

Spectral Constraints on Theories of Colored Particles and Gravity

Aaron Hillman¹, Yu-tin Huang^{2,3}, Laurentiu Rodina⁴, and Justinas Rumbutis²

¹Walter Burke Institute for Theoretical Physics, *Caltech*, Pasadena, California 91125, USA

²Department of Physics and Center for Theoretical Physics, *National Taiwan University*, Taipei 10617, Taiwan

³Physics Division, *National Center for Theoretical Sciences*, Taipei 10617, Taiwan

⁴Beijing Institute of Mathematical Sciences and Applications (BIMSA), Beijing, 101408, China



(Received 23 November 2024; revised 24 April 2025; accepted 10 July 2025; published 6 August 2025)

In this Letter, we consider effective field theories for light fields transforming under the fundamental or adjoint representation of a continuous group. Assuming tree-level completions, we demonstrate that, in the presence of gravity, crossing symmetry combined with twice-subtracted sum rules leads to constraints on the irreducible representations that the ultraviolet degrees of freedom must populate. A spectrum is allowed only if its low energy projection contains the graviton pole. Beautifully, the graviton pole is the anchor of our argument, not an obstruction.

DOI: [10.1103/k23f-y47w](https://doi.org/10.1103/k23f-y47w)

Introduction—Gravitational scattering has long been understood to be highly constrained. This understanding is an amalgam of everything from sharp results such as the Weinberg-Witten theorem to intuitions and standard lore such as that regarding the fate of global symmetries in quantum gravity. Much of the work on the relation between low energy physics and consistency of quantum gravity comes from the swampland program (see Refs. [1–4] for review), largely consisting of conjectures about possible configurations of low energy EFT due to constraints from string theory. On the other hand, bootstrap programs have made much progress in recent years constraining low energy EFT (including gravitational ones) from UV consistency on general grounds [5–25], though only very recently approaching the subject of symmetries [26–28]. A complete understanding of gravitational EFTs requires the proper treatment of the graviton pole arising from massless graviton exchange, which has long posed a major obstacle for traditional bootstrap approaches based on the forward limit. Reference [29] introduced a method to systematically incorporate the graviton pole through smearing the amplitude against suitable test functions, demonstrating that the graviton pole generically weakens the resulting bounds on Wilson coefficients.

In this Letter, we present the first constraints for scattering in the presence of gravity and gauge or global symmetries on general grounds. This amounts to a constraint on the sets of irreps of the symmetry groups, which

are permitted to be absent from the UV spectrum, based purely on consistency with Lorentz invariance, the internal symmetry group, crossing symmetry, and Regge boundedness. Notably, positivity of the spectral density is not required to derive these equations. In a beautiful irony, the classic t -channel graviton pole is a feature of our argument, not a bug. When applied to tree-level completions, these constraints lead to the following claim: consistency with gravity at low energies permits only certain patterns of irreps to be absent from the UV spectrum of a theory of colored particles.

We consider the $2 \rightarrow 2$ massless scattering amplitude $\mathcal{A}^{i_1 i_2 i_3 i_4}(s, t)$, where i 's label the representation of the external state under some symmetry group $\mathcal{G}$ and $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$ are the Mandelstam invariants. On general grounds the amplitude satisfies $\lim_{s \rightarrow \infty} \mathcal{A}^{i_1 i_2 i_3 i_4}(s, t)/|s^2| = 0$ for $t < 0$ [29,30]. This implies that one can set up twice subtraction sum rules for the amplitude, which take the form

$$\frac{8\pi G_N}{-t} \delta^{i_1 i_4} \delta^{i_2 i_3} + \dots = 2 \int_{M^2}^{\infty} \frac{ds'}{s'^3} \text{Im}[\mathcal{A}^{i_1 i_2 i_3 i_4}(s', t)], \quad (1)$$

where $\dots$ represents contributions from low energy physics, including higher-dimensional operators and loops from light states. M^2 is the scale at which heavy states of the UV theory appear. We can decompose both sides of the above in terms of projectors $\mathbb{P}_R$ of a given channel, representing the exchange of states transforming in the $\{R\}$ representation. Note that since the pole from gauge mediator scales as s^1 , it will not be captured by our twice subtraction sum rules. Therefore in our analysis there is no distinction between global or local symmetry, we only assume that it is not broken. If the symmetry is gauged, we also assume the EFT is not confined.

