

Universal Transport at Lifshitz Metal-Insulator Transitions in Two Dimensions

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We study the charge transport across a band-tuned metal-insulator transition in two dimensions. For high temperatures T and chemical potentials μ far from the transition point, conduction is ballistic and the resistance $R(T)$ verifies a simple one-parameter scaling relation. Here, we explore the limits of this semiclassical behavior and study the quantum regime beyond, where scaling breaks down. We analytically evaluate the simplest Feynman diagram of the linear-response conductivity $\sigma = 1/R$ of a parabolic band endowed with a finite lifetime. Our formula shows excellent agreement for experiments for a field-tuned $\text{MoTe}_2/\text{WSe}_2$ moiré bilayer, and can capture the quantum effects responsible for breaking the one-parameter scaling. We go on to discuss a fascinating prediction of our model: The resistance at the quantum-critical band-tuned Lifshitz point ($\mu = T = 0$) has the *universal value*, $R_L = (2\pi h)/e^2$, per degree of freedom, in congruence with experiment. Furthermore, we investigate whether two-dimensional metal-insulator transitions driven by strong electron correlations or disorder can also be classified by their quantum-critical resistance and come up with an, in principle, complete assignment of the transition mechanism.

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Introduction—Metal-insulator transitions (MIT) in two-dimensional materials have been studied intensely (and controversially) for decades [1–4]. The consensus is that the phenomenon involves a formidable interplay of disorder and electronic correlations. Hence, when a change in carrier density triggers a transition, pinpointing its microscopic driver—strong electron-electron interactions leading to Mott-Wigner localization [5–8], strong disorder causing Anderson localization [9,10], or a more conventional bandlike Lifshitz transition [11–14]—remains far from trivial. An observable directly probing the metal-insulator transition is the resistance and much can be learned from its temperature profile $R(T)$ [15]. Approximating charge carriers as ballistic, a semiclassical treatment of conduction [16] predicts that $R(T)$ verifies a simple one-parameter scaling law [1,4]: $R(T) = R_c(T) \times \mathcal{F}_{\text{sgn}(\mu)}(T_0(\mu)/T)$. Here, $R_c(T)$ is the critical resistance profile at the transition that we shall identify with the origin of the chemical potential, $\mu = 0$. Then, when scaling temperature T with the μ -dependent parameter $T_0(\mu)$, the ratio $R(T)/R_c(T)$ will collapse onto one of two universal curves, $\mathcal{F}_{\text{sgn}(\mu)}$, distinguishing whether the system is a metal ($\mu > 0$: $\mathcal{F}_+$) or an insulator ($\mu < 0$: $\mathcal{F}_-$). This means that, while

conduction in the metal and in the insulator will be different ($\mathcal{F}_+ \neq \mathcal{F}_-$), both regimes are controlled by a *single*, common energy scale, $T_0(\mu)$. In traditional 2D systems displaying an MIT—foremost metal-oxide-semiconductor field-effect transistors [17,18] (MOSFET) and 2D heterostructure quantum-well (QW) devices [19–23]—deviations from this scaling behavior are experimentally nearly absent.

However, with the emergence of intrinsically two-dimensional systems, such as transition-metal dichalcogenides (TMD), MITs can now be perused with unprecedented precision by applying external electric fields [24–26]. In Fig. 1 we display a key experiment [27,28]: Adding two holes to a $\text{MoTe}_2/\text{WSe}_2$ moiré bilayer via gating depletes its first valence band (filling factor $f = 2$). Then, the bandwidth of the first and second moiré valence band is tuned with an external electrical displacement, that manipulates the gap or overlap between the bands, leading to a band insulator to metal transition of the Lifshitz type. The resistances across that transition (colored dots in Fig. 1) verify the above *one*-parameter scaling (dashed lines) in a regime limited to high temperatures and sufficiently deep into the metal phase. For $T/T_0 \lesssim 2$, however, the resistances no longer collapse onto the semiclassical prediction and an ill-understood nonuniversal behavior emerges—suggesting that *multiple* energy scales are at play.

