

Kerr Black Hole Dynamics from an Extended Polyakov Action

N. Emil J. Bjerrum-Bohr^{1,2,*} Gang Chen^{1,2,†} Chenliang Su^{2,‡} and Tianheng Wang^{3,§}

¹Center of Gravity, *Niels Bohr Institute*, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

²Niels Bohr International Academy, *Niels Bohr Institute*, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

³Center for Theoretical Physics, *Seoul National University*, 1 Gwanak-ro, Gwanak-gu, 08826, Seoul, South Korea



(Received 4 May 2025; revised 21 October 2025; accepted 28 January 2026; published 19 February 2026)

We examine a hypersurface model for the classical dynamics of spinning black holes. Under specific, rigid geometric constraints, it reveals an intriguing solution resembling expectations for the Kerr Black hole three-point amplitude. We explore various generalizations of this formalism and outline potential avenues for applying it to analyze the attraction of spinning black holes.

DOI: [10.1103/k5g2-q38q](https://doi.org/10.1103/k5g2-q38q)

Introduction—Modeling spinning black holes, particularly those with significant classical spin, presents a formidable theoretical challenge deeply linked to the complex behavior of strongly coupled extended bodies. A prominent example of a spinning black hole is the Kerr black hole [1], which, in the thorough analysis of Israel [2], was shown to be isomorphic to a hypersurface exhibiting a disk topology. It motivates the exploration of the hypersurface covariant action principle for spinning black holes, which we will pursue here.

In spinning black holes, the small-spin limit permits a perturbative framework utilizing the multipole expansion of an extended body. This approach is encapsulated in the well-established Mathisson-Papapetrou-Dixon equations [3–7], which have been extensively discussed in the literature. Recent advancements in scattering amplitudes [8–50] and worldline formulations [51–66], particularly pertaining to spinning point particles, have shown consistency with the multipole expansion for lower spin orders.

Regardless, the situation becomes more complex for black holes exhibiting large spins—typically associated with superheavy masses in strong coupling regimes. Here, the limitations of the spin expansion become pronounced, and to achieve a reliable description applicable to both finite spin and finite gravitational coupling, one that exceeds the point-particle approximation while preserving a covariant coupling, it is necessary to elucidate an appropriate action principle.

The analogy with the strong-coupling properties of matter in quantum chromodynamics is noteworthy in this context. It has yielded the Veneziano formula [67], which extends hadronic interactions beyond the confines of perturbative quantum chromodynamics by employing concepts such as crossing symmetry and Regge behavior. The governing dynamics of such behavior are nowadays rephrased through the Nambu-Goto and Polyakov 1 + 1-dimensional string theory actions [68–70], as well as higher-dimensional extensions (see, e.g., Refs. [71–74]).

Inspired by such ideas, this Letter takes the first step toward exploring a new, comprehensive gravitational framework for characterizing strongly coupled spinning objects in a covariant way from an action principle. The model we propose is one with a rigid internal structure of a hypersurface with specific geometric constraints on the topology of $S^2 \times \mathbb{R}$ and dynamics governed by the equations of motion of a generalized Polyakov action. The metric and the Riemann tensor mediate the interaction between this hypersurface and the target spacetime. In this framework, the only additional assumption required is minimal couplings between fields that respect the fundamental symmetry of the system and the spin supplemental condition.

As we will see, this model effectively reproduces the Kerr three-point amplitude to arbitrary spin orders in a dimension-agnostic manner and, as a spinoff, accommodates a diverse array of effective generalized worldline theories. Thus, it opens numerous interesting new research avenues for characterizing spin dynamics, including explorations of the new action principle as a fundamental model for spin in gravitational systems that could provide new insights into higher multiplicity amplitudes, such as the Compton amplitude with large classical spin, which is needed for precision analysis of dynamics in binary merger events.

This Letter is organized as follows: first, we explore hypersurfaces in flat and curved spacetimes. Next, we describe how the Kerr three-point amplitude at arbitrary spin orders can be situated within this analytical framework.

*Contact author: bjbohr@nbi.dk

†Contact author: gang.chen@nbi.ku.dk

‡Contact author: chenliang.su@nbi.ku.dk

§Contact author: tianhengwang@snu.ac.kr

Finally, we discuss several extensions of the formalism before concluding with our findings and highlighting interesting open questions for further exploration.

Modeling the dynamics of a spinning black hole on a hypersurface—Aiming to model a classical spinning black hole covariantly and at all spin orders while accounting for symmetry and physical requirements, we are inspired to consider an action principle based on a higher-dimensional extension of the Polyakov action integrated over the coordinates of a $(2 + 1)$ hypersurface as follows:

$$C \int_{\mathbb{M}^2 \times \mathbb{R}} d^3\sigma \sqrt{\gamma} (\partial_a Z^\mu \partial_b Z^\nu \gamma^{ab} \eta_{\mu\nu}), \quad (1)$$

where C is a constant with dimension and $\mathbb{M}^2$ denotes a compact closed Riemann surface, which together with $\mathbb{R}$ are parametrized by the coordinate $\sigma^a = (\tau, \vec{\sigma})$, while $Z^\mu(\sigma)$ denotes the spacetime coordinate of a given point in the worldvolume. Unless mentioned otherwise, we use the worldline convention for coordinates. This action will preserve general diffeomorphism invariance on the worldvolume and the target space, and we will employ a rigid worldvolume metric γ^{ab} that is τ independent and respects the rotational symmetries expected for the spinning object, i.e., resembling a disk topology. The metric we work with has the following spherically symmetric generic form:

$$ds^2 = d\tau^2 - a_w^2 (d\theta^2 + f(\theta) d\phi^2), \quad (2)$$

where $f(\theta)$ is an arbitrary function and a_w is a constant with “length” dimension; we note that the necessity of a nontrivial worldvolume metric is a characteristic feature of this higher-dimensional framework and has no direct counterpart in traditional worldline and string theories.

