

Universal Bound on the Duration of a Kination Era

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We show that primordial adiabatic curvature fluctuations generate an instability of the scalar field sourcing a kination era. We demonstrate that the generated higher Fourier modes constitute a radiationlike component dominating over the kination background after about 11 e-folds of cosmic expansion. Current constraints on the extra number of neutrino flavors ΔN_{eff} thus imply the observational bound of approximately 10 e-folds, representing the most stringent bound to date on the stiffness of the equation of state of the pre-Big Bang–nucleosynthesis universe.

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Introduction—The equation of state (EoS) of the universe between the end of inflation and the onset of Big Bang nucleosynthesis (BBN), defined by the pressure-to-energy density ratio $\omega = p/\rho$, remains unconstrained [1]. While the standard model assumes radiation domination with $\omega \simeq 1/3$, a period of *kination*, where the energy density is dominated by the kinetic energy of a fast-rolling scalar field, corresponds to $\omega = 1$ [2–6]. Kination can occur right after inflation in quintessential inflation models [7,8], in string compactifications [9–11], or well after reheating during the radiation era, due to a scalar field rotating along a flat direction [12–20]. The latter has been proposed to explain the baryon asymmetry or axion dark matter [21–28]. Kination may also arise from scalar field oscillations in steep potentials $V(\phi) \propto \phi^p$ with $p \gg 4$ [29–32], modified gravity [33], or small-scale anisotropic stress [34–37]. With energy density scaling as $\rho \propto a^{-6}$, kination naturally ends when it becomes subdominant to standard model (SM) radiation, without requiring decay. A well-studied cosmological implication of kination is the enhancement of primordial gravitational wave backgrounds

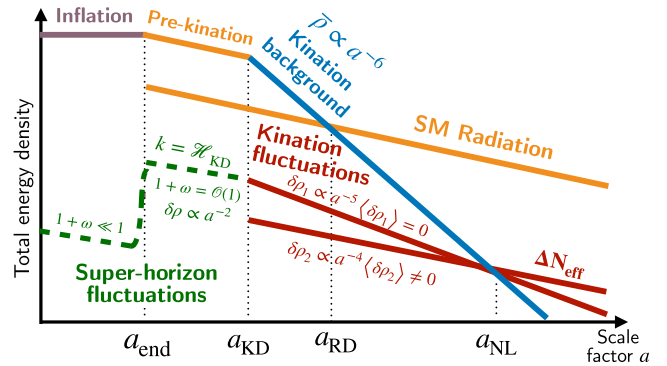


FIG. 1. During kination, inflationary curvature fluctuations (green) source kination fluctuations (red), which either terminate the kination era if $a_{\text{RD}} > a_{\text{NL}}$ or contribute to N_{eff} if $a_{\text{RD}} < a_{\text{NL}}$. The former is excluded by Eq. (31). We consider kination (blue) either immediately after inflation (purple) if $a_{\text{KD}} = a_{\text{end}}$, as in quintessential inflation scenario [7,8], or after unspecified post-inflationary dynamics (orange), which is modeled to be radiationlike for simplicity and a conservative result. $\delta\rho_1$ and $\delta\rho_2$ are the linear and quadratic corrections to the kination background defined in Eqs. (25) and (26). The latter implies that kination fluctuations have a nonvanishing volume average and contributes to ΔN_{eff} . Superhorizon energy fluctuations (green) evolve as $\delta\rho_k \propto (1 + \omega)\mathcal{R}_k/a^2$, see Eqs. (18) and (22). A sudden rise occurs at the end of inflation when $1 + \omega = 2\epsilon/3$ initially $\ll 1$ grows to $\mathcal{O}(1)$, with ϵ being the slow-roll parameter [85]. We picture the superhorizon fluctuations in dashed since they depend on the gauge, here chosen to be the Comoving gauge. We show the mode $k = \mathcal{H}_{\text{KD}}$ that re-enters the horizon when kination starts since it is the mode that dominates the contribution to the volume average $\langle \delta\rho_2 \rangle$ [cf. Eqs. (27) and (28)] and therefore to ΔN_{eff} . The duration of kination is $N_{\text{KD}} \equiv \ln(a_{\text{RD}}/a_{\text{KD}})$. The quantities a_{end} , a_{KD} , a_{RD} , a_{NL} are the scales factors at the end of inflation, start of kination, end of the kination, and start of nonlinear regime.