Here, we develop an analytical theory that describes conduction across and arbitrarily close to the MIT. The theory's results (red solid lines in Fig. 1) encompass the semiclassical high- T limit, while also providing an excellent description of the experimental near-critical

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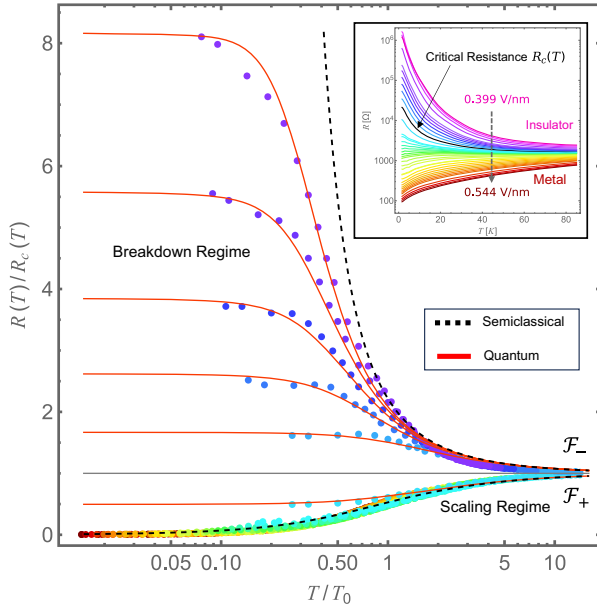


FIG. 1. Metal-insulator transition in MoTe₂/WSe₂ moiré bilayer. Tuning an electric displacement field D , the bilayer near full-filling ($f = 2$) undergoes an MIT. The inset shows the experimental data of Li *et al.* [27] from Ref. [28]. Dividing resistances $R(T)$ by the critical line $R_c(T)$ (black curve in inset, $D = 0.437$ V/nm) and scaling temperature with a field-dependent T_0 (see Sec. V in Ref. [29]) yields the main panel data points from $D = 0.544$ V/nm (red dots) to $D = 0.427$ V/nm (purple dots). At high T , resistances collapse onto a metallic $\mathcal{F}_+$ or insulating $\mathcal{F}_-$ branch (dashed lines), verifying one-parameter scaling. Near the quantum critical point, $R_c(T = 0)$, the experiment strongly deviates from this semiclassical prediction, while agreeing excellently with our quantum theory (red solid lines) for all temperatures.

conduction in the quantum regime beyond. Our analysis reveals (i) at low temperatures, the electronic scattering rate Γ turns into a relevant energy scale, invalidating one-parameter scaling; (ii) strikingly, we find the transport of any band-tuned system at the Lifshitz point, $R(\mu = 0, T \rightarrow 0) = R_L = (2\pi\hbar)/e^2$ to be *universal*, in congruence with the bilayer experiment; and (iii) we discuss more broadly how the quantum critical resistance in 2D MITs may be related to the underlying transition mechanism. We motivate that R_L provides a sufficient but not necessary condition for identifying transitions driven predominately by interactions.

Modeling the phase transition—We model the metal-insulator transition in a 2D system by a single parabolic band, $\epsilon_{\mathbf{k}} = (\hbar^2 k^2/2m)$, with the reduced Planck constant $\hbar$, the band-mass m , and Brillouin-zone momentum $\mathbf{k}$ (see Sec. IV in Ref. [29] for the case of nonparabolic bands). This dispersion is realizable in MOSFETs and quantum-well devices and approximates well the transport-dominating band-edges in van-der-Waals semiconductors, such as TMDs. By adjusting the chemical potential, μ , the system

can be tuned from a metal ($\mu > 0$) to a band insulator ($\mu < 0$). Moving through the critical value, $\mu = 0$, where the chemical potential is touching the band edge, the system undergoes a Lifshitz transition as the Fermi surface disappears [11–14]. To allow for quantitative comparisons with specific materials, we multiply the conductance with an integer N_d to account for the electron’s spin, the presence of multiple bands, and the possibility of multiple valleys in the Brillouin zone. We model electronic scattering by endowing the dispersion with a finite lifetime τ , which can depend on temperature and, in principle, on the chemical potential. The scattering rate $\Gamma = \hbar/(2\tau)$ effectively mimics effects of disorder and electronic correlations. However, we make no attempt to describe its precise microscopic origin. Deliberately assuming a static scattering rate further precludes capturing metal-insulator transitions of the Mott-Wigner type.