Since the description of the spinning black hole depends nontrivially on the worldvolume metric, we are led to fix $f(\theta) = \sin^2(\theta)$ using physical constraints. For instance, we insist that a suitable candidate metric must satisfy the free equation of motion following from Eq. (1) and admit solutions that match weak-field expectations, including τ independence (see Supplemental Material [75] for more details). We arrive at a free action in the following form:

$$S_0 = -\frac{m}{8\pi a_w^2} \int_{\mathbb{S}^2 \times \mathbb{R}} d^3\sigma \sqrt{\gamma} (\partial_a Z^\mu \partial_b Z^\nu \gamma^{ab} \eta_{\mu\nu} + 1), \quad (3)$$

where we denote the hypersurface $\mathbb{S}^2 \times \mathbb{R}$ and have included a mass term normalized consistently with the requirement of reparametrization invariance of τ so that we reach the point mass action in the limit $a_w \rightarrow 0$. The equation of motion of $Z^\mu(\sigma)$ is readily derived by varying the action (3),

$$\square Z^\mu = \gamma^{ab} (\partial_a \partial_b Z^\mu - \Gamma_{ab}^c \partial_c Z^\mu) = 0, \quad (4)$$

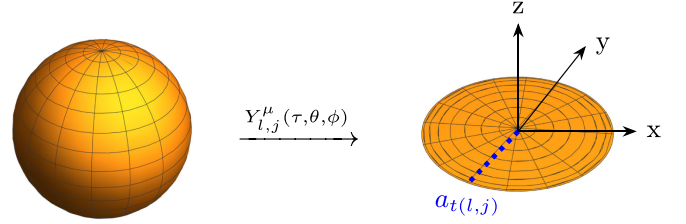


FIG. 1. Spinning hypersurface from worldvolume (left) to the disk in target spacetime (right), with coordinate axes labeled. The radius of the black hole is $a_{t(l,j)}$.

where Γ_{ab}^c denotes the nontrivial connection due to γ_{ab} , with nonvanishing components $\Gamma_{\theta\phi}^\phi = \Gamma_{\phi\theta}^\theta = \cot(\theta)$ and $\Gamma_{\phi\phi}^\theta = -\sin(\theta) \cos(\theta)$.

The solution to the free equation of motion controls the classical trajectory of a given point on the hypersurface. The solution separates into a center-of-mass coordinate and a relative coordinate compared to the center of mass, i.e., $Z_0^\mu = X_0^\mu + Y_0^\mu$. Here $X_0^\mu = x^\mu + v^\mu \tau$ gives the straight-line trajectory of the center of mass. For the relative coordinate, we denote it Y_0^μ and write it in terms of spherical harmonics as

$$Y_0^\mu = \sum_{l>j\geq 0} Y_{l,j}^\mu, \\ Y_{l,j}^\mu = a_w \left(c_{l,j} \beta_x^\mu \cos\left(\frac{\sqrt{l(l+1)}}{a_w} \tau + j\phi\right) + c'_{l,j} \beta_y^\mu \sin\left(\frac{\sqrt{l(l+1)}}{a_w} \tau + j\phi\right) \right) \mathcal{P}_l^j(\cos(\theta)), \quad (5)$$

where $\mathcal{P}_l^j$ is the harmonic Legendre polynomial and β_x, β_y are unit vectors in flat spacetime. It is natural to impose $c_{l,j} = c'_{l,j}$ to keep the rotational symmetry.

A feature of our model is that different modes $Y_{l,j}^\mu$ are associated with the physics of the spinning object we analyze. We will interpret these modes as maps from the worldvolume to solutions on a spinning disk in the target space of the radius $a_{t(l,j)} \equiv a_w c_{l,j}$ (see Fig. 1). For example, $Y_{1,1}^\mu$ is the single covering map to the top and bottom surface of the spinning disk, while $Y_{2,1}^\mu$ is a double covering while still consistent with the starting point that the source of a Kerr black hole is a disk [2]. We interpret the finite size of the spinning disk in the target space as a result of the balance between rotation and the hypersurface inertial tension. [Although interesting, it is beyond the scope of this Letter to consider superpositions of multimodes with different frequencies or wrapping numbers that clash with classical principles. Likewise, we observe that modes such as $Y_{l,0}^\mu$ depend on only the variables (τ, θ) , making them dimension reducing. Given our expectation that the source of a Kerr-like black hole is inherently two dimensional, we will not address such modes either.]

Using the solution for Y_0^μ , the angular momentum due to the rotation around the center of mass is

$$S^{\mu\nu} = \frac{ma_w^2}{4\pi a_w^2} \int_{\mathbb{S}^2} d\theta d\phi \sin(\theta) (Y_{l,j}^\mu \partial_\tau Y_{l,j}^\nu - Y_{l,j}^\nu \partial_\tau Y_{l,j}^\mu) \\ = \frac{\sqrt{l(l+1)}(l+j)!(ma_w)}{(2l+1)(l-j)!} c_{l,j}^2 (\beta_x^\mu \beta_y^\nu - \beta_y^\mu \beta_x^\nu). \quad (6)$$

Identifying (6) with the standard spin tensor $S^{\mu\nu} = m\epsilon^{\mu\nu\rho\lambda} v_\rho a_\lambda$ (we impose the spin supplementary condition, $v \cdot \beta_x = v \cdot \beta_y = 0$ for orthogonal β_x and β_y),

$$S^{\mu\nu} = m|a|(\beta_x^\mu \beta_y^\nu - \beta_y^\mu \beta_x^\nu), \\ v^\mu v^\nu - \eta^{\mu\nu} - \frac{a^\mu a^\nu}{|a|^2} = \beta_x^\mu \beta_x^\nu + \beta_y^\mu \beta_y^\nu. \quad (7)$$