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(GWBs), generated by inflation [12–15,33,38–65], cosmic strings [13,15,16,64,66–74], or first-order phase transitions [1,15,64,75]. For a large tensor-to-scalar ratio r , the enhanced inflationary tensor modes give a large contribution to the number of exotic relativistic degrees of freedom ΔN_{eff} . The Planck bound $\Delta N_{\text{eff}} \lesssim 0.34$ (2σ) [76] converts to an upper limit on the number of kination Hubble expansion e-folding $N_{\text{KD}} \lesssim 12.2 + 0.5 \ln(0.036/r)$, e.g., Ref. [15].

In this Letter, we demonstrate that kination not only enhances tensor modes but also amplifies scalar modes, which contribute to ΔN_{eff} as well. Using $\Delta N_{\text{eff}} \lesssim 0.34$ (2σ), we place the most stringent bound to date on the duration of kination, $N_{\text{KD}} \lesssim 10$ e-folds. While the previous bound on N_{KD} from tensor modes strongly depends on the tensor-to-scalar ratio r , the new upper bound—derived in this Letter—is independent of r and solely set by the amplitude of the curvature power spectrum $\mathcal{P}_{\mathcal{R}}(k)$.

Primordial metric fluctuations—We consider the cosmological scenario shown in Fig. 1. We are interested in the impact of such kination era on the power spectra of primordial curvature $\mathcal{P}_{\mathcal{R}}(k)$ and tensor modes $\mathcal{P}_t(k)$ [77]. The amplitude and spectral index of the curvature power spectrum are measured by Planck at the pivot scale $k_\star \equiv 0.05 \text{ Mpc}^{-1}$ to be [78]

$$\mathcal{P}_{\mathcal{R}}(k_\star) \simeq 2.099(29) \times 10^{-9}, \quad n_s \simeq 0.965(4). \quad (1)$$

The nonobservation of primordial B modes in cosmic microwave background (CMB) anisotropies by BICEP/Keck [79] provides the most stringent upper bound on the tensor-to-scalar ratio at pivot scale $k_\star$:

$$r \equiv \mathcal{P}_t(k_\star)/\mathcal{P}_{\mathcal{R}}(k_\star) \lesssim 0.036, \quad (2)$$

leading to a bound on the inflation scale when $k_\star$ exit the horizon $H_\star \simeq 1.8 \times 10^{13} \text{ GeV} \sqrt{r/0.036}$. The power spectra at $k \gg k_\star$ are unknown and require modeling. In this Letter, we explore two possibilities. The first considers the curvature and tensor power spectra [80],

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{8\pi^2} \frac{1}{\epsilon_k} \left(\frac{H_k}{M_{\text{pl}}} \right)^2, \quad \mathcal{P}_t(k) = \frac{2}{\pi^2} \left(\frac{H_k}{M_{\text{pl}}} \right)^2, \quad (3)$$

predicted by one of the most strongly supported inflation models [78,79,81]—the *Starobinsky* inflation [82–84]; see Supplemental Material [85]. Here H_k and ϵ_k are the Hubble and slow-roll parameters when the comoving scale k exits the horizon. The second possibility is that the spectra can be well-approximated by power laws,

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}(k_\star) (k/k_\star)^{n_s(k)-1}, \quad (4)$$

$$\mathcal{P}_t(k) = r \mathcal{P}_{\mathcal{R}}(k_\star) (k/k_\star)^{n_t(k)}, \quad (5)$$

with the spectral indices n_s and n_t truncated at first-order in $\ln(k)$,

$$n_s(k) = n_s(k_\star) + \alpha_s \ln(k/k_\star), \quad (6)$$

$$n_t(k) = n_t(k_\star) + \alpha_t \ln(k/k_\star). \quad (7)$$

Additionally, the slow-roll assumption leads to the following consistency relations [87]:

$$n_t = -\frac{r}{8} \left(2 - n_s - \frac{r^2}{256} \right), \quad \alpha_t = n_t(n_t + 1 - n_s). \quad (8)$$