For this electronic structure, we derive an exact expression for the leading-order longitudinal linear-response electrical conductance σ , per spin, band, and valley degree of freedom. The corresponding Feynman diagram (that neglects vertex corrections) yields [43,44]

$$\sigma(T) = \frac{he^2}{2V} \sum_{\mathbf{k}} \int_{-\infty}^{\infty} d\omega \left(-\frac{\partial f}{\partial \omega} \right) [A_{\mathbf{k}}(\omega) v_{\mathbf{k}}^x]^2 \quad (1)$$

with the electron’s charge e , Planck’s constant h , the unit-cell volume V , the Fermi function $f(\omega)$, and the Fermi velocity $v_{\mathbf{k}}^x = \hbar^{-1} \partial/\partial k_x \epsilon_{\mathbf{k}}$ in the x direction. For the static scattering rate considered, the spectral function has a Lorentzian line shape: $A_{\mathbf{k}}(\omega) = -1/\pi \text{Im}(\omega + \mu - \epsilon_{\mathbf{k}} - i\Gamma)^{-1}$. Then, the ω integral in Eq. (1) can be evaluated analytically using contour-integration [44–47]. In the current case, the momentum summation can also be performed explicitly, leading to our central result

$$\sigma(\mu, T) = \frac{e^2}{2\pi h} \left\{ \frac{\pi k_B T}{\Gamma} \ln 2\pi + \frac{\pi \mu}{2\Gamma} + \text{Re} \Psi_0 \left(\frac{1}{2} + \frac{1}{2\pi k_B T} (\Gamma - i\mu) \right) - \frac{2\pi k_B T}{\Gamma} \text{Re} \ln \Gamma \left(\frac{1}{2} + \frac{1}{2\pi k_B T} (\Gamma - i\mu) \right) \right\}, \quad (2)$$

where $\ln \Gamma(z)$ is the natural logarithm of the Gamma function, $\Psi_0(z) = d/dz \ln \Gamma(z)$ is the digamma function [48], and is k_B Boltzmann’s constant. This exact expression for the conductance without vertex-corrections makes no assumption on the magnitude of scattering: it is valid for transport in the ballistic and the diffusive limit, as well as in between. The effective mass of the band does not occur explicitly in our expression, which allows for a general application to any 2D parabolic band system.

For context, we revisit the ballistic, semiclassical limit, previously derived by Fratini *et al.* [16]. Considering T/Γ

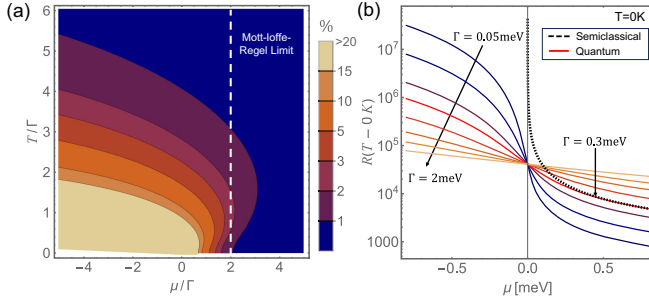


FIG. 2. Quantifying transport in the quantum regime. (a) Relative deviation of the quantum resistance $R(\mu, T)$ from the semiclassical limit $|1 - R^{sc}(\mu, T)/R(\mu, T)|$ for a constant scattering rate Γ . Approaching the Lifshitz transition at low temperatures from the metallic side, semiclassical behavior breaks down, broadly consistent with the Mott-Ioffe-Regel criterion (dashed line). The semiclassical approximation is accurate deep in the metal, but, also, for arbitrary chemical potential, when the temperature is high enough. (b) At $T = 0$ the semiclassical resistance diverges when reaching the insulator at $\mu = 0$ (dashed); in our quantum theory (colored lines) lifetime effects assure it is finite for all values of μ . Note the isosbestic point at $\mu = 0$.

and $|\mu|/\Gamma \gg 1$, Eq. (2) reduces to (see Sec. III in Ref. [29])

$$\sigma^{sc}(\mu, T) = \frac{e^2}{2h} \left(\frac{k_B T}{\Gamma} \log \left[1 + e^{\frac{\mu}{k_B T}} \right] \right). \quad (3)$$

Manifestly, the approximate resistance $R^{sc} = 1/\sigma^{sc}$ now obeys the scaling relation $R^{sc}(\mu, T) = R^{sc}(0, T) \times \mathcal{F}_{\text{sgn}(\mu)}(\mu/T)$ with $\mathcal{F}_{\text{sgn}(x)}(x) = 1/\log_2[1 + e^{\text{sgn}(x)/x}]$.