We have spin length related to the radius of the $\mathbb{S}^2$ as follows

$$|a| = \frac{\sqrt{l(l+1)}(l+j)!}{(2l+1)(l-j)!} a_w c_{l,j}^2. \quad (8)$$

Extending this flat-space model to a generic curved background is straightforward. In principle, one can add any covariant coupling that respects the fundamental symmetries of the system. However, we have excluded certain types of terms due to physical reasons. For instance, to avoid introducing a potential term in the flat-space limit that spoils the free equation of motion, we exclude interactions of the form $[(DZ)^2]^n$ for $n \geq 2$, where $(DZ)^2 \equiv \partial_a Z^\mu \partial_b Z^\nu \gamma^{ab} \mathcal{G}_{\mu\nu}(Z)$ and $\mathcal{G}_{\mu\nu}(Z)$ is the *background* metric at the point Z^μ . Explicit polynomial couplings in Z^μ are also excluded in favor of first-order derivative coupling of Z^μ to avoid coupling to the field X^μ .

Our basic assumption is that each geometric point in $\mathbb{S}^2$ couples minimally to a background metric in a simple way, either directly or to the local Riemann tensor $\mathcal{R}_{\mu\nu\rho\lambda}(Z)$. (We do not consider higher-dimensional interactions, i.e., higher derivatives of the metric or higher powers of the curvature.) The worldvolume geometry comes into play via the worldvolume metric γ^{ab} and its respective Riemann tensor $\mathcal{Q}^{abcd}$, with nonvanishing components

$$\mathcal{Q}^{\theta\phi\theta\phi} = -\mathcal{Q}^{\phi\theta\theta\phi} = -\mathcal{Q}^{\theta\phi\phi\theta} = \mathcal{Q}^{\phi\theta\phi\theta} = \frac{-1}{a_w^6 \sin^2(\theta)}.$$

Thus, starting from the minimal action, taking into account the above considerations, we include the following additional couplings, which respect both the target space and the worldvolume diffeomorphism invariance:

$$S = -\frac{m}{8\pi a_w^2} \int_{\mathbb{S}^2 \times \mathbb{R}} d^3\sigma \sqrt{\gamma} \left[(DZ)^2 + 1 + a_w^2 \mathcal{R}_{\mu\nu\rho\lambda}(Z) \right. \\ \times \partial_a Z^\mu \partial_b Z^\rho \partial_c Z^\nu \partial_d Z^\lambda \left(\gamma^{ab} \gamma^{cd} \left(\sum_{j=0}^{\infty} \xi_{2j+1} [(DZ)^2]^j \right) \right. \\ \left. \left. + a_w^2 \mathcal{Q}^{abcd} \left(\sum_{j=0}^{\infty} \xi_{2j+2} [(DZ)^2]^j \right) \right) \right]. \quad (9)$$

To compute the classical three-point one-graviton amplitude using this action, we consider a perturbation around a flat metric $\mathcal{G}_{\mu\nu}(Z_0) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(Z_0)$ with $\kappa = \sqrt{32\pi G_N}$ and identify the amplitude $A_3(v, a, k)$ from the action using

$$\frac{ie^{ik \cdot x}}{(2\pi)^4} 2\pi \delta(mv \cdot k) A_3(v, a, k) = -2h_{\mu\nu}(k) \frac{\delta S}{\delta h_{\mu\nu}(k)} \Big|_{h_{\mu\nu} \rightarrow 0}. \quad (10)$$

In order to evaluate this equation we rewrite $h_{\mu\nu}(Z_0)$ using its Fourier transform $h_{\mu\nu}(k)$

$$h_{\mu\nu}(Z_0) = \int \frac{d^4 k}{(2\pi)^4} h_{\mu\nu}(k) e^{ik \cdot X_0(\tau)} e^{ik \cdot Y_0(\sigma)}, \quad (11)$$

and take the outgoing on-shell graviton field to be a plane wave. [Without loss of generality, we will assume the graviton polarization tensor is a product of two vector polarizations $\epsilon_\mu(k) \epsilon_\nu(k)$]. Working to the leading classical post-Minkowskian order, it is helpful to define

$$A_3^{(l,j)}(v, a, k) \equiv A_3(v, a, k) \Big|_{Y_0^\mu \rightarrow Y_{l,j}^\mu}, \quad (12)$$

and expand the various $e^{ik \cdot Y_0(\sigma)}$ factors everywhere into powers of $[k \cdot Y_0(\sigma)]$. We note that it is straightforward to evaluate the worldvolume integral at a given order in $Y_0(\sigma)$ and then perform the resummation after integration.

Starting from the Polyakov part in Eq. (9), i.e., ignoring all the Riemann tensor couplings, we illustrate the calculation of the (1,1)-mode three-point amplitude at the first post-Minkowskian order as follows:

$$2\pi i \delta(mv \cdot k) A_{3,p}^{(1,1)}(v, a, k) = \frac{\kappa m}{4\pi} \int d\tau e^{ik \cdot v\tau} \int_0^\pi \sin(\theta) d\theta \\ \times \int_0^{2\pi} d\phi \epsilon_\mu \epsilon_\nu (2v^\mu \partial_\tau Y_{1,1}^\nu + v^\mu v^\nu \\ + \partial_a Y_{1,1}^\mu \partial^a Y_{1,1}^\nu) e^{ik \cdot Y_{1,1}}. \quad (13)$$