For α_s , we consider the 98% posterior distribution derived from the Bayesian analysis of ~ 300 models of single-field slow-roll inflation over the latest cosmological data [88],

$$\alpha_s = -3.65_{-5.35}^{+3.20} \times 10^{-4}. \quad (9)$$

We accounted for a factor of 1/2 due to different definitions of α_s in this Letter and [88].

ΔN_{eff} bound from tensor modes—The already known upper bound on the duration of kination comes from the enhanced tensor modes, e.g., [15]. Primordial tensor modes with dimensionless power spectrum $\mathcal{P}_t(k)$ lead to a GWB with energy fraction at late times [89],

$$\Omega_{\text{GW}}^0(k) = \left(\frac{a_{\mathbf{k}} \mathcal{H}_{\mathbf{k}}}{a_0 \mathcal{H}_0} \right)^2 \frac{\mathcal{P}_t(k)}{24}, \quad (10)$$

where a_0 and $a_{\mathbf{k}}$ are the scale factors today and at horizon reentry $k = \mathcal{H}_{\mathbf{k}}$ with $\mathcal{H} = aH$ being the comoving Hubble factor. Modes reentering during the kination era ($H_k \propto a_{\mathbf{k}}^{-3}$) yield a slope k^1 in the GW spectrum,

$$\Omega_{\text{GW}}^0(k) = \Omega_{\text{rad}}^0 \mathcal{G}_k \frac{\mathcal{P}_t(k)}{24} \begin{cases} k/k_{\text{RD}}, & k_{\text{RD}} < k < k_{\text{KD}}, \\ 1, & k < k_{\text{RD}}, \end{cases} \quad (11)$$

where $\Omega_{\text{rad}}^0 \simeq 9.1 \times 10^{-5}$ is today's radiation energy fraction [90]. The quantity $\mathcal{G}_k$ accounts for the changes in relativistic degrees of freedom,

$$\mathcal{G}_k \equiv \left(\frac{g_{\star,k}}{g_{\star,0}} \right) \left(\frac{g_{s,0}}{g_{s,k}} \right)^{\frac{4}{3}}, \quad (12)$$

where $g_\star$ and g_s are the numbers of relativistic degrees of freedom contributing to the energy and entropy densities, respectively. Different subscripts denote the times of today (“0”), the mode reentry (“k”), the beginning of the kination era (“KD”), and the end of the kination era (“RD”). The contribution of tensor modes to ΔN_{eff} reads [85]:

$$\Delta N_{\text{eff,t}} \simeq 0.12 \int_{k_{\text{RD}}}^{k_{\text{KD}}} d \log k \left(\frac{\mathcal{G}_k}{0.39} \right) \left(\frac{k}{k_{\text{RD}}} \right) \mathcal{P}_t(k), \quad (13)$$

where $k_{\text{KD}} = \mathcal{H}_{\text{KD}}$ and $k_{\text{RD}} = \mathcal{H}_{\text{RD}}$. Assuming the power-law $\mathcal{P}_t(k)$ in Eq. (5) with $N_{\text{KD}} \gg 1$ and $\mathcal{G}_k \simeq 0.39$, Eq. (13) becomes,

$$\Delta N_{\text{eff},t} \simeq \frac{0.12}{1 + n_t(k_{\text{KD}})} \mathcal{P}_t(k_{\text{KD}}) \exp(2N_{\text{KD}}), \quad (14)$$

where N_{KD} is the number of e-folds of the kination era,

$$N_{\text{KD}} \equiv \ln(a_{\text{RD}}/a_{\text{KD}}). \quad (15)$$

Note that $k \propto a_k^{-2}$ during kination era, leading to $N_{\text{KD}} = 0.5 \ln(k_{\text{KD}}/k_{\text{RD}})$. Using the BAO + CMB constraint $\Delta N_{\text{eff}} \lesssim 0.34$ at 2σ [76], Eq. (14) converts into an upper bound on the duration of kination,