Metal-insulator transition in a moiré bilayer—To validate our theory, we model the experimental data of the field-tuned MIT in the $\text{MoTe}_2/\text{WSe}_2$ bilayer near full-filling ($f = 2$) [27] displayed in Fig. 1 (colored dots) with Eq. (2) (red lines). Following Fratini *et al.* [16], we use the ansatz, $\Gamma(T) = \Gamma_0 + bT$. This scattering rate reproduces the experimental low- T behavior: a critical resistance with negative slope (required for the scaling collapse) and a linear-in- T resistance in the metallic phase (see Fig. 2 and Sec. II.2 in Ref. [29]). We make no attempt at identifying the microscopic origin of the scattering, which could derive from conventional electron-phonon effects or exotic “Planckian” scattering, observed in several strongly correlated systems [49]. Using $\Gamma_0 = 0.3$ meV and $b = 0.005$ meV/K we find outstanding agreement with the scaled experimental data. We clearly see in Fig. 1 that, for large (scaled) temperatures, $T/T_0 \gtrsim 2$, the above branches $\mathcal{F}_+$ and $\mathcal{F}_-$ (dashed lines) describe the data of, both, the metallic and the insulating phase accurately, as found previously [16,28]. At lower temperatures and close to the Lifshitz transition, however, the behavior is qualitatively different: the excellent agreement with Eq. (2) for experimentally available temperatures suggests that in the metal, instead of vanishing, the scaled resistance

$R(T)/R_c(T)$ extrapolates to a finite residual value for $T \rightarrow 0$. In the insulator, instead of diverging as predicted semiclassically, the scaled resistance also saturates.

Limits of a semiclassical description—Deviations from semiclassical scaling in the quantum regime imply that, besides the chemical potential μ , the scattering rate Γ has become a relevant energy scale. To quantify the quantum regime we calculate the relative deviation of the full quantum resistance formula to its semiclassical limit $|1 - R^{sc}/R|$ in Fig. 2(a). Under the assumption of a constant scattering rate, both, Eq. (2) and Eq. (3) only depend on the two dimensionless ratios μ/Γ and $k_B T/\Gamma$. But only in the approximate conductance, Eq. (3), do these dependencies factorize, allowing for scaling: $\sigma^{sc}(\mu, T) = \sigma^{sc}(0, T/\Gamma)/\mathcal{F}(\mu/T)$. Instead, in the exact conductance, Eq. (2), the interplay of μ/Γ and $k_B T/\Gamma$ can be nontrivial and highly nonlinear.

Deep into the metallic phase ($\mu/\Gamma \gtrsim 2$) the semiclassical expression provides an excellent approximation all the way down to $T = 0$. However, as the system approaches the band-tuned Lifshitz transition, deviations start to occur at low temperatures, becoming especially pronounced in the insulating phase. This finding is consistent with the phenomenological Mott-Ioffe-Regel criterion [50,51], which states that for transport in lattice systems to be ballistic, the distance between scattering events cannot be smaller than the interatomic separation. For the electron gas this requirement corresponds to $\mu/(2\Gamma) \gtrsim 1$ [16], which is indicated by the dashed vertical line in Fig. 2(a).

Transport at zero temperature—The effect of lifetime-broadening is most striking at $T = 0$ where thermal broadening is absent. Namely our new theory suggests that the residual ($T \rightarrow 0$) conductance changes *continuously* through the field-tuned metal-insulator transition. An asymptotic expansion of Eq. (2) yields

$$\sigma(\mu, 0) = \frac{e^2}{2\pi h} \left\{ 1 + \frac{\mu}{\Gamma} \left[\frac{\pi}{2} + \arctan \left(\frac{\mu}{\Gamma} \right) \right] \right\}. \quad (4)$$

A plot of the resistance ($1/\sigma$) in Fig. 2(b), shows that the residual resistance is indeed a uniformly continuous (and differentiable) function of the chemical potential. As a consequence, unlike the semiclassical prediction (dashed line), the resistance at $T = 0$ remains finite in the insulating phase. The microscopic origin of this resistance saturation is the presence of lifetime-broadened, incoherent spectral weight in the gap [44], despite the absence of thermal broadening, as discussed previously for correlated narrow-gap semiconductors [45,52].

