta)] \cos(\Delta), \quad (\text{A2})$$

$$S_1(E) = -16\Delta_p \Delta_{21} J_r [\sin(\Delta)/\Delta - \cos(\Delta)] \sin(\Delta), \quad (\text{A3})$$

where $I = c_{13}^2 s_{13}^2 s_{23}^2$ and $J_r = c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23}$ are rephasing invariant mixings and we use the standard short notation $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, with θ_{ij} the neutrino mixing angles. Also $\Delta_{21} = (\Delta m_{21}^2 L/4E)$, $\Delta_p = (aL/4E)$, and $\Delta \equiv \Delta_{31} = (\Delta m_{31}^2 L/4E)$. We remind the reader that, in agreement with these equations, $A_1(E)$ had to be Δ -odd, $C_1(E)$ Δ -even, and $S_1(E)$ Δ -odd. We point out the preeminence of the most

interesting $A_1(E)$ term proportional to the potential phase shift Δ_p .

The observable $A_p^{(CPT,CP)}(E)$ asymmetry is written in Eq. (5) of the text in terms of the two functions, $A_2(E)$ and $C_2(E)$. They are calculated in the discussed perturbative expansion, with the following result:

$$A_2(E) = 16\Delta_p I \cos(2\theta_{23}) [\sin(\Delta)/\Delta - \cos(\Delta)] \sin(\Delta) \quad (\text{A4})$$

$$C_2(E) = 16\Delta_p J_r \Delta_{21} [\cos(2\theta_{23}) \sin^2(\Delta) + [\cos^2(\Delta) - \sin^2(\Delta)/\Delta^2] + 2s_{23}^2 [\sin(\Delta)/\Delta - \cos(\Delta)] \sin(\Delta)/\Delta]. \quad (\text{A5})$$

We remind the reader that $A_2(E)$, in agreement with these equations, had to be Δ -odd whereas $C_2(E)$ was Δ -even. The small values of the most interesting $A_2(E)$ term have to be noticed. The reason is seen in Eq. (A4) for a (2, 3) mixing near its maximum. More information on the data generated for this research can be found in Ref. [39].