$$N_{\text{KD}} \lesssim 12.2 + 0.5 \ln\left(\frac{0.036}{r}\right) + 0.5 \ln\left(\frac{\Delta N_{\text{eff},t}}{0.34}\right) - 0.5 n_t(k_{\text{KD}}) \ln\left(\frac{k_{\text{KD}}}{k_\star}\right) + 0.5 \ln(1 + n_t(k_{\text{KD}})). \quad (16)$$

A slightly weaker bound can be obtained from BBN, i.e., $\Delta N_{\text{eff}} < 0.54$ (2σ) [91]. The N_{KD} bound in Eq. (16) depends on the tensor-to-scalar ratio r . In a specific inflationary scenario, the value of r could theoretically be much smaller than the BICEP/Keck bound $r \ll 0.036$ [79], resulting in an upper bound significantly less stringent than Eq. (16). Inflaton potentials capable of producing arbitrarily small values of r can be designed [92,93]. Even in top-down constructions such as Kähler moduli inflation, values as low as $r \sim 10^{-10}$ are predicted [94,95]. We now discuss the effects of kination on scalar perturbations.

Growth of scalar modes during kination—In the absence of anisotropic stress, the metric perturbation in the conformal Newtonian gauge reads [80],

$$ds^2 = a^2 \{ (1 + 2\Phi) d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}] dx^i dx^j \}, \quad (17)$$

where Φ and h_{ij} are the scalar and tensor fluctuations. The gauge-invariant comoving curvature perturbation $\mathcal{R}$ can be expressed in terms of Φ [80],

$$-\mathcal{R} \equiv \Phi + \frac{2}{3} \frac{(\mathcal{H}^{-1}\Phi' + \Phi)}{1 + \omega} \xrightarrow{[k\eta \ll 1]} \frac{5 + 3\omega}{3 + 3\omega} \Phi \xrightarrow{[\omega=1]} \frac{4}{3} \Phi. \quad (18)$$

In terms of the Fourier mode $\Phi_{\mathbf{k}} = \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} \Phi(\mathbf{x})$, the linear perturbation of Einstein's equations implies [80]

$$\Phi_{\mathbf{k}}''(\eta) + \frac{6(1 + \omega)}{1 + 3\omega} \frac{1}{\eta} \Phi_{\mathbf{k}}'(\eta) + \omega k^2 \Phi_{\mathbf{k}}(\eta) = 0. \quad (19)$$

The solution of Eq. (19) during kination ($\omega = 1$) is

$$\Phi_{\mathbf{k}}(k\eta) = \frac{2\Phi_{\mathbf{k}}(0)J_1(k\eta)}{k\eta} \xrightarrow{[k\eta \gg 1]} -\frac{3\mathcal{R}_{\mathbf{k}}(0)\cos(k\eta + \varphi)}{\sqrt{2\pi}(k\eta)^{3/2}}, \quad (20)$$

where φ is a phase. We deduce the time-averaged scalar power spectrum during kination,

$$\mathcal{P}_{\Phi}(k, \eta) = \frac{9\mathcal{P}_{\mathcal{R}}(k)}{4\pi(k\eta)^3}. \quad (21)$$

Einstein's equation along 00 leads to Poisson's equation,

$$-k^2 \Phi_{\mathbf{k}} = 4\pi G a^2 \bar{\rho} \delta_{\mathbf{k}}, \quad (22)$$

where $\delta_{\mathbf{k}} \equiv (\rho_{\mathbf{k}} - \bar{\rho})/\bar{\rho}$ is the contrast density in the Comoving gauge [96]. Hence, scalar metric fluctuations $\Phi_{\mathbf{k}}(\eta)$ unavoidably generate energy density fluctuations $\delta_{\mathbf{k}}$ for the dominant background fluid. Using Eq. (22) into Eq. (21) gives rise to the following energy density power spectrum:

