

Programming Coherent and Quantum Light with a Free-Electron Wavepacket

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The pursuit of programmable light sources with high coherence and spectral purity hinges on establishing a precise set of phase relationships in light-matter interactions. Here, we demonstrate that the quadratic dispersion of a freely propagating electron wave packet serves as a programmable quantum medium. Prepared in a coherent momentum-state ladder via a single laser interaction, the electron subsequently undergoes deterministic phase evolution during free propagation—an intrinsic process that compiles its quantum state into two distinct output channels. This mechanism, quantified by a quantum bunching factor, enables (i) Talbot-resonant bunching, where the electron density self-structures into subcycle combs with tunable harmonic selectivity, and (ii) coherent phase transfer of the programmed quadratic phase to light, generating nonclassical photon states such as multicomponent Schrödinger cat states via energy-resolved electron readout. This quadratic-phase programming provides a drift-section platform for Talbot-selected coherent short-wavelength emission and nonclassical light synthesis based on electron self-interference.

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The quest for intense, coherent radiation and complex photonic quantum states drives modern ultrafast science and quantum technology. This pursuit has spurred diverse physical mechanisms, from high-harmonic generation [1–4] and free-electron lasers (FELs) [5,6] to their envisioned quantum counterparts [7–9]. Underpinning these approaches is a common physical requirement: the establishment of a well-defined periodic coherence, whether manifested as electron density modulation or as structured phase-space coherence in the photonic field.

In advanced seeded FELs, including high-gain harmonic generation (HG) [10] and echo-enabled harmonic generation (EEHG) [11,12], such modulation is imposed externally through cascaded laser modulators and magnetic chicanes. While powerful, these approaches couple modulation depth to spectral bandwidth and achieve high harmonics through multistage phase-space manipulation,

at the cost of increased complexity and susceptibility to collective instabilities. A fundamentally different route has been envisioned in the quantum FEL, where spectral purity arises from quantum-recoil-dominated gain, but this demands an exceptionally narrow initial electron energy spread [8,13], trading operational robustness for coherence. This raises a central question: can high-purity, programmable structuring arise intrinsically from the coherent evolution of a quantum wave packet itself, without external modulators or stringent state preparation?

While recent studies have exploited modulated free electrons for resonant quantum interactions [14] and nonclassical light generation [15], the propagation-induced phase has remained largely unexplored as a programmable resource. Here, we harness this propagation-induced quadratic phase to control both harmonic-selective electron bunching and quantum-state synthesis. This mechanism is rooted in the Talbot effect, a fundamental wave phenomenon in which quadratic dispersion gives rise to periodic self-imaging and fine wave packet structures [16]. The Talbot effect has recently been demonstrated for electron wave packets [17–19], primarily in the context of attosecond pulse-train formation [20–22]. In this Letter, we leverage this underutilized degree of freedom by using Talbot evolution as a built-in compiler for programmable quantum-state synthesis.

As shown in Fig. 1(a), the electron is first prepared in a coherent momentum-state ladder via the interaction of

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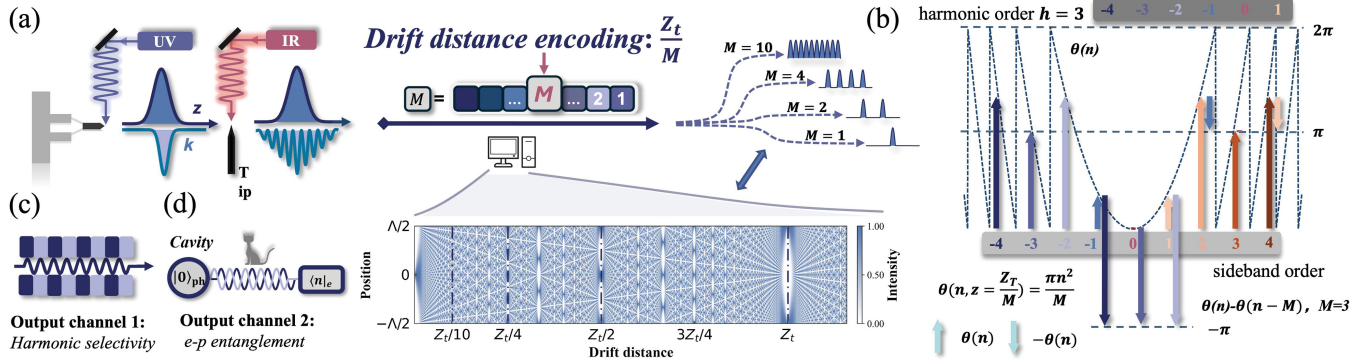