Any term with an odd power of $Y_{1,1}^\mu$ in the ϕ integral is canceled out (including terms involving $\partial_a Y_{1,1}^\mu$). For even powers, the ϕ integral is readily evaluated. Each sequence-pair tensor structure can be rewritten using the identification provided by Eqs. (6) and (7). Schematically, we have the types of rewritings

$$[\partial_\tau Y_0^\mu Y_0^\nu] \rightarrow [S^{\mu\nu}]$$

$$[Y_0^\mu Y_0^\nu], \quad [\partial_b Y_0^\mu \partial_b Y_0^\nu] \rightarrow \left[v^\mu v^\nu - \eta^{\mu\nu} - \frac{a^\mu a^\nu}{|a|^2} \right], \quad (14)$$

where $b \in \{\tau, \theta, \phi\}$. We observe the presence of the desired kinematic variables. The ϕ -integral value is independent of τ ; this renders the τ integral trivial and gives a factor $2\pi\delta(k \cdot v)$ as expected. For the contribution from $2v^\mu \partial_\tau Y_{1,1}^\nu$, only the odd power terms of Y_0 in the series expansion of $e^{ik \cdot Y_0}$ are nonvanishing. Using Eq. (8), the total contribution from these terms is $i(v \cdot \varepsilon)(k \cdot S \cdot \varepsilon) \mathcal{I}_{P,o}^{(1,1)}$. Carrying out the θ integral, we arrive at

$$\mathcal{I}_{P,o}^{(1,1)} = \frac{3|a|^2 \cosh\left(\frac{a_w c_{1,1} k \cdot a}{|a|}\right)}{(a_w c_{1,1})^2 (k \cdot a)^2} - \frac{3|a|^3 \sinh\left(\frac{a_w c_{1,1} k \cdot a}{|a|}\right)}{(a_w c_{1,1})^3 (k \cdot a)^3}. \quad (15)$$

Considering a radius of the disk in the target space $a_w c_{1,1}$ as a Kerr black hole $|a|$ [recall $c_{1,1} = (3/2\sqrt{2})$ from Eq. (8)], we arrive at functions such as $\cosh(k \cdot a)$ and $\sinh(k \cdot a)$. Similarly, the terms with the common factor $v^\mu v^\nu + \partial_a Y_{1,1}^\mu \partial_b Y_{1,1}^\nu \gamma^{ab}$ contribute only to the even orders in spin. The same procedure yields

$$\mathcal{I}_{P,e}^{(1,1)} = \frac{(17(a \cdot k)^2 + 27) \sinh(a \cdot k) - 27a \cdot k \cosh(a \cdot k)}{8(a \cdot k)^3}. \quad (16)$$

It follows that the amplitude from the Polyakov part is

$$A_{3,P}^{(1,1)}(v, a, k) = -i\kappa(mv \cdot \varepsilon)(mv \cdot \varepsilon \mathcal{I}_{P,e}^{(1,1)} + ik \cdot S \cdot \varepsilon \mathcal{I}_{P,o}^{(1,1)}). \quad (17)$$

We see that the minimal Polyakov action term completely determines $A_3^{(l,j)}$ up to $\mathcal{O}(|a|)$. It follows from the fact that interactions involving $\mathcal{R}_{\mu\nu\rho\sigma}$ contribute only at order $\mathcal{O}(|a|^2)$ and higher, as can be seen from spin counting: $a_w, Y_{1,j}^\mu$ each carry spin power one, while $\partial_a Z_0^\mu$ gives no extra spin power. These contributions from Riemann tensor couplings can be evaluated analogously and also lead to $\cosh(k \cdot a)$ and $\sinh(k \cdot a)$ for the $Y_{1,1}^\mu$ mode in the amplitude. We note that higher-order modes, which are not analyzed in detail here, give rise to distinct entire functions, such as generalized hypergeometric functions (see Supplemental Material [75]).

We find that (9) evaluated for the (1,1)-mode suffices to give the three-point amplitude of any such black hole at the leading post-Minkowskian order. To see this, we write the amplitude following from the generic action in a spin-expanded form up to $\mathcal{O}(|a|^s)$ and find that any amplitude admitting a regular spin expansion of only $(k \cdot a)$ can be

reproduced by a unique set of $\xi_{1,2,\dots,s}$. We have verified this up to $s = 99$.

We only list the final results for the first few terms in (9), i.e., $\xi_{1,2,4}$ to be arbitrary and all other interaction terms vanishing. The amplitude organized by a basis of entire functions then is

$$A_3^{(1,1)}(v, a, k) = (-i\kappa) \sum_{i=1}^5 c_i B_i, \quad (18)$$

where $c_1 = (25/8)\xi_1 + \frac{9}{4}\xi_2 - (9/32)\xi_4$, $c_2 = (17/8) - (79/8)\xi_1 - \frac{9}{4}\xi_2 + (369/16)\xi_4$, $c_3 = (27/32)(24\xi_1 - 81\xi_4 - 4)$, $c_4 = 3\xi_1$, $c_5 = 3 - 9\xi_1$ and basis elements B_i are listed in Table I. The ξ_2, ξ_4 terms only contribute to even powers in spin due to the presence of ϱ^{abcd} . Interestingly, we have discovered that we can recover a new representation of the Kerr black hole three-point amplitude (following from an extended Polyakov action with minimal coupling to the background) by fixing the coupling constants ξ_i to be $\xi_1 = \frac{1}{3}$, $\xi_2 = -(1/3^4)$, $\xi_4 = (4/3^4)$, $\xi_i = 0$ for $i = 3$ or $i > 4$. Thus, we get