$$\mathcal{P}_{\delta}(k, \eta) \simeq \frac{4}{9} \left(\frac{k}{\mathcal{H}}\right)^4 \mathcal{P}_{\Phi}(k, \eta) = \frac{8}{\pi} \left(\frac{k}{\mathcal{H}}\right) \mathcal{P}_{\mathcal{R}}(k), \quad (23)$$

where we used Friedmann's equation $3\mathcal{H}^2 = 8\pi G \bar{\rho} a^2$, and $\mathcal{H} = 2/(1 + 3\omega)\eta = 1/2\eta$ during the kination domination. Using $\eta \propto a^{(1+3\omega)/2}$, we see that fluctuations grow like $\delta \propto a$ during kination. We suppose the kination era to be induced by the kinetic term of massless homogeneous scalar field condensate that gets fragmented into higher modes $\phi = \bar{\phi} + \delta\phi$ such that its energy density,

$$\bar{\rho}_{\phi} = \frac{1}{2} \left(\frac{\bar{\phi}'}{a}\right)^2, \quad (24)$$

receives linear and quadratic corrections $\delta\rho_{\phi} = \delta\rho_1 + \delta\rho_2$:

$$\delta\rho_1 \simeq \bar{\phi}' \delta\phi' / a^2, \quad (25)$$

$$\delta\rho_2 \simeq \frac{1}{2} \left(\frac{\delta\phi'}{a}\right)^2 + \frac{1}{2} \left(\frac{\nabla\delta\phi}{a}\right)^2 \simeq \left(\frac{\delta\phi'}{a}\right)^2, \quad (26)$$

where we have neglected faster decaying terms containing $\Phi_{\mathbf{k}}$. The gradient term in Eq. (26) is equal to the kinetic term since $\delta\phi_{\mathbf{k}} \propto \Phi_{\mathbf{k}}(\eta) \propto \cos(k\eta)$. Upon volume averaging the linear term vanishes $\langle\delta\rho_1\rangle = 0$, while the second term is equal to

$$\langle\delta\rho_2\rangle = \langle(\delta\rho_1)^2/2\bar{\rho}_{\phi}\rangle = \frac{\bar{\rho}_{\phi}}{2} \int_{k_{\text{RD}}}^{k_{\text{KD}}} d\log k \mathcal{P}_{\delta}(k, \eta), \quad (27)$$

where we integrate over all fluctuation modes entering during the kination era in the second step. Plugging $\mathcal{P}_{\delta}(k, \eta)$ in Eq. (23) into Eq. (28), we obtain the fluctuation over background kination energy ratio,

$$\Omega_{\delta_\phi}(k) \equiv \left\langle \frac{\delta\rho_\phi}{\bar{\rho}_\phi} \right\rangle = \frac{4}{\pi} \int_k^{k_{\text{KD}}} d \log \tilde{k} \frac{\tilde{k}}{k} \mathcal{P}_{\mathcal{R}}(\tilde{k}), \quad (28)$$

which we evaluated at the time when $\mathcal{H}(a_k) \equiv k$. The equation of motion for kination fluctuations is

$$\delta\phi''_{\mathbf{k}} + 2\mathcal{H}\delta\phi'_{\mathbf{k}} + k^2\delta\phi_{\mathbf{k}} = 4\Phi'_{\mathbf{k}}\bar{\phi}', \quad (29)$$

where the source term is negligible for subhorizon modes [85]. The solution, $\delta\phi \propto a^{-1}$, indicates that kination fluctuations redshift like radiation $\delta\rho_2 \propto a^{-4}$. The equation of motion in Eq. (29) includes all orders in $\delta\phi_{\mathbf{k}}$ (only $\Phi_{\mathbf{k}}$ was treated as a perturbation). Hence, the expression for the fluctuation energy density in Eq. (29) and the fact that it redshifts like radiation hold for both the linear and non-perturbative regimes $\Omega_{\delta_\phi} < 1$ and $\Omega_{\delta_\phi} > 1$ [97]. Note that our derivation of the kination fluctuation relies on the scalar field description, which has been considered in the radiation background in [26] and kination era in [11]. We postpone a full derivation from the perfect fluid formalism for future work. We now discuss the two novel bounds on the duration of kination which follow from Eq. (28).