FIG. 1. The programmable free-electron wave packet phase-programming scheme. (a) An electron wave packet interacts with an optical near-field via PINEM, forming a coherent momentum ladder that undergoes quadratic-phase evolution during free propagation, manifesting the Talbot self-imaging effect (inset). (b) Phase-matching picture for harmonic order $h = 3$. At the fractional Talbot distance $z = Z_T/M$, the quadratic phase $\theta(n, z) = \pi n^2/M$ causes sidebands separated by M to acquire a fixed relative phase modulo 2π , leading to the constructive addition of the corresponding bunching harmonic while suppressing off resonant orders. (c) Output channel 1: the Talbot-microbunched electron wave packet acts as a harmonic-selective source or seed, with dominant radiation at the Talbot-selected harmonics. (d) Output channel 2: the quadratic phase is coherently transferred to an optical mode via energy-resolved projective readout, generating multicomponent Schrödinger cat states.

photon-induced near-field electron microscopy (PINEM) [20,22–27]. Its subsequent free propagation, governed solely by quadratic dispersion, programs this initial state into two distinct output channels. First, the electron density self-structures into attosecond pulse trains [20,24] with Talbot-resonant harmonic selectivity, enabling high-harmonic electron bunching without external seeding or filtering. This constitutes a quantum analog to beam microbunching, yet emerges from intrawave packet interference rather than collective amplification. Second, coherent phase transfer to a quantum cavity mode, conditioned on energy-resolved projective readout, generates multicomponent optical Schrödinger cat states [28,29]. Both the harmonic order of the attosecond emission and the number of cat-state components are controlled by a single classical parameter: the free-propagation distance. This work establishes a unified testbed for Talbot-selected coherent short-wavelength emission and measurement-conditioned nonclassical-light synthesis.

Quadratic-phase programming—The quadratic dispersion of a free electron wave packet provides the essential resource for this programmable platform. When prepared in a coherent superposition of discrete momentum states, the electron accumulates a deterministic quadratic phase during free propagation—a process that functions as a built-in compiler, controlled solely by the drift distance z . Instead of relying on external modulators or material nonlinearities, our approach leverages this intrinsic yet underutilized resource to sculpt both the density profile of the electron and the quantum state of an interacting radiation field—an approach we term *quadratic-phase programming*.

The programming protocol begins by encoding a coherent momentum-state ladder onto an electron wave packet. As sketched in Fig. 1(a), this can be realized through the

PINEM interaction [20,23]. In this process, the electron traverses an optical near field driven at frequency ω with vacuum wavelength $\lambda_0 = 2\pi c/\omega$, and coherently absorbs or emits integer numbers of photons. This interaction produces a phase-coherent ladder of photon-spaced energy sidebands, which can be written in momentum space as $\psi(z, t=0) = \sum_{l=-N}^N c_l e^{i(p_0 + lh)z/\hbar}$. Here p_0 is the central momentum and c_l is the complex amplitude of the l th momentum sideband. The sideband spacing $\hbar k$, with $k = \omega/v_0$, defines the natural spatial period of the electron modulation, $\Lambda = 2\pi/k = 2\pi v_0/\omega = \beta\lambda_0$, which is shorter than the vacuum optical wavelength $\lambda_0 = 2\pi c/\omega$ by the factor $\beta = v_0/c$.

After this preparation step, the electron propagates freely over a distance $z = v_0 t$. Free propagation does not require any additional optical element; instead, the electron’s own quadratic dispersion imprints a sideband-dependent phase. Since the energy of the l th sideband is approximately $E_l \simeq E_0 + v_0 \hbar k + (l \hbar k)^2 / (2m_e \gamma^3)$, the relative phase between two sidebands separated by h quanta is

$$\Delta\phi_{l,h}(z) = \frac{(E_{l+h} - E_l)z}{\hbar v_0} = \frac{2\pi l h z}{Z_T} + \phi_h, \quad (1)$$

where $Z_T = 2\pi m_e v_0^3 \gamma^3 / \hbar \omega^2$ is the electron Talbot distance, and ϕ_h collects all terms that depend only on h but not on l (see Supplemental Material, Sec. I [30]).