$$S_{\text{Kerr}} = -\frac{m}{8\pi a_w^2} \int_{\mathbb{S}_2 \times \mathbb{R}} d^3\sigma \sqrt{\gamma} \left[(\mathcal{D}Z)^2 + 1 \right. \\ \left. + a_w^2 \mathcal{R}_{\mu\nu\rho\lambda}(Z) \partial_a Z^\mu \partial_b Z^\nu \partial_c Z^\rho \partial_d Z^\lambda \right. \\ \left. \times \left(\frac{1}{3} \gamma^{ab} \gamma^{cd} - \frac{1}{3^4} a_w^2 \varrho^{abcd} + \frac{4}{3^4} a_w^2 \varrho^{abcd} (\mathcal{D}Z)^2 \right) \right], \quad (19)$$

which amazingly lands us directly on the three-point Kerr amplitude (see Refs. [8–11,13]),

$$A_{3,\text{Kerr}}^{(1,1)}(v, a, k) = \text{diagram} = -i\kappa(B_1 + B_4) \\ = -i\kappa(mv \cdot \varepsilon) \left(mv \cdot \varepsilon \cosh(k \cdot a) + ik \cdot S \cdot \varepsilon \frac{\sinh(k \cdot a)}{k \cdot a} \right). \quad (20)$$

Singularities in general spinning hypersurface—Here we study the physical effects arising from the Kerr action in Eq. (19) and the corresponding amplitudes. For comparison, we allow the lower-dimensional couplings involving the Riemann tensor to have arbitrary coefficients $\xi_{1,2,4}$. We find that only the Kerr action yields the simple-pole divergence in the bending angle of the scattering with a test particle and maintains an unaltered stable vacuum solution of Z_0^μ , in contrast to an action with generic $\xi_{1,2,4}$. The bending angle is computed at the first post-Minkowskian order with impact parameter b in the “aligned-spin” configuration $v_1 \cdot a = v_2 \cdot a = 0$ (where v_1, v_2 denote the velocities of the spinless test particle and the spinning black hole, respectively). We have

TABLE I. We define $\mathcal{A} \equiv \arctan(|a|/r)$, $\mathcal{A}_b \equiv (|b|/|a|)\arctanh(|a|/|b|)$, $W(r) \equiv [1 + (r^2/|a|^2)]\mathcal{A} - (r/|a|)$, $\mathcal{S} \equiv [\sinh(k \cdot a)/(k \cdot a)]$, $\mathcal{C} \equiv \cosh(k \cdot a)$. In the bending angle and metric, we omit the overall factor $\kappa^2\sqrt{s}/(16\pi|b|(y^2-1))$ and $G_N m$, respectively, where $s = m_1^2 + m_2^2 + 2m_1m_2y$ and $y = v_1 \cdot v_2$. We also have used the definition: $ds_{\text{flat}}^2 \equiv [dt^2 - (|a|^2\cos^2(\theta) + r^2)/((|a|^2 + r^2))dr^2 - (|a|^2\cos^2(\theta) + r^2)d\theta^2 - (|a|^2 + r^2)\sin^2(\theta)d\phi^2]$.

	$B_1 = m^2(v \cdot \varepsilon)^2 \mathcal{C}$	$B_2 = m^2(v \cdot \varepsilon)^2 \mathcal{S}$	$B_3 = m^2(v \cdot \varepsilon)^2 [\partial_{k,a}/(k \cdot a)] \mathcal{S}$	$B_4 = i(mv \cdot \varepsilon)(k \cdot S \cdot \varepsilon) \mathcal{S}$	$B_5 = i(mv \cdot \varepsilon)(k \cdot S \cdot \varepsilon)[\partial_{k,a}/(k \cdot a)] \mathcal{S}$
χ	$[(2y^2 - 1) b ^2/(b ^2 - a ^2)]$	$(2y^2 - 1)\mathcal{A}_b$	$(y^2 - \frac{1}{2})[(b ^2/ a ^2) \times (1 - \mathcal{A}_b) + \mathcal{A}_b]$	$[2y\sqrt{y^2 - 1} a b /(b ^2 - a ^2)]$	$2y\sqrt{y^2 - 1}(\mathcal{A}_b - 1)(b / a)$
$ds_{(1)}^2$	$\{2r/[r^2 + a ^2\cos^2(\theta)]\}$ $[ds_{\text{flat}}^2 - 2dt^2]$	$(2/ a)\mathcal{A}$ $[ds_{\text{flat}}^2 - 2dt^2]$	$[(1 - 3\cos^2(\theta))W(r)/2 a] - (\sin^2(\theta)\mathcal{A}/ a)$ $[ds_{\text{flat}}^2 - 2dt^2]$	$\{4 a rsin^2(\theta)/[r^2 + a ^2\cos^2(\theta)]\}$ $[d\phi dt]$	$2\sin^2(\theta)W(r)$ $[d\phi dt]$

explored a large number of terms in the general action (9) and for generic $\xi_{1,2,4}$, the spin-resummed bending angle at this order is

$$\chi_{\text{IPM}}^{(1,1)}(\xi_1, \xi_2, \xi_4) = \sum_{i=1}^5 c_i \chi_{\text{IPM}}^{(1,1)}(B_i), \quad (21)$$

where $\chi_{\text{IPM}}^{(1,1)}(B_i)$ denotes the contribution from the basis element B_i . We find that the singularities always lie at $|a| = |b|$. For the Kerr action in Eq. (19), only B_1 and B_4 contribute, and the result agrees with [14,15,40,42,76,77]. For generic $\xi_{1,2,4}$, the resulting bending angle has logarithmic divergences. Further analysis of the bending angle involving higher modes is presented in Supplemental Material [75].

The spin-resummed bending angle at the second post-Minkowskian order is presented in Supplemental Material [75]. Similarly, we find that the singularities always lie at $|a| = |b|$. The second post-Minkowskian bending angle for Kerr agrees with [33,42], also free of logarithmic divergences. For generic $\xi_{1,2,4}$, the existence of logarithmic divergences persists.