Universal transport at the quantum critical point—We now combine the above understanding to reveal a fascinating feature of the 2D band-tuned Lifshitz transition. At the Lifshitz transition, $\mu = k_B T = 0$, the resistance is independent of any system parameter including the scattering rate, the carrier density, and the effective mass [cf. Eq. (4) and

Fig. 2(b)] and has (per spin, band, and valley degree of freedom) the universal value

$$R_L = (2\pi h)/e^2. \quad (5)$$

We will refer to R_L as the “Lifshitz resistance.” This result implies that for any 2D system which undergoes a band-tuned (“BT”) metal-insulator transition, the resistance at their quantum critical point $R_c^{BT} = R(\mu = 0, T = 0)$ should take the same unique value R_L , even across materials with different scattering mechanisms. To verify this prediction, we confront it with the experimental data from the ($f = 2$) $\text{MoTe}_2/\text{WSe}_2$ system [27]. Since experiments are performed at finite temperatures (the experimental base temperature is $T = 1.6$ K) and for a discrete set of displacement fields only, we assess the compatibility of our prediction by extracting a confidence interval for the experimental critical resistance $R_c^{\text{exp}}(T \rightarrow 0)$ (see Sec. VI in Ref. [29] for details). We find an estimated range of $0.3R_L \leq R_c^{\text{exp}} \leq 1.2R_L$ (per degree of freedom) meaning that our prediction of universal quantum critical transport is compatible with the experiment of Li *et al.* [27]. This encouraging result calls for future measurements which further approach the Lifshitz quantum-critical point, reaching lower temperatures and sampling a finer set of displacement fields to reduce the error bar for the $\text{MoTe}_2/\text{WSe}_2$ bilayer along with the identification of other systems which could exhibit a continuously band-tuned MIT.

Identifying the transition mechanism using R_c —A natural question to ask is whether the value of R_L can be used to distinguish band-tuned transitions from MITs driven by other mechanisms. More precisely, is there a relation to the critical resistances R_c^D and R_c^I of transitions dominantly driven by disorder (“D”) or interactions (“I”)? Charge carriers in 2D systems experience a complex interplay of disorder and interactions. Determining which dominates driving the MIT cannot always be identified experimentally, especially when the MIT is induced by changing the carrier density on a sample-by-sample basis [53–57]. Therefore, a simple descriptor, such as R_c , could be a useful tool to classify the complex physics behind a given 2D MIT.

In highly disordered systems, the low-energy electronic states may undergo Anderson localization and the system becomes insulating when the chemical potential drops below the mobility edge μ_c [9,58,59]. We expect our theory for the residual conduction, Eq. (4), to apply for $0 < \mu_c \leq \mu$, that is, in the metallic phase above the mobility edge. Since the resistance drops monotonously with decreasing chemical potential $R_c^D = R(\mu = \mu_c, T = 0) < R(\mu = 0, T = 0) = R_L$, we hypothesize that the Lifshitz resistance provides an *upper bound* for the critical resistance $R_c^D < R_L$ of a predominately disorder-driven MIT. This statement again assumes that vertex corrections can be neglected. Indeed, to first approximation, ladder-type corrections can be accounted for by a renormalized scattering rate (see, e.g., Ref. [43]), which then has no

influence on R_L that is independent of Γ . In turn, weak localization effects have been shown to be suppressed near the MIT in relevant 2D systems [1,60].

However for a predominately interaction-driven MIT, a similar constraint on R_c^I cannot be defined. In fact, at a Mott-Wigner/Hubbard transition the electronic effective mass diverges and this is accompanied by a strongly dynamical scattering rate $\Gamma(\omega)$ that engenders a vanishing quasiparticle lifetime on the former Fermi surface [6,8]. This violates the approximation of a static Γ , made in the derivation of Eq. (2), hence our theory can provide no indication on the value of R_c^I .

Therefore, we propose a sufficient but not necessary criterion, that if $R_c > R_L$ then this signifies the predominant influence of interactions in a 2D metal-insulator transition. For $R_c \leq R_L$, however, nothing conclusive about the origin of the MIT can be deduced. Indeed, we predict that Lifshitz and interaction-driven transitions may exhibit $R_c = R_L$, while both disorder and interaction driven MITs can occur at values $R_c < R_L$. However, if we combine the R_L criterion with the additional information on whether or not the system’s effective mass diverges through the transition, these ambiguities are resolved and an in principle complete assignment of the dominant transition mechanism is achieved (see also Sec. VIII in Ref. [29]).