Equation (1) gives the central physical picture. Each density harmonic h is formed by the interference of many sideband pairs $(l, l+h)$. The l -independent phase ϕ_h only shifts the overall phase of that harmonic, whereas the term $2\pi l h z / Z_T$ controls whether the different sideband pairs add in phase or cancel each other. Therefore, the drift distance acts as a built-in momentum-space phase mask: by

choosing z/Z_T , one programs the phase relation among all sideband-pair interference paths. In this way, ordinary free-space propagation compiles the initially prepared momentum comb into a spatially bunched electron wave packet with selected harmonic content.

To quantify this programming, we employ the bunching factor $b_f(h)$. In classical beam physics, this factor characterizes the amplitude of the h th spatial Fourier harmonic of the ensemble electron density modulation [10,11]. In the quantum description, it extends naturally to a free-electron wave packet and captures the periodic order in its probabilistic density: $b_f(h) = |\langle \psi | e^{ihkz} | \psi \rangle|$ [41,42], with ψ denoting the electron wave function. This bunching factor directly captures the degree of spatiotemporal coherence that a quantum electron wave packet can impart onto radiation at the corresponding wavelength, namely a free-space wavelength λ_0/h in a phase-matched radiator, while the corresponding electron-density period is $\Lambda/h = \beta\lambda_0/h$.

This quantum bunching, together with the programmed phase in Eq. (1), captures the basic compilation mechanism:

$$\begin{aligned} b_f(h, z) &= |\langle \psi(z) | e^{ihkz} | \psi(z) \rangle| \\ &= \left| \sum_{l, l'} c_l^* c_{l'} e^{i(E_l - E_{l'})z/\hbar v_0} \langle l | e^{ihkz} | l' \rangle \right| \\ &= \left| \sum_l c_l^* c_{l+h} e^{i(E_l - E_{l+h})z/\hbar v_0} \right| \\ &= \left| \sum_l c_l^* c_{l+h} e^{i2\pi l h z / Z_T} \right|. \end{aligned} \quad (2)$$

This expression makes the Talbot selection rule explicit. At a fractional Talbot distance $z = Z_T/M$, the phase factor becomes $e^{i2\pi l h / M}$. Therefore, for $h = qM$ with $q \in \mathbb{Z}$, all sideband-pair contributions are phase matched and add constructively, giving a pronounced bunching peak. For other harmonics, the phase winds across the sideband index l , leading to cancellation. Thus, by choosing the drift distance, the free-electron wave packet is compiled into a harmonic-selective density grating. This provides two output channels: tailored electron bunching with intrinsic spectral filtering [channel 1, Fig. 1(c)] and nonclassical photonic-state synthesis through coherent interaction [channel 2, Fig. 1(d)].

Talbot-enhanced high-harmonic bunching—Building on this programmable scheme, we first demonstrate how the electron’s own quantum state can be sculpted into a radiation-relevant density modulation. This channel realizes a high-contrast, harmonic-selective attosecond electron bunching structure, providing a quantum analog of the classically microbunched beams used in advanced free-electron-radiation schemes. Unlike multistage classical approaches, where harmonic generation and selection rely

on external beamline elements, dispersive sections, or gain-based filtering, *Talbot-enhanced high-harmonic bunching* (TEHHB) arises directly from intrawave packet interference in a single programmed electron state.

As illustrated in Fig. 1(b), the resonance structure of Eq. (2) leads to a simple Talbot selection rule. At fractional Talbot distances $z = Z_T/M$ [43], only harmonic orders satisfying $h = qM$ rephase constructively, while off resonant harmonics are suppressed. Consequently, when the bunched electron subsequently couples to a phase-matched radiation channel, the dominant emission is expected at the Talbot-selected harmonics. The fundamental selected radiation channel corresponds to $h = M$, namely $\lambda_{\text{emit}} = \lambda_0/M$, with higher selected channels at $\lambda_{\text{emit}} = \lambda_0/(qM)$ for integer q . Thus, the drift distance provides a direct and programmable route from free-electron quadratic-phase evolution to high-harmonic bunching and coherent radiation.