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The central logic of the argument is simple: we can identify vectors in the space of projectors $\mathbb{P}_R$ which, when contracted, remove all but some chosen set of irreps $\{\bar{R}\}$, while keeping the t -channel graviton pole on the low energy side. It is then inconsistent for the imaginary part of the amplitude to have only irreps $\{\bar{R}\}$ in the UV, as this would require

$$\frac{8\pi G_N}{-t} + \dots = 0. \quad (2)$$

Since the $G_N \neq 0$, the above is inconsistent. (The presence of the graviton pole has also been leveraged to constrain the Regge limit [18,31], as well as the other way around [32].) We demonstrate this phenomenon in the case of scattering fundamental and adjoint scalars. In particular, in the case of $SO(n)$ adjoint scalars, it is inconsistent to have only singlets and adjoints in the UV. As we will demonstrate, the minimal set of irreps that are consistent with our constraints consists of the singlet, adjoint, and an anti-symmetric tensor. Note that such constraints do not appear for field theory completions, since a massive scalar exchange

$$\mathcal{A}^{i_1 i_2 i_3 i_4}(s, t) = \frac{\mathbb{P}_R^{i_1 i_2 i_3 i_4}}{s - M^2} + \text{perm}, \quad (3)$$

is a perfectly UV complete amplitude for any R .

These are constraints on the imaginary part of the amplitude in the UV. When graviton loops are not suppressed, then the eikonal amplitude will automatically satisfy the constraints, since as shown in [32], the imaginary part of the latter can reproduce the t -channel pole through the dispersion relations. Generalizing to the scattering of colored states, the eikonal contribution corresponds to a t -channel singlet, which when projected onto s - and u -channel projectors, produces all irreps.

However, if the amplitude unitarizes at tree level, by which we mean the tree-level amplitude satisfies twice subtraction dispersion relations with positive residues, then these constraints become a statement on the spectrum of new states. Indeed one can verify that gauge amplitudes in closed string theories satisfy these constraints. For example, consider the gluon amplitude in the heterotic string:

$$\mathcal{A}_{\text{Het}} = s^2 \left[\frac{1}{M_P^2} \left(\frac{\mathbb{P}_1^s}{s(1+s)} + \frac{\mathbb{P}_1^t}{t(1+t)} + \frac{\mathbb{P}_1^u}{u(1+u)} \right) + \frac{g_{\text{YM}}^2}{3} \left(\frac{\mathbb{P}_5^s - \mathbb{P}_5^t}{st} + \frac{\mathbb{P}_5^t - \mathbb{P}_5^u}{tu} + \frac{\mathbb{P}_5^u - \mathbb{P}_5^s}{us} \right) \right] \Gamma_{\text{str}}, \quad (4)$$

where $\mathbb{P}_1^s$ and $\mathbb{P}_5^s$ represents the singlet and adjoint projector in s channel, respectively, and

$$\Gamma_{\text{str}} = -\frac{\Gamma[-s]\Gamma[-t]\Gamma[-u]}{\Gamma[s]\Gamma[u]\Gamma[t]}. \quad (5)$$

The 10-dimensional partial wave expansion of the residue at $s = 1$, denoted as $a[\ell, R]$, where ℓ is the spin and R the irrep of $SO(n)$, is given as

$$\begin{aligned} a[0, 1] &= \frac{36 + n - n^2}{9M_P^2(n-1)n} + \frac{g_{\text{YM}}^2(n-2)}{(n-1)n}, \\ a[0, 2] &= \frac{4}{M_P^2(n-2)} + \frac{g_{\text{YM}}^2(n-4)}{2(n-2)}, \\ a[0, 3] &= \frac{2}{M_P^2} - \frac{g_{\text{YM}}^2}{2}, \quad a[0, 4] = g_{\text{YM}}^2 + \frac{2}{M_P^2}, \\ a[1, 5] &= \frac{g_{\text{YM}}^2}{2}, \quad a[2, 1] = \frac{1}{9M_P^2}. \end{aligned} \quad (6)$$

The projectors for the irreps are given in Supplemental Material [33]. As one can see, already at the first massive level, 5 out of the 6 irreps that can be exchanged between adjoints in a $2 \rightarrow 2$ process are present. The positivity of these partial waves in fact puts constraints on the rank of the gauge group, which was explored in [34].