Another critical direction to consider is the metric, given the hypersurface's interaction with gravity. We start with the Einstein equation expanded up to the first post-Minkowskian order,

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} = 8\pi G_N T^{\mu\nu} = 8\pi G_N \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}(x)}, \quad (22)$$

where

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa^2 \mathfrak{h}_{1\mu\nu}(x) + \mathcal{O}(\kappa^4)$$

$$T^{\mu\nu}(x) = T_0^{\mu\nu}(x) + \mathcal{O}(\kappa).$$

At this order, $\mathfrak{h}_{1\mu\nu}$ contains only the leading-order stationary contribution. Fourier transforming Eq. (22) to momentum space (see Refs. [78–80] for spinless examples), we find it is directly related to $A_3^{(1,1)}$ [81],

$$\tilde{\mathfrak{h}}_1^{\mu\nu}(q) = \frac{\tilde{T}_0^{\mu\nu}(q) - \frac{1}{2}\tilde{T}_0^\rho{}_\rho(q)\eta^{\mu\nu}}{2q^2}$$

$$= \frac{2\pi\delta(mv \cdot q)}{2q^2} \sum_{\text{spin sum}} \frac{A_3^{(1,1)}(v, a, q)}{-i\kappa} \varepsilon^{*\mu} \varepsilon^{*\nu}. \quad (23)$$

Here we work in the de Donder gauge, and ε^* denotes the conjugate of ε . The first post-Minkowskian stationary metric in position space is then given by

$$\mathfrak{h}_1^{\mu\nu}(x) = \int \frac{d^4q}{(2\pi)^4} e^{-iq \cdot x} \tilde{\mathfrak{h}}_1^{\mu\nu}(q). \quad (24)$$

Explicit expressions are listed in Table I. The metric always satisfies the Einstein equation above for any $\xi_{1,2,4}$.

As shown in Table I, entire functions B_1 , B_4 that are present in the Kerr amplitude give rise to the ring singularity in the metric, while the rest [B_2 , B_3 , B_5 in Eq. (18)] contribute only to the regular part of the metric at the first post-Minkowskian order. Interestingly, the generated Kerr metric $\mathfrak{h}_1^{\mu\nu}$ (only $B_1 + B_4$ in Table I) vanishes as $r \rightarrow 0$ if the ring singularity is appropriately regularized, suggesting the flat-space solution of Z_0^μ remains stable and unaffected by $\mathfrak{h}_1^{\mu\nu}$ at this order. For a more general action, $\mathfrak{h}_1^{\mu\nu}$ receives contributions from B_2 , B_3 , B_5 , which do not vanish as $r \rightarrow 0$ and can modify the flat-space solution. This observation may corroborate the stability of the Kerr black hole from a novel viewpoint.

Conclusion and outlook—This Letter takes a first step in exploring a new hypersurface model, in which the action preserves diffeomorphism invariance across the worldvolume and the target spacetime, yielding solutions that rely exclusively on variables such as velocity, classical spin, and mass. We note that the spin tensor arises naturally from the rotational angular momentum of the Kerr disk [2] around its center of mass (in contrast, the worldsheet action in [18] introduces the spin degree of freedom extrinsically via the Newman-Janis shift). As we have observed, this framework effectively encapsulates the external states necessary for an in-depth characterization of the Kerr black hole, offering interesting avenues for further study. The pivotal result of our analysis is the successful generation of the classical

three-point Kerr amplitude to arbitrary spin order directly from the model's equation of motion. We hope that this constructive result will enhance the theoretical precision of the computational analysis of observations of binary events.

For instance, by expanding in action's relative coordinate, we can distill the action into a worldline formulation comprising an infinite number of terms beyond minimal coupling. While our hypersurface action involves only metric and Riemann tensor couplings, derivatives of the Riemann tensor and multiple Riemann tensor couplings naturally emerge from this expansion, opening intriguing new avenues for worldline computations.

Within the framework established for the three-point amplitude, it is crucial to investigate how to use this action to derive high-multiplicity classical amplitudes by solving the corresponding extended equations of motion. The major task is to solve both the hypersurface coordinate equation of motion and the Einstein equation Eq. (22). The above equation of motion can be solved perturbatively in both spin parameter $|a|$ and gravity coupling κ order by order. One can thus extract the high-multiplicity amplitude perturbatively from the outgoing gravitational wave solution in the presence of multiple incoming plane waves or partial waves (see, i.e., Ref. [61] for related discussions on Compton amplitude). The advantage of the hypersurface equation of motion is that it obviates the need for a multipole expansion or the introduction of multipole moments, while still yielding an amplitude that depends only on the spin and mass of the black hole. We leave a detailed discussion in an upcoming publication (see also the illustrative discussions in Supplemental Material [75]). To verify the obtained amplitude, we examine the factorization behavior and use the model's contact terms to gain new insights, for example, in the context of existing Compton amplitude bootstrap techniques [34,38,44].

Given that our framework encompasses intricate characteristics of extended bodies, it also facilitates the extraction of the multipole expansion from the energy-momentum tensor (see, i.e., Ref. [61] using disk source). It opens a new avenue for analytically addressing the Mathisson-Papapetrou-Dixon equations within the post-Minkowskian expansion framework and for conducting numerical analyses in scenarios characterized by strong gravitational coupling. Such investigations go beyond the scope of this presentation. Still, they are vital as they will likely serve as future cross-validation of the theoretical framework we envision for higher-multiplicity amplitudes.

Besides identifying a correspondence with the amplitude associated with a Kerr black hole, we emphasize that the proposed hypersurface model for spinning black holes extends to a broad class of interactions in arbitrary dimensions, potentially leading to a new understanding of the complex behavior of extended bodies that could be relevant for future astrophysical phenomenology and precision observations.