Investigating the R_c criterion—To explore this criterion we extract the “critical resistance ranges” (per degree of freedom) from a variety of 2D systems that were driven through a MIT. Where clear prior evidence for one predominant mechanism of the MIT was found, the system is labeled appropriately. However, in many systems due to the highly nontrivial interplay between interactions and disorder no conclusive cause for the MIT has previously been identified; therefore, these systems are labeled as “disorder + interaction” driven.

The critical resistance ranges plotted in Fig. 3 show broadly the expected pattern, that while transitions driven predominately by interactions (blue symbols) span a huge range of R_c values, transitions where disorder should also play an important role in the MIT lie around or below the value of R_L (cyan, green symbols).

The systems with the largest R_c values [points (a) $\text{SiGe}/\text{Si}/\text{SiGe}$ QW [19] and (c) ($f = 1$) $\text{MoTe}_2/\text{WSe}_2$ [27] in Fig. 3 occurring at $R_c \sim 10R_L$ and $R_c \sim 37R_L$ respectively] display transitions that are very predominantly interaction driven, with the electronic effective mass in both systems diverging before the transition [19,27]. Therefore, we hypothesize that for $R_c \gg R_L$ the system will be very strongly interaction driven; however, the appearance of clear experimental signatures may make our criterion less necessary for such identification. It is in the intermediate regime $R_c \sim R_L$ where interactions and disorder are on a more equal footing that our criterion may be most useful in identifying when interactions are more relevant for the transition. For example, our criterion supports the proposal [1] that despite contributions from disorder, it is electronic

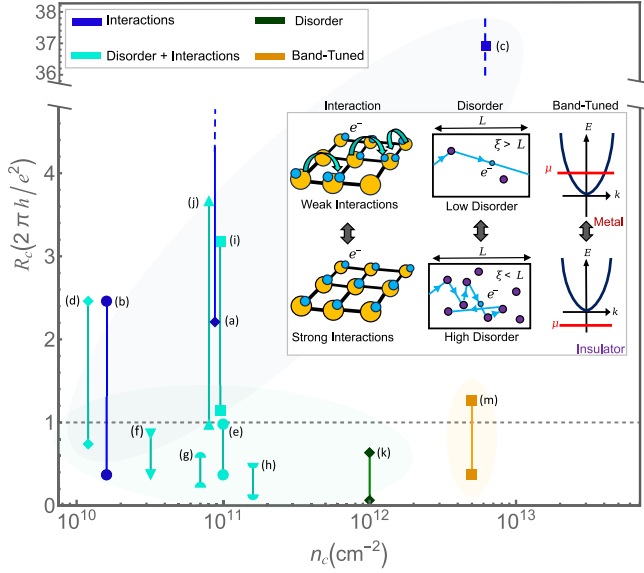


FIG. 3. Comparison of experimental critical resistance ranges with R_L . Extracted critical resistance ranges from several experiments—(a) SiGe/Si/SiGe QW [19], (b) ZnO [61], (c) MoTe₂/WSe₂ (half-filled, $f = 1$) [27], (d) GaAs [20], (e) SiGe [21], (f) Si [22], (g) AlAs [23], (h) WSe₂ [53], (i) Si MOSFET [17], (j) Si MOSFET [18], (k) Graphene [62], (m) MoTe₂/WSe₂ ($f = 2$) [27]—per system degree of freedom (cf. Sec. VII in Ref. [29]), plotted against the carrier density n_c at criticality. Data points are colored by the predominant driver of the MIT. In the intermediate regime where this assignment is unclear, systems are labeled “disorder + interactions.” The Lifshitz resistance R_L , is shown as a horizontal gray dashed line. Dashed vertical lines indicate ranges beyond the limits of the graph. A clear critical range for system (c) could not be estimated; however, the transition takes place near the point indicated.

interactions that are the protagonists in driving the transition of the two Si MOSFET devices [points (i) [17] and (j) [18] in Fig. 3], as their critical resistance ranges ($R_c > R_L$) lie above the upper bound for a predominantly disordered transition. For the remaining intermediate disorder + interaction systems our criterion is unable to conclusively classify the transitions. Having established that R_c can be a useful tool for identifying interaction-driven 2D MITs in some situations, further exploration is warranted into how the value of R_c for an arbitrary system will be affected as the relative strength of disorder and electronic correlations are adjusted, experimentally as well as in theoretical descriptions of, e.g., the Anderson-Hubbard model.

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Data availability—The data supporting this study’s findings are available within the article.

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