These conclusions are also visible with SDPB numerical bootstrap methods used to bound EFT couplings [29,35]. We indeed find that particular irrep configurations must be present in order to have a strictly nonzero G_N . Utilizing SDPB also allows us to test scenarios where the massless spin-2 is in some other irrep, and we find that only having it in the singlet gives consistent results.

The dispersive representation for colored S matrix—Expansion basis for colored amplitudes: At low energies, the amplitude is a meromorphic function with simple massless poles, reflecting the exchange of gravitons. We will assume that gravity is unitarized at tree level, and hence the massless branch cuts from the graviton loops are suppressed. The EFT amplitude can then be organized as

$$\mathcal{A}_{\text{Low E}}^{i_1 i_2 i_3 i_4} = \mathcal{A}_{\text{Grav}}^{i_1 i_2 i_3 i_4} + B_{\text{poly}}^{i_1 i_2 i_3 i_4}, \quad (7)$$

where

$$\mathcal{A}_{\text{Grav}}^{i_1 i_2 i_3 i_4} = 8\pi G_N \left(\frac{(t-u)^2}{s} \delta^{i_1 i_2} \delta^{i_3 i_4} + \text{Perm} \right). \quad (8)$$

The “ B ” functions are polynomials in Mandelstam variables and expanded in suitable color basis, if these operators arise from symmetry preserving UV physics.

For colored amplitudes, the partial wave expansion is a double expansion in kinematic and color space [34]. In particular, we have

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \sum_{J,R} n_J^{(D)} f_{J,R}(s) \mathbb{P}_R^{i_1 i_2 i_3 i_4} \mathbb{G}_J^{(D)} \left(1 + \frac{2t}{s} \right), \quad (9)$$

where $n_J^{(d)} = \{[(4\pi)^{(d/2)}(d+2J-3)\Gamma(d+J-3)]/\pi\Gamma(d-2/2)\Gamma(J+1)\}$, $f_{J,R}(s)$ are the partial wave coefficients, and in addition to the expansion in Gegenbauer polynomials $\mathbb{G}_J^{(D)}$, we have expanded in the s -channel projectors $\mathbb{P}_R^{i_1 i_2; i_3 i_4}$, which are the set of orthogonal, irreducible tensors in the indices i_i , corresponding to the exchange of irreducible representations R . More details can be found in [36]. For $\text{SO}(n)$ and $\text{SU}(n)$ the number of independent projectors, denoted as q , are n independent. For $\text{SO}(n)$, $q = 3$ for fundamental i_i 's, corresponding to the singlet, symmetric traceless, and the antisymmetric. In the s channel they take the form

$$\begin{aligned}\mathbb{P}_1^{i_1 i_2; i_3 i_4} &= \frac{\delta^{i_1 i_2} \delta^{i_3 i_4}}{n}, \\ \mathbb{P}_2^{i_1 i_2; i_3 i_4} &= \frac{1}{2} \left(\delta^{i_1 i_4} \delta^{i_2 i_3} + \delta^{i_1 i_3} \delta^{i_2 i_4} - \frac{2}{n} \delta^{i_1 i_2} \delta^{i_3 i_4} \right), \\ \mathbb{P}_3^{i_1 i_2; i_3 i_4} &= \frac{1}{2} (\delta^{i_1 i_4} \delta^{i_2 i_3} - \delta^{i_1 i_3} \delta^{i_2 i_4}).\end{aligned}\quad (10)$$