Fluctuation domination—When $\Omega_{\delta_\phi}(k) > 1$ in Eq. (28), the energy density of the radiationlike kination fluctuations becomes larger than the energy density of the kination background. This leads to kination era to terminate and be replaced by a radiation era after a duration N_{KD} which we now determine. Assuming $\mathcal{P}_{\mathcal{R}}(k)$ to be given by the power law in Eq. (4), Eq. (28) becomes for $N_{\text{KD}} = 0.5 \ln(k_{\text{KD}}/k_{\text{RD}}) \gg 1$

$$\Omega_{\delta_\phi}(k_{\text{RD}}) \simeq \frac{4}{\pi n_s(k_{\text{KD}})} \mathcal{P}_{\mathcal{R}}(k_{\text{KD}}) \exp(2N_{\text{KD}}). \quad (30)$$

We conclude that the kination era cannot physically last longer than

$$N_{\text{KD}} \lesssim 9.9 + 0.5 \ln\left(\frac{2.099 \times 10^{-9}}{\mathcal{P}_{\mathcal{R}}(k_\star)}\right) + 0.5 \ln n_s(k_{\text{KD}}) + 0.5(1 - n_s(k_{\text{KD}})) \ln\left(\frac{k_{\text{KD}}}{k_\star}\right). \quad (31)$$

This is not an observational constraint but a theoretical bound on the duration of kination, arising from radiation-like fluctuations dominating over the kination background. We now assume that kination terminates before reaching the limit in Eq. (31) due to the presence of SM radiation and discuss the contribution of kination fluctuations Ω_{δ_ϕ} to ΔN_{eff} , as illustrated in Fig. 1.

ΔN_{eff} bound from scalar modes—If no entropy injection occurs, the kination fluctuations contribute to the number of effective degree of freedom at the onset of BBN as [85]

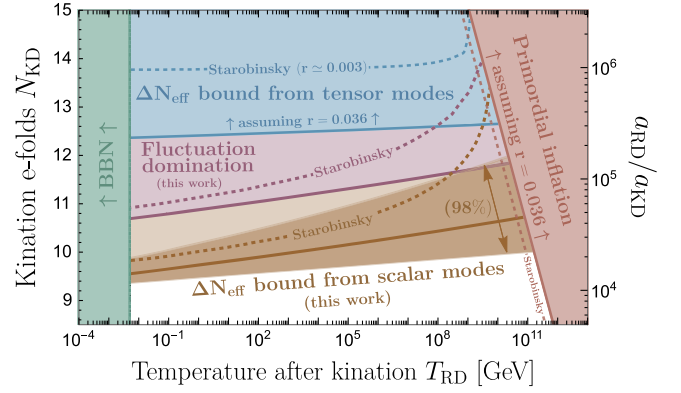


FIG. 2. Observational bounds on the duration of a kination era due to the linear growth of tensor [Eq. (16) and blue region] and scalar modes [Eq. (33) and brown region] violating the observational constraints $\Delta N_{\text{eff}} \lesssim 0.34$ (2σ) [76]. The purple region is a physical upper limit on the kination duration due to domination by the radiation-like fluctuations, cf. Eq. (31). The power spectra $\mathcal{P}_t(k)$ and $\mathcal{P}_{\mathcal{R}}(k)$ are either given by the power-laws in Eqs. (4) and (5) for the solid lines or by the predictions from Starobinsky inflation model in Eq. (3) for the dashed lines. The brown arrow indicates the uncertainty in the spectral-index running α_s in Eq. (9), for extrapolating $\mathcal{P}_{\mathcal{R}}(k)$ above the CMB pivot scale $k_\star = 0.05 \text{ Mpc}^{-1}$. The solid brown and purple lines assumes α_s to be given by the mean of Eq. (9). In the green region, kination would end after the onset of BBN around $T_{\text{BBN}} \simeq 5 \text{ MeV}$. In the red region, kination would start at a Hubble scale larger than the value H_{end} predicted in Starobinsky inflation (dashed) or its maximal value allowed by the BICEP/Keck bound $r \lesssim 0.036$ [79] (solid).