Figure 2 shows the spatiotemporal evolution of the programmed electron wave packet, calculated using experimentally relevant parameters: an electron energy $E_e = 200$ keV and a photon-sideband spacing of 1.55 eV, corresponding to a Talbot distance of $Z_T \approx 238.8$ mm. The initial state comprises approximately $N \approx 150$ populated sidebands [25], with $|c_l|^2$ following a Gaussian envelope [28]. The free propagation is sampled over the range $0 \leq z \leq 1.12Z_T$. As shown in Fig. 2(a), the electron probability density exhibits the characteristic periodic revivals associated with the Talbot effect, and its evolving periodicity is captured by $b_f(h, z)$ [Fig. 2(b)]. At fractional distances $z = Z_T/M$, the density profiles partition into M sharply localized attosecond peaks per electron modulation period Λ , forming the interference pattern known as “quantum carpet” [43,44], as shown in the representative frames of Fig. 2(c). For the parameters used here, the $M = 30$ case corresponds to $z = Z_T/30 = 7.96$ mm. The same Talbot index selects the primary radiation wavelength $\lambda_{\text{emit}} = \lambda_0/30 \approx 26.7$ nm, along with its associated harmonic family at integer multiples of this order. Correspondingly, the bunching spectrum [Fig. 2(d)] becomes highly selective, exhibiting peaks only at harmonic orders that are integer multiples of M , while off resonant orders are suppressed—consistent with the Talbot phase matching condition in Eqs. (1) and (2).

The peak width $\Delta x \sim \Lambda/N$ reaches the attosecond scale for these parameters and is ultimately bounded by the Fourier constraints set by the populated sideband bandwidth $\Delta p \sim N\hbar k$. It demonstrates that the coherent electron wave packet can deterministically self-organize into a periodic train of attosecond bunches emerging purely from intrinsic single-particle quantum interference rather than collective amplification.

The origin of this sharp spectral selection is illustrated in Fig. 3, which examines the momentum-space correlation $\psi^*(p)\psi(p + \hbar k)$ that constitutes the summation in

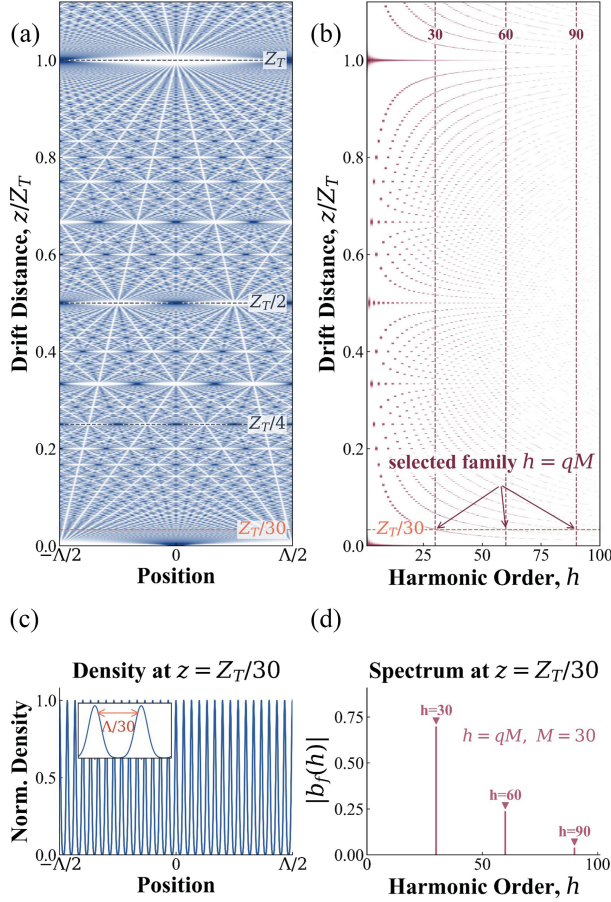


FIG. 2. Intrinsic Talbot self-imaging and harmonic bunching of a free-electron wave packet. The position coordinate is the comoving longitudinal coordinate normalized by the electron modulation period Λ . (a) Spatiotemporal evolution of electron density showing periodic revivals. Fractional Talbot distances ($z = Z_T/M$) yield subcycle foci separated by Λ/M ; the full revival appears at $z = Z_T$. (b) Bunching factor evolution with period Z_T . (c),(d) Density distribution and bunching spectrum at $z = Z_T/30$, demonstrating subcycle localization and selective enhancement of harmonics spaced by order 30.