For $\text{SU}(n)$, we consider $i_1 \bar{i}_2 i_3 \bar{i}_4$ scattering in which we have two irreps, adjoint and singlet, in the s (or t) channel:

$$\mathbb{P}_{1\bar{i}_2 i_4}^{i_1 i_3} = \delta_{i_4}^{i_1} \delta_{i_2}^{i_3} - \frac{\delta_{i_4}^{i_1} \delta_{i_2}^{i_3}}{n}, \quad \mathbb{P}_{2\bar{i}_2 i_4}^{i_1 i_3} = \frac{\delta_{i_4}^{i_1} \delta_{i_2}^{i_3}}{n}, \quad (11)$$

and two different irreps, antisymmetric and symmetric, in u channel:

$$\mathbb{P}_{3\bar{i}_2 i_4}^{i_1 i_3} = \frac{\delta_{i_4}^{i_1} \delta_{i_2}^{i_3} - \delta_{i_2}^{i_1} \delta_{i_4}^{i_3}}{2}, \quad \mathbb{P}_{4\bar{i}_2 i_4}^{i_1 i_3} = \frac{\delta_{i_4}^{i_1} \delta_{i_2}^{i_3} + \delta_{i_2}^{i_1} \delta_{i_4}^{i_3}}{2}. \quad (12)$$

For adjoint external states $q = 6$ and 7 for $\text{SO}(n)$ and $\text{SU}(n)$, respectively. We present the explicit form for $\text{SO}(n)$ in Supplemental Material [33]. We simply note that the first and fifth operator correspond to the singlet and adjoint representation, respectively.

Twice subtraction sum rules: We now derive the dispersive representation for the low energy EFT coefficients. Starting with the standard contour in the complex s plane and assuming that the integral at infinity with two subtractions vanishes:

$$\oint_{\infty} \frac{ds'}{2\pi i(s'-s)} \frac{\mathcal{A}^{i_1 i_2 i_3 i_4}(s', t)}{s'(s'+t)} = 0, \quad (13)$$

using s , u crossing symmetry $\mathcal{A}^{i_1 i_2 i_3 i_4}(-s'-t, t) = \mathcal{A}^{i_1 i_3 i_2 i_4}(s', t)$, (13) can be written as

$$\begin{aligned}(\text{Res}_{s'=0} + \text{Res}_{s'=-t} + \text{Res}_{s'=s}) \frac{\mathcal{A}^{i_1 i_2 i_3 i_4}(s', t)}{(s'-s)s'(s'+t)} \\ = \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \left(\frac{\text{Im} \mathcal{A}^{i_1 i_2 i_3 i_4}(s', t)}{(s'-s)} + \frac{\text{Im} \mathcal{A}^{i_1 i_3 i_2 i_4}(s', t)}{(s'+t+s)} \right).\end{aligned}\quad (14)$$

Now we substitute (7) and (9) in the equation above:

$$\begin{aligned}\frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(s, t)}{s(s+t)} - \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(0, t)}{st} + \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(-t, t)}{(s+t)t} - \frac{8\pi G n}{t} \mathbb{P}_1^{i_2 i_3; i_4 i_1} \\ = n_J^{(D)} \sum_{JR} \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \\ \times \left(\frac{\mathbb{P}_R^{i_1 i_2; i_3 i_4}}{(s'-s)} + \frac{\mathbb{P}_R^{i_1 i_3; i_2 i_4}}{(s'+t+s)} \right) \mathbb{G}_J^{(D)}(1+2t/s').\end{aligned}\quad (15)$$

Despite appearance, the lhs of the above is analytic except for the t -channel graviton pole in the singlet channel $\mathbb{P}_1^{i_2 i_3; i_4 i_1}$. Note that if the symmetry is gauged, there would be a massless vector pole that scales as s^1 . This will not be captured by the twice subtraction sum rules, and thus here there is no distinction between global and gauge symmetry in the following discussion.