Acknowledgments—We thank Andreas Brandhuber, Paolo Di Vecchia, Yutin Huang, Jung-Wook Kim, and Marcos Skowronek for discussions. The work of N. E. J. B.-B. and G. C. was supported in part by DFF Grant No. 1026-00077B, and by The Center of Gravity, which is a Center of Excellence funded by the Danish National Research Foundation under Grant No. 184. G. C. has also received funding from the European Union Horizon 2020 research and innovation program under the Marie Skłodowska-Curie Grant Agreement No. 847523 INTERACTIONS, as well as support in part by VILLUM Foundation (Grant No. VIL37766). Guangzhou Elite Project supported C. S. through Grant No. JY202211. T.W. was supported by the National Research Foundation of Korea (NRF) Grant No. NRF RS-2024-00351197.

Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] R. P. Kerr, *Phys. Rev. Lett.* **11**, 237 (1963).
 - [2] W. Israel, *Phys. Rev. D* **2**, 641 (1970).
 - [3] M. Mathisson, *Acta Phys. Polon.* **6**, 163 (1937).
 - [4] A. Papapetrou, *Proc. R. Soc. A* **209**, 248 (1951).
 - [5] W. G. Dixon, *Proc. R. Soc. A* **314**, 499 (1970).
 - [6] W. G. Dixon, *Proc. R. Soc. A* **319**, 509 (1970).
 - [7] W. G. Dixon, *Phil. Trans. R. Soc. A* **277**, 59 (1974).
 - [8] J. Vines, *Classical Quantum Gravity* **35**, 084002 (2018).
 - [9] N. Arkani-Hamed, T.-C. Huang, and Y.-t. Huang, *J. High Energy Phys.* **11** (2021) 070.
 - [10] A. Guevara, A. Ochirov, and J. Vines, *J. High Energy Phys.* **09** (2019) 056.
 - [11] M.-Z. Chung, Y.-T. Huang, J.-W. Kim, and S. Lee, *J. High Energy Phys.* **04** (2019) 156.
 - [12] D. A. Kosower, B. Maybee, and D. O'Connell, *J. High Energy Phys.* **02** (2019) 137.
 - [13] W.-M. Chen, M.-Z. Chung, Y.-t. Huang, and J.-W. Kim, *J. High Energy Phys.* **08** (2022) 148.
 - [14] A. Guevara, A. Ochirov, and J. Vines, *Phys. Rev. D* **100**, 104024 (2019).
 - [15] N. Arkani-Hamed, Y.-t. Huang, and D. O'Connell, *J. High Energy Phys.* **01** (2020) 046.
 - [16] R. Aoude, K. Haddad, and A. Helset, *J. High Energy Phys.* **05** (2020) 051.
 - [17] M.-Z. Chung, Y.-t. Huang, J.-W. Kim, and S. Lee, *J. High Energy Phys.* **05** (2020) 105.
 - [18] A. Guevara, B. Maybee, A. Ochirov, D. O'Connell, and J. Vines, *J. High Energy Phys.* **03** (2021) 201.
 - [19] W.-M. Chen, M.-Z. Chung, Y.-t. Huang, and J.-W. Kim, *J. High Energy Phys.* **08** (2022) 148.
 - [20] A. Brandhuber, G. Chen, G. Travaglini, and C. Wen, *J. High Energy Phys.* **10** (2021) 118.
 - [21] D. Kosmopoulos and A. Luna, *J. High Energy Phys.* **07** (2021) 037.
 - [22] M. Chiodaroli, H. Johansson, and P. Pichini, *J. High Energy Phys.* **02** (2022) 156.