Eq. (2). For an off resonant harmonic (e.g., $h = 2$ at $z = Z_T/30$), the programmed phase varies rapidly with sideband index l , so contributions from different sideband pairs accumulate mismatched phases and interference destructively [Figs. 3(a) and 3(b)]. In contrast, for the resonant harmonic ($h = 30$), the phase is uniform across all dominant contributions, with $\exp(i2\pi l) \equiv 1$, yielding coherent alignment and a pronounced peak in b_f [Figs. 3(c) and 3(d)]. This comparison shows that harmonic selectivity emerges precisely when sideband pairs satisfy the Talbot phase-matching condition, directly linking the observed spectral purity to the deterministic quantum interference (Fig. 2).

This mechanism differs fundamentally from classical EEHG and HGHG schemes, where collective phase-space manipulation first generates multiple harmonics and the

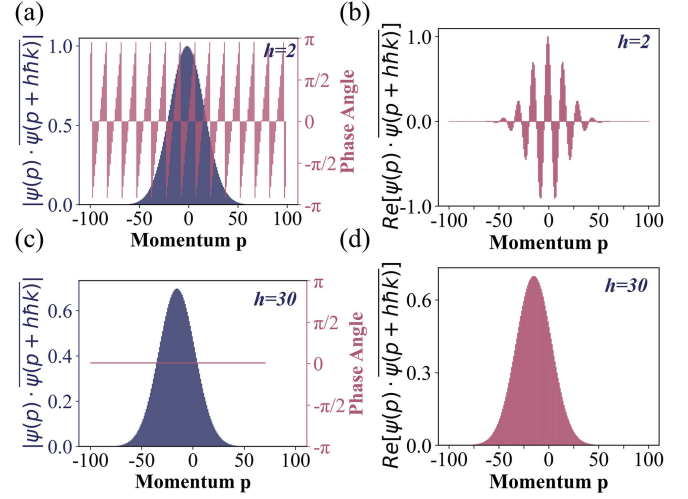


FIG. 3. Momentum-space origin of selective harmonic bunching. (a),(b) At off resonant order $h = 2$, phase oscillations among momentum sidebands cause destructive interference, suppressing bunching. (c),(d) At resonant order $h = 30$, phase alignment ($\Delta\phi \approx 0$ modulo 2π) enables constructive interference and pronounced enhancement. This phase-matching condition underpins the order-selective dynamics observed in Fig. 2.

downstream radiator or gain bandwidth subsequently selects the target one. In TEHHB, by contrast, harmonic selectivity is embedded at the quantum-state-preparation stage: deterministic Talbot evolution rephases the selected harmonic family while suppressing off resonant orders through destructive interference. This single-electron interference mechanism provides spectrally selective bunching before radiation or gain narrowing [30].

Synthesizing optical cat states—Quadratic-phase programming is not restricted to structuring the electron wave packet itself; rather, it represents a coherent phase resource that can be transferred to external degrees of freedom. Crucially, the phase mask $\phi_{l,h} = 2\pi lh_z/Z_T$ can be coherently transferred from the momentum-space lattice onto a radiation field via interaction and measurement. This idea casts the electron not only as a compiler of its own probability density, but also as a programmable phase mediator that extends the programming paradigm from wave packet self-imaging to nonclassical photon state synthesis.

We now examine the second output channel of this programmable scheme: using the modulated electron to synthesize nonclassical states of light. The electron is coupled to a single quantized optical mode (e.g., a high- Q cavity initially in vacuum), and energy-resolved projective readout of the outgoing electron conditionally transfers the programmed quadratic phase to the photon-number amplitudes of the light field, as illustrated in Fig. 1(d). The energy-conserving scattering operator can be written as $\hat{S} = \exp(g\hat{b}\hat{a}^\dagger - g^*\hat{b}^\dagger\hat{a})$ [18,28,45], where $\hat{a}^\dagger$ ($\hat{a}$) creates (annihilates) a cavity photon mode, $\hat{b}$ ($\hat{b}^\dagger$) lowers (raises) the electron energy ladder by one

sideband, and g is the dimensionless electron-photon coupling strength.