Color projectors: To solve Eq. (15), the color factors should be cast into a common basis. We can simply use the t -channel projector basis. Denoting $\mathbb{P}_R^{i_2 i_3; i_4 i_1} = \mathbb{P}_R^t$, $\mathbb{P}_R^{i_1 i_2; i_3 i_4} = \mathbb{P}_R^s$, $\mathbb{P}_R^{i_1 i_3; i_2 i_4} = \mathbb{P}_R^u$, we introduce the conversion matrix,

$$\mathbb{P}^s = M_{s,t} \mathbb{P}^t, \quad \mathbb{P}^u = M_{u,t} \mathbb{P}^t. \quad (16)$$

For example, for the $\text{SO}(n)$ fundamental representation, M_{st} is given by

$$\begin{pmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{(n-1)(n+2)}{2n} & \frac{n-2}{2n} & -\frac{n+2}{2n} \\ \frac{n-1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad (17)$$

where the first, second, and third columns correspond to the singlet, symmetric, and antisymmetric representation, respectively. M_{ut} is the same as M_{st} with an additional minus sign in the third column. For $\text{SO}(n)$ adjoint representation, the 6×6 matrices are given in Eqs. (S2) and (S3) of Supplemental Material [33]. For $\text{SU}(n)$ there are fewer projectors than invariant tensors, so we use $(\delta_{i_2}^{i_1} \delta_{i_4}^{i_3}, \delta_{i_4}^{i_1} \delta_{i_2}^{i_3})$ as the basis of the color factor space. In this basis the s and u projectors are given, respectively, as

$$\begin{pmatrix} -\frac{1}{n} & 1 \\ \frac{1}{n} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}. \quad (18)$$

Implications of gravity—Crucially, after projecting on to independent basis Eq. (15) leads to q equalities, one for each basis element. The rhs of Eq. (15) now takes the form

$$\sum_J \int_{M^2}^{\infty} ds' \text{Im}[\mathbf{f}_J(s')] (M_{st} h_{s,J}(s', t) + M_{ut} h_{u,J}(s', t)), \quad (19)$$

TABLE I. The vectors v that satisfies Eq. (21) for each irrep in the exchange of $\text{SO}(n)$ and $\text{SU}(n)$ fundamentals.

Group	Irreps	v
$\text{SO}(n)$	$\{1\}$	$\{1, [2 - n/(n-1)(2+n)], 0\}$
	$\{2\}$	$(1, -1, 0)$
	$\{3\}$	$[1, (1/n-1), 0]$
$\text{SU}(n)$	$\{1\}$	$[1, (1/n)]$
	$\{2\}$	$(1, 0)$
	$\{3\}$	$(1, 1)$
	$\{4\}$	$(1, -1)$

where each $\mathbf{f}_J(s')$ is a q -dimensional vector, and we include the kinematic dependence in the factors

$$h_{z,J}(s', t) = \frac{n_J^{(D)} \mathbb{G}_J^{(D)}(1 + 2t/s')}{\pi s'(s' + t)(s' - z)}. \quad (20)$$

Now consider a q -dimensional vector $v = (1, \dots)$ such that

$$[M_{st}v]_i = [M_{ut}v]_i = 0, \quad (21)$$

where i is one of the q irreps. Basically, one is considering the complement to the i th projector. If such an exclusion vector exists, then when multiplied to both sides of Eq. (15), after converted into the $\mathbb{P}'$ basis, we arrive at

$$\begin{aligned} \frac{8\pi G_N}{-t} + \text{Poly}(t) &= \sum_{j \neq i} \sum_J \int_{M^2}^\infty ds' \text{Im}[f_{J,j}(s')] \\ &\times ([M_{st}v]_j h_{s,J}(s', t) + [M_{ut}v]_j h_{u,J}(s', t)). \end{aligned} \quad (22)$$

The fact that the first component of v is 1 guarantees that the graviton pole will appear on the lhs. Importantly, if we set $f_{J,j} = 0$ for all $j \neq i$, the rhs vanishes and we reach an inconsistency since the gravitational coupling is nonzero. This immediately rules out such spectrum for gravity.

Fundamental and adjoint matter: Let us start with the case of *fundamental* representation. We find that for each irrep we can find its corresponding vector v , which is listed in Table I. Importantly, the vector v exists for all n . Therefore, we see that fundamental matter in $\text{SO}(n)$ or $\text{SU}(n)$ demands a UV completion of gravity with at least two irreps [37].