- [23] Y. F. Bautista, A. Guevara, C. Kavanagh, and J. Vines, *J. High Energy Phys.* **03** (2023) 136.
- [24] L. Cangemi, M. Chiodaroli, H. Johansson, A. Ochirov, P. Pichini, and E. Skvortsov, *Phys. Rev. Lett.* **131**, 221401 (2023).
- [25] A. Ochirov and E. Skvortsov, *Phys. Rev. Lett.* **129**, 241601 (2022).
- [26] P. H. Damgaard, K. Haddad, and A. Helset, *J. High Energy Phys.* **11** (2019) 070.
- [27] Z. Bern, A. Luna, R. Roiban, C.-H. Shen, and M. Zeng, *Phys. Rev. D* **104**, 065014 (2021).
- [28] J. Vines, J. Steinhoff, and A. Buonanno, *Phys. Rev. D* **99**, 064054 (2019).
- [29] B. Maybee, D. O'Connell, and J. Vines, *J. High Energy Phys.* **12** (2019) 156.
- [30] K. Haddad, *Phys. Rev. D* **105**, 026004 (2022).
- [31] Z. Bern, D. Kosmopoulos, A. Luna, R. Roiban, and F. Teng, *Phys. Rev. Lett.* **130**, 201402 (2023).
- [32] R. Aoude, K. Haddad, and A. Helset, *J. High Energy Phys.* **07** (2022) 072.
- [33] P. H. Damgaard, J. Hoogeveen, A. Luna, and J. Vines, *Phys. Rev. D* **106**, 124030 (2022).
- [34] N. E. J. Bjerrum-Bohr, G. Chen, and M. Skowronek, *J. High Energy Phys.* **06** (2023) 170.
- [35] M. Bianchi, C. Gambino, and F. Riccioni, *J. High Energy Phys.* **08** (2023) 188.
- [36] Y. F. Bautista, *Phys. Rev. D* **108**, 084036 (2023).
- [37] W.-M. Chen, M.-Z. Chung, Y.-t. Huang, and J.-W. Kim, *J. High Energy Phys.* **12** (2022) 058.
- [38] N. E. J. Bjerrum-Bohr, G. Chen, and M. Skowronek, *Phys. Rev. Lett.* **132**, 191603 (2024).
- [39] S. De Angelis, P. P. Novichkov, and R. Gonzo, *Phys. Rev. D* **110**, L041502 (2024).
- [40] A. Brandhuber, G. R. Brown, G. Chen, J. Gowdy, and G. Travaglini, *J. High Energy Phys.* **02** (2024) 026.
- [41] G. Chen and T. Wang, *J. High Energy Phys.* **12** (2025) 213.
- [42] G. Chen, J.-W. Kim, and T. Wang, *Phys. Rev. D* **111**, L021701 (2025).
- [43] A. Brandhuber, G. R. Brown, G. Chen, G. Travaglini, and P. Vives Matasan, *J. High Energy Phys.* **12** (2024) 039.
- [44] G. Chen and N. E. J. Bjerrum-Bohr, *Proc. Sci. ICHEP2024* (2025) 793.
- [45] L. Cangemi, M. Chiodaroli, H. Johansson, A. Ochirov, P. Pichini, and E. Skvortsov, *Phys. Rev. Lett.* **133**, 071601 (2024).
- [46] N. E. J. Bjerrum-Bohr, G. Chen, Y. Miao, and M. Skowronek, *J. High Energy Phys.* **07** (2024) 242.
- [47] L. Bohnenblust, L. Cangemi, H. Johansson, and P. Pichini, *J. High Energy Phys.* **07** (2025) 261.
- [48] Y. F. Bautista, Y.-T. Huang, and J.-W. Kim, *Phys. Rev. D* **111**, 044043 (2025).
- [49] D. Akpinar, F. Febres Cordero, M. Kraus, A. Smirnov, and M. Zeng, *Phys. Rev. Lett.* **135**, 041602 (2025).
- [50] A. Brandhuber, G. R. Brown, P. Pichini, G. Travaglini, and P. Vives Matasan, *J. High Energy Phys.* **08** (2024) 188.
- [51] M. Levi and J. Steinhoff, *J. High Energy Phys.* **09** (2015) 219.
- [52] G. Mogull, J. Plefka, and J. Steinhoff, *J. High Energy Phys.* **02** (2021) 048.
- [53] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, *J. High Energy Phys.* **01** (2022) 027.
- [54] F. Comberiati and L. de la Cruz, *J. High Energy Phys.* **03** (2023) 068.
- [55] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, *Phys. Rev. Lett.* **126**, 201103 (2021).
- [56] Z. Liu, R. A. Porto, and Z. Yang, *J. High Energy Phys.* **06** (2021) 012.
- [57] G. U. Jakobsen and G. Mogull, *Phys. Rev. Lett.* **128**, 141102 (2022).
- [58] G. Menezes and M. Sergola, *J. High Energy Phys.* **10** (2022) 105.
- [59] F. Febres Cordero, M. Kraus, G. Lin, M. S. Ruf, and M. Zeng, *Phys. Rev. Lett.* **130**, 021601 (2023).
- [60] F. Alessio and P. Di Vecchia, *Phys. Lett. B* **832**, 137258 (2022).
- [61] T. Scheopner and J. Vines, *J. High Energy Phys.* **12** (2024) 060.
- [62] M. Ben-Shahar, *J. High Energy Phys.* **03** (2024) 108.
- [63] J.-W. Kim and J. Steinhoff, *J. High Energy Phys.* **07** (2023) 042.
- [64] G. U. Jakobsen, G. Mogull, J. Plefka, B. Sauer, and Y. Xu, *Phys. Rev. Lett.* **131**, 151401 (2023).
- [65] R. Gonzo and C. Shi, *Phys. Rev. Lett.* **133**, 221401 (2024).
- [66] K. Haddad, G. U. Jakobsen, G. Mogull, and J. Plefka, *J. High Energy Phys.* **02** (2025) 019.
- [67] G. Veneziano, *Nuovo Cimento A* **57**, 190 (1968).
- [68] L. Brink, P. Di Vecchia, and P. S. Howe, *Phys. Lett.* **65B**, 471 (1976).
- [69] S. Deser and B. Zumino, *Phys. Lett.* **65B**, 369 (1976).
- [70] A. M. Polyakov, *Phys. Lett.* **103B**, 207 (1981).
- [71] J. Polchinski, in *Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96): Fields, Strings, and Duality* (1996), pp. 293–356, [arXiv:hep-th/9611050](https://arxiv.org/abs/hep-th/9611050).
- [72] R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, *Phys. Rev. Lett.* **102**, 191301 (2009).
- [73] N. E. J. Bjerrum-Bohr and C. B. Jepsen, *Phys. Rev. D* **110**, L081902 (2024).
- [74] P. Charalambous, S. Dubovsky, and M. M. Ivanov, *J. High Energy Phys.* **06** (2025) 180.
- [75] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/k5g2-q38q> for discussions on gravitational Compton amplitudes and alternative worldvolume metrics, and explicit examples of computed bending angles for various models.
- [76] E. T. Newman and A. I. Janis, *J. Math. Phys. (N.Y.)* **6**, 915 (1965).
- [77] J.-H. Kim, J.-W. Kim, and S. Lee, *J. High Energy Phys.* **08** (2024) 080.
- [78] S. Mougiakakos and P. Vanhove, *Phys. Rev. D* **103**, 026001 (2021).
- [79] P. H. Damgaard and K. Lee, *Phys. Rev. Lett.* **132**, 251603 (2024).
- [80] S. Mougiakakos and P. Vanhove, *Phys. Rev. Lett.* **133**, 111601 (2024).
- [81] We thank Yuexiang Zhang for discussions on this point and for sharing his calculations for cross-checking.