For a single incoming electron sideband and the optical vacuum, the action of $\hat{S}$ gives

$$\hat{S}|n, 0\rangle = \sum_{m=0}^{\infty} e^{-|g|^2/2} \frac{g^m}{\sqrt{m!}} |n - m, m\rangle,$$

which explicitly shows that creating m photons is accompanied by lowering the electron energy by $m\hbar\omega$. For an incident electron wave packet carrying the programmed Talbot phase, linear superposition of the above process yields the joint electron-photon state

$$|\Psi_f\rangle = \sum_{n=-\infty}^{\infty} |n\rangle_e \otimes \sum_{m=0}^{\infty} c_{n+m} e^{i\pi(n+m)^2/M} \times e^{-|g|^2/2} \frac{g^m}{\sqrt{m!}} |m\rangle_{\text{ph}}.$$

The index structure of this joint state indicates that the component with photon number m in the outgoing electron sideband n originates from the incoming electron component $n + m$. Measuring the outgoing electron sideband n' therefore selects the corresponding row of the joint state and maps the precompiled Talbot phase onto the optical Fock-state amplitudes. Up to normalization, the conditional photon state reads as

$$|\phi_{n'}\rangle = \mathcal{N}_{n'} \sum_{m=0}^{\infty} c_{n'+m} e^{i\pi(n'+m)^2/M} e^{-|g|^2/2} \frac{g^m}{\sqrt{m!}} |m\rangle_{\text{ph}} \quad (3)$$

where m is the photon number in the Fock basis $|m\rangle_{\text{ph}}$, $c_{n'+m}$ is the inherited electron-sideband amplitude, and $\mathcal{N}_{n'}$ normalizes the conditioned state. Physically, Eq. (3) characterizes measurement-conditioned Talbot phase transfer: generating m photons necessitates an electron transition from sideband $n' + m$ to n' . The m -photon Fock component thereby inherits the amplitude $c_{n'+m}$ and the programmed quadratic phase $e^{i\pi(n'+m)^2/M}$. While the coupling strength g dictates the multiphoton exchange probability, the inherited phase term drives the quantum interference between Fock components. Because different values of the detected sideband index n' act as selective filters that project the electron wave packet onto distinct, phase-shifted optical states, the protocol is therefore measurement conditioned rather than based on probabilistic multiphoton heralding [46,47].

The conditional state $|\phi_{n'}\rangle$ constitutes a coherent superposition of multiple phase-rotated components in optical phase space, i.e., a programmable multicomponent Schrödinger cat state. The programming parameter M , set by the drift distance $z = Z_T/M$, determines the number of distinct coherent state components in the superposition. Figure 4 presents the Wigner quasiprobability distributions for synthesized states with varying M . These separated

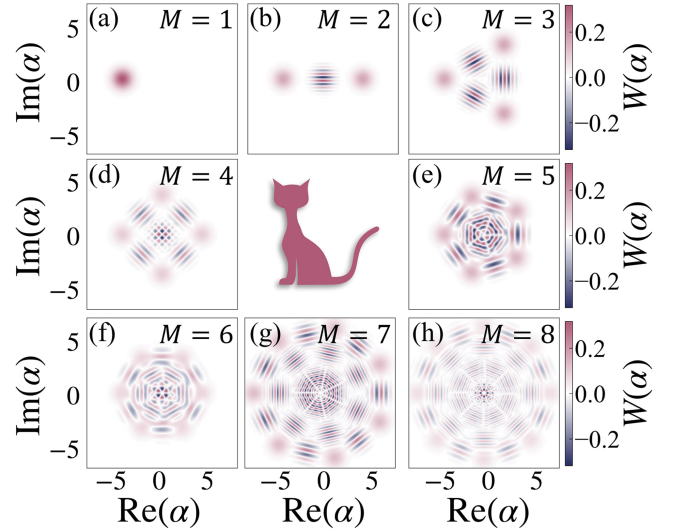


FIG. 4. Preparation of Schrödinger-cat states mediated by Talbot-modulated free electrons. (a)–(f) Wigner quasiprobability distributions of the generated photonic states for Talbot indices ranging from $M = 1$ to $M = 6$, respectively, with the coupling strength fixed at $g = 3$. The phase-space structures clearly reveal coherent superpositions of M distinct coherent components, including Schrödinger-cat-like states. (g),(h) Wigner quasiprobability distributions for $M = 7$ and $M = 8$, respectively, with an increased coupling strength $g = 5$.