For adjoint matter, we can show that at least three irreps are required for a general n . In fact, we find the only possible UV completion with three irreps consists of the singlet, adjoint, and antisymmetric representations, denoted as $\{1, 5, 6\}$ in our notation [Supplemental Material [33] Eq. (S1)]. For all other three-irrep completions we can find exclusion vectors v , provided in Supplemental Material [33], Table I. To establish that completions with only two representations are not possible, it remains to show that any

 TABLE II. The vectors v that satisfies Eq. (21) for two irreps of $\text{SO}(n)$.

Irreps	v
$\{1, 5\}$	$[1, 0, (-4 + n/3), (1 - n/3), 0, 0]$
$\{1, 6\}$	$\{1, [4 - 2n - 3n^2 + n^3/2(-2 + n)],$ $-[(-3 + n)n^2/2(-2 + n)], 0, 0, 0\}$
$\{5, 6\}$	$[1, -1 + n, (-3 + n/2)n, 0, 0, 0]$

two-representation subset of $\{1, 5, 6\}$ is forbidden. The corresponding vectors for this case are listed in Table II. Since no two-irrep completions are allowed, this automatically implies no single-irrep completions are possible. Additionally, some four-representation completions can also be ruled out. Specifically, for $n = 4$, we find exclusion vectors for the sets $\{1, 2, 4, 6\}$ and $\{1, 3, 4, 5\}$, which are presented in Supplemental Material [33], Table II.

Importantly, the spectrum with only singlets and adjoints is ruled out. That is, the UV completion must include irreps outside of that of the low energy states. Let us compare this to a scenario where we can access the massless pole from the gauge boson. This would require one-subtraction, which can be justified for specific setups [28,30]. Then we can repeat the same analysis, and find that all single irrep completions are ruled out *except* for the adjoint, see Supplemental Material [33], Table III. Indeed, starting with the tree-level field theory amplitude

$$\mathcal{A}^{\text{FT}} = \mathbb{P}_5^s \frac{t-u}{s} + \mathbb{P}_5^t \frac{u-s}{t} + \mathbb{P}_5^u \frac{s-t}{u}, \quad (23)$$

we can deform the s -channel term as [38]

$$\frac{t-u}{s} \rightarrow \frac{t-u}{s(t-M^2)(u-M^2)} + \frac{t-u}{(s+M^2)(t-M^2)(u-M^2)}, \quad (24)$$

with similar deformations for the other two channels. It is straightforward to see that the amplitude now satisfies one subtraction. Importantly, while there are now t poles contributing to $\mathbb{P}_5^s$, when combined with contributions from $\mathbb{P}_5^u$ they become proportional to $\mathbb{P}_5^t$ via the Jacobi identities of the projector. Note that this is not a “physical” deformation, as it is plagued by tachyons, nonlocal residues, and infinite number of massless higher odd spins, but it still satisfies our starting assumptions. It will be interesting to find a more physical solution. We also note that attempting an analogous UV completion for gravity, replacing adjoint exchange with singlet exchange, leads to the I-channel singlet generating additional irreps in the s and u channels, which unlike the gauge theory example, do not cancel out.

Graviton poles as other irreps—It is tempting to ask if the spin-2 massless pole being in a singlet plays an

important role. It is known that consistent factorization on massless poles in the four-particle amplitude forces any self-interacting massless spin-2 states to be singlets [39,40]. However, one might entertain the scenario that the spin-2 state in question does not have self-interaction. Utilizing numerical approaches [6,29] to study bounds on EFT coefficients from Eq. (1), we find in $D \geq 5$ that the allowed regions for EFT coefficients collapses whenever the pole is nonsinglet. We note that, unlike the results of the previous section, this fact requires the application of positivity, as well as crossing symmetry. We leave a more comprehensive study of bounds, including the case of $D = 4$ which requires special care [29], for future work.