$$\Delta N_{\text{eff},s} = 2.9 \left(\frac{\mathcal{G}_k}{0.39} \right) \Omega_{\delta_\phi}(k_{\text{RD}}), \quad (32)$$

where $\Omega_{\delta_\phi}(k)$ is given by Eq. (28). The constraint $\Delta N_{\text{eff}} \lesssim 0.34$ at 2σ [76] can be recast as

$$N_{\text{KD}} \lesssim 8.8 + 0.5 \ln\left(\frac{\Delta N_{\text{eff},s}}{0.34}\right) + 0.5 \ln\left(\frac{2.099 \times 10^{-9}}{\mathcal{P}_{\mathcal{R}}(k_\star)}\right) + 0.5(1 - n_s(k_{\text{KD}})) \ln\left(\frac{k_{\text{KD}}}{k_\star}\right) + 0.5 \ln n_s(k_{\text{KD}}). \quad (33)$$

The bound in Eq. (33) is more restrictive than both Eqs. (16) and (31). We compare the three bounds in Fig. 2. In contrast to the already-known bound in Eq. (16), the two novel bounds in Eqs. (31) and (33) are independent of the—unmeasured—value of the tensor-to-scalar ratio r .

Conclusion—We have studied how cosmological perturbations evolve during a kination era. At linear order, fluctuations grow linearly with the scale factor, $\delta \propto a$ like matter fluctuations in agreement with previous findings—see, e.g., Refs. [26,101]. Accounting for non-linear corrections, we demonstrated that fluctuations of

the scalar driving kination have a nonvanishing volume average $\langle \delta \rangle \neq 0$ cf. Eq. (27), unlike matter fluctuations due to mass conservation [102,103], and redshift like radiation. Consequently, a kination era induced by a scalar field must end after approximately 11 e-folds when these fluctuations become nonperturbative and induce a radiationlike equation of state (EoS); see the purple curves in Fig. 2 [104].

These radiationlike fluctuations also contribute to the effective number of neutrino species ΔN_{eff} . Planck 2σ limit, $\Delta N_{\text{eff}} \lesssim 0.34$ [76], currently imposes the strongest constraint on kination, restricting its duration to $N_{\text{KD}} \lesssim 10$ e-folds; see Eq. (33) and the brown curves in Fig. 2. Unlike the already-known bound from tensor modes in Eq. (16), the new limits from fluctuation domination and scalar modes in Eqs. (31) and (33) derived in this Letter are independent of the unknown value of the tensor-to-scalar ratio r . Future measurements, such as CMB-S4 with a projected sensitivity of $\Delta N_{\text{eff}} \lesssim 0.05$ [106,107], could further tighten the bounds on N_{KD} by about one e-fold.

The conservative bounds presented in this Letter rely on the assumption that the scalar field acquires the primordial curvature perturbations from the inflaton, which requires that the scalar field either inherits its energy density from the inflaton or is the inflaton itself, as demonstrated in Fig. 1 [108].

We considered for simplicity scenarios where kination is preceded by either a radiation era or inflation [7]. Scalar field theories leading to a radiation EoS preceding kination are discussed in [33]. On the other hand, kination in rotating axion models [13–20], which is disconnected from inflation and preceded by a matter era, would amplify scalar perturbations more strongly, leading to a more stringent ΔN_{eff} bound. Our bound should therefore be seen as conservative. Moreover, if fluctuations become massive due to the scalar field condensate acquiring a mass, the constraints derived in this Letter would be further tightened, or the scalar fluctuations could account for dark matter [109,110]. We leave for future works the study of kination fluctuations starting from the stress-energy tensor of a perfect fluid instead of the Lagrangian of a scalar field. This would, allow to possibly extrapolate our results to any eras with stiff EoS $\omega > 1/3$.

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Data availability—The data that support the findings of this Letter are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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