lobes correspond to phase-rotated coherent components, whereas the oscillatory fringes between them arise from off diagonal coherence and would disappear from an incoherent mixture. These patterns thus unambiguously exhibit nonclassical interference fringes and discrete nodal structures characteristic of cat states, with the number of major lobes scaling directly with M . Additional fidelity, purity, and photon-loss benchmarks for finite sideband width and phase noise are given in the Supplemental Material [30].

The role of the transferred quadratic phase is analogous to Kerr-type fractional revival: in a Kerr medium, a coherent state acquires a photon-number-dependent phase, $e^{-i\chi t m^2}$, which can reorganize it into a Schrödinger-cat state at suitable times [48,49]. Unlike Kerr, atom-cavity, dissipative, or probabilistic optical schemes [46,47,50,51], our approach does not require the optical field to accumulate a large nonlinear phase in a material medium. Instead, the quadratic phase is first generated by free-electron dispersion and then transferred to the photon Fock basis through electron-photon interaction. Thus, the non-Gaussian phase resource is precompiled in the electron wave function and controlled by the drift distance. Energy-resolved electron detection mainly selects the phase-space orientation of the conditioned state, rather than acting as a success-or-failure herald [30].

Conclusion—We demonstrate a programmable free-electron wave packet platform for Talbot-selected high-harmonic bunching and measurement-conditioned

quantum-light synthesis through intrinsic dispersive evolution. By embedding spectral selectivity into quadratic-phase programming, this approach harnesses deterministic, lossless phase accumulation as an effective phase-programming resource, transforming external filtering into self-compiled, narrow-band electron modulation. Operating as a dual-channel system controlled by drift distance z , the programmed phase kernel $\exp(i2\pi lhz/Z_T)$ yields distinct outputs: when measured in real space, it acts as an interference filter generating an attosecond electron comb (channel 1); when projected onto momentum sidebands, the phase coherently transfers to photon Fock states, producing programmable M -component Schrödinger cat states (channel 2). This unified framework suggests a broader template for quantum-state synthesis in dispersive wave systems. For free electrons specifically, the scheme provides a phase-programming mechanism for coherent short-wavelength emission and nonclassical light synthesis, utilizing electron self-interference in a drift section. Beyond the explicit demonstration of optical cat states, the protocol provides a framework for future nanophotonic electron-light platforms [31,52]. By treating free electrons as quantum registers rather than mere radiation sources, this scheme opens pathways toward hybrid quantum information processing [19,32] architectures for programmable quantum-state preparation, coherent light generation, and ultrafast spectroscopy.

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Data availability—The data that support the findings of this article are available from the corresponding author upon reasonable request.

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End Matter

In this section, based on the quantum interference framework established above, we construct a robustness analysis of quadratic-phase programming against realistic experimental imperfections. Through quantitative examination of stochastic phase noise and finite sideband energy spread under the experimental conditions used in Figs. 2 and 3 ($E_e = 200$ keV, $N \approx 150$ sidebands, $z = Z_T/30$), we demonstrate how Talbot-enabled harmonic selectivity emerges as a resilient component within this programmable scheme. The Supplementary Material further provides an experimental roadmap and tolerance estimates for center-energy jitter, drift-length detuning, transverse alignment, finite electron counts, and optical loss in the quantum-light channel.

The robustness of this programmable scheme rests on preserving relative phase coherence between sidebands, rather than absolute phase stability. Consequently, a global phase ramp—such as the $n\theta$ term imprinted in a standard PINEM interaction [26]—induces only a spatial translation of the entire electron wave packet, leaving all intersideband phase relationships intact (see Supplemental Material [30]).