Conclusions—We have shown that the presence of a graviton pole, together with its coupling to colored states, imposes sharp constraints on the UV spectrum of tree-level amplitudes that satisfy crossing symmetry and twice-subtracted dispersion relations. Specifically, these constraints require the inclusion of some minimal sets of irreducible representations (irreps) in the spectrum. Our framework also invites further exploration of the completeness hypothesis, which posits that all irreps allowed by the gauge symmetry must be present in the spectrum [41–45]. Since starting with the fundamental or adjoint, consistency requires the presence of a new irreps, the scattering of the latter will presumably require further new irreps. A key question is whether the only possible outcome of this iterative process is the presence of all irreps. On the other hand, we see for scalar $SO(n)$ adjoint scattering that the minimal set of UV irreps consists of a singlet, adjoint and antisymmetric representation. It will be interesting to see if such spectrum can be realized with an explicit UV model.

The presence of higher spin states, with $J > 1$, will lead to new constraints from crossing symmetry and positivity. These extra constraints are needed for obtaining two-sided bounds on EFT couplings [6,25,46], and for scattering of identical scalars imply the presence of an infinite number of higher spins [8], when any $J \geq 2$ are present. It will be interesting to study whether such arguments further constrain the allowed irreps that can appear.

Acknowledgments—We thank Simon Caron-Huot and Alexander Zhiboedov for discussions. The work of J. R. is supported by Taiwan NSTC Grant No. 112-2811-M-002 -054 -MY2, while Y.-t. H. is supported by Taiwan NSTC Grant No. 112-2628-M-002 -003 -MY3. L. R. is supported by the Beijing Natural Science Foundation International Scientist Project No. IS24014, and the National Natural Science Foundation of China General Program No. 12475070. A. H. is supported by the Department of Energy (Grant No. DE-SC0011632) and by the Walter Burke Institute for Theoretical Physics.

[1] T. D. Brennan, F. Carta, and C. Vafa, *Proc. Sci. TASI2017* (2017) 015 [arXiv:1711.00864].

- [2] E. Palti, *Fortschr. Phys.* **67**, 1900037 (2019).
- [3] M. van Beest, J. Calderón-Infante, D. Mirfendereski, and I. Valenzuela, *Phys. Rep.* **989**, 1 (2022).
- [4] M. Graña and A. Herráez, *Universe* **7**, 273 (2021).
- [5] N. Arkani-Hamed, T.-C. Huang, and Y.-t. Huang, *J. High Energy Phys.* **05** (2021) 259.
- [6] S. Caron-Huot and V. Van Duong, *J. High Energy Phys.* **05** (2021) 280.
- [7] Y.-t. Huang, J.-Y. Liu, L. Rodina, and Y. Wang, *J. High Energy Phys.* **04** (2021) 195.
- [8] L.-Y. Chiang, Y.-t. Huang, W. Li, L. Rodina, and H.-C. Weng, *J. High Energy Phys.* **03** (2022) 063.
- [9] L.-Y. Chiang, Y.-t. Huang, W. Li, L. Rodina, and H.-C. Weng, *arXiv:2201.07177*.
- [10] L.-Y. Chiang, Y.-t. Huang, L. Rodina, and H.-C. Weng, *J. High Energy Phys.* **05** (2024) 102.
- [11] A. Guerrieri, J. Penedones, and P. Vieira, *Phys. Rev. Lett.* **127**, 081601 (2021).
- [12] S. Caron-Huot, Y.-Z. Li, J. Parra-Martinez, and D. Simmons-Duffin, *J. High Energy Phys.* **05** (2023) 122.
- [13] S. Caron-Huot, Y.-Z. Li, J. Parra-Martinez, and D. Simmons-Duffin, *Phys. Rev. D* **108**, 026007 (2023).
- [14] S. Caron-Huot and Y.-Z. Li, *J. High Energy Phys.* **02** (2025) 115.
- [15] J. Tokuda, K. Aoki, and S. Hirano, *J. High Energy Phys.* **11** (2020) 054.
- [16] L. Alberte, C. de Rham, S. Jaitly, and A. J. Tolley, *Phys. Rev. D* **102**, 125023 (2020).
- [17] L. Alberte, C. de Rham, S. Jaitly, and A. J. Tolley, *Phys. Rev. Lett.* **128**, 051602 (2022).
- [18] C. de Rham, S. Jaitly, and A. J. Tolley, *Phys. Rev. D* **108**, 046011 (2023).
- [19] S. D. Chowdhury, K. Ghosh, P. Haldar, P. Raman, and A. Sinha, *SciPost Phys.* **13**, 051 (2022).
- [20] Z. Bern, D. Kosmopoulos, and A. Zhiboedov, *J. Phys. A* **54**, 344002 (2021).
- [21] C. Beadle, G. Isabella, D. Perrone, S. Ricossa, F. Riva, and F. Serra, *arXiv:2407.02346*.
- [22] F. Bertucci, J. Henriksson, B. McPeak, S. Ricossa, F. Riva, and A. Vichi, *J. High Energy Phys.* **12** (2024) 051.
- [23] B. Bellazzini, G. Isabella, S. Ricossa, and F. Riva, *Phys. Rev. D* **109**, 024051 (2024).
- [24] B. Bellazzini, M. Riembau, and F. Riva, *Phys. Rev. D* **106**, 105008 (2022).
- [25] B. Bellazzini, J. E. Miró, R. Rattazzi, M. Riembau, and F. Riva, *Phys. Rev. D* **104**, 036006 (2021).
- [26] B. McPeak, M. Venuti, and A. Vichi, *arXiv:2310.06888*.
- [27] J. Albert and L. Rastelli, *J. High Energy Phys.* **09** (2024) 039.
- [28] J. Albert, J. Henriksson, L. Rastelli, and A. Vichi, *J. High Energy Phys.* **09** (2024) 172.
- [29] S. Caron-Huot, D. Mazac, L. Rastelli, and D. Simmons-Duffin, *J. High Energy Phys.* **07** (2021) 110.
- [30] K. Häring and A. Zhiboedov, *SciPost Phys.* **16**, 034 (2024).
- [31] T. Noumi and J. Tokuda, *J. High Energy Phys.* **06** (2023) 032.
- [32] K. Häring and A. Zhiboedov, *arXiv:2410.21499*.
- [33] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/k23f-y47w> provides the explicit

form of projectors for various irreducible representations used in the text, as well as the explicit vectors used to derive the constraints in the text.

- [34] B. Bachu and A. Hillman, [arXiv:2212.03871](#).
- [35] D. Simmons-Duffin, *J. High Energy Phys.* **06** (2015) 174.
- [36] P. Cvitanovic, *Group Theory: Birdtracks, Lie's, and Exceptional Groups* (Princeton University Press, Princeton, NJ, 2020).
- [37] It might seem that $SO(2)$ or $U(1)$ is a contradiction, since in [26] it was shown that it is allowed to have a UV completion of fundamental representation scattering [$U(1)$ charge ± 1] with just charge 0 exchanges. However, in this case there are still two distinct charge 0 exchanged states (parity even and parity odd) which in $SO(2)$ language corresponds to two distinct representations.
- [38] We consider this deformation in place of the standard string theory completion, as the latter necessarily includes additional irreps beyond the adjoint. In contrast, our goal is to construct a UV completion involving adjoint states only.
- [39] P. Benincasa and F. Cachazo, [arXiv:0705.4305](#).
- [40] D. A. McGady and L. Rodina, *Phys. Rev. D* **90**, 084048 (2014).
- [41] J. Polchinski, *Int. J. Mod. Phys. A* **19**, 145 (2004).
- [42] T. Banks and N. Seiberg, *Phys. Rev. D* **83**, 084019 (2011).
- [43] D. Harlow and H. Ooguri, *Commun. Math. Phys.* **383**, 1669 (2021).
- [44] B. Heidenreich, J. McNamara, M. Montero, M. Reece, T. Rudelius, and I. Valenzuela, *J. High Energy Phys.* **09** (2021) 203.
- [45] T. Rudelius and S.-H. Shao, *J. High Energy Phys.* **12** (2020) 172.
- [46] A. J. Tolley, Z.-Y. Wang, and S.-Y. Zhou, *J. High Energy Phys.* **05** (2021) 255.