Stochastic phase noise, however, directly randomizes the relative phase $\arg(c_n)$, perturbing the precise destructive interference designed to suppress off resonant harmonics. As shown in Fig. 5(a), this raises a noise floor across the bunching spectrum while the resonantly enhanced harmonics (e.g., $h = 30$ at $z = Z_T/30$) remain dominant. The signal-to-noise ratio decreases monotonically with increasing disorder but remains substantial even under strong noise [Fig. 5(b)], demonstrating that while stochastic noise smears the sharp destructive interference minima, it does not erase the fundamental harmonic selectivity encoded by the quadratic phase resonance.

Finite sideband width presents a second practical limitation, introducing a distribution of longitudinal velocities that causes phase slippage during propagation. Here the FWHM denotes the energy width of an individual PINEM sideband, $\Delta E_{\text{sb}}^{\text{FWHM}}$, centered at $E_l = E_0 + l\hbar\omega$; it is distinct from the total populated ladder bandwidth $\sim N\hbar\omega$ and originates from the longitudinal energy spread of the incident electron wave packet or ensemble. Using the conservative ultrafast transmission electron microscopy (UTEM) or PINEM benchmark $\Delta E_{\text{sb}}^{\text{FWHM}} = 0.6$ eV $\simeq 0.39\hbar\omega$, the finite width reduces the resonant bunching amplitude but preserves the Talbot-selected harmonic family at $h = 30, 60, 90, \dots$ [Figs. 5(a) and 5(c)]. This behavior reflects the fact that

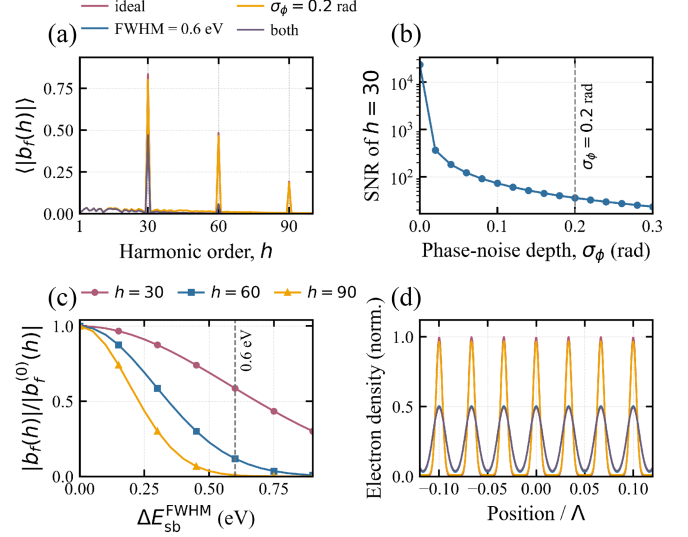


FIG. 5. Robustness of Talbot-enhanced bunching under finite PINEM-sideband width and stochastic sideband phase noise at $z = Z_T/30$. (a) Bunching spectrum for the ideal case, $\Delta E_{\text{sb}}^{\text{FWHM}} = 0.6$ eV, $\sigma_\phi = 0.2$ rad, and the combined nonideal case; dashed guides mark $h = 30, 60, 90$. (b) Spectral SNR of the $h = 30$ peak versus phase-noise depth, evaluated with the 0.6 eV sideband-width benchmark. (c) Normalized selected harmonic amplitudes versus individual sideband FWHM, showing that higher selected orders are more sensitive to energy spread while the selected family remains fixed. (d) Enlarged real-space density over the comoving coordinate x/Λ using the same color coding as in (a), showing that the Talbot microbunch train remains visible in the combined nonideal case.

a fixed energy offset mainly adds a linear phase across the sideband ladder, shifting the microbunching pattern in time; incoherent averaging reduces contrast without moving the selected harmonics. The real-space microbunch train therefore remains visible even when finite sideband width and phase noise are applied together [Fig. 5(d)].

These results confirm that while long-range self-imaging may be progressively washed out, the phase coherence responsible for harmonic selection is preserved under standard experimental conditions. The scheme does not require cryogenic isolation or coherence beyond existing ultrafast electron microscopy, supporting the feasibility of observing Talbot-selected bunching under existing UTEM or PINEM conditions.