Bootstrapping Six-Gluon QCD Amplitudes

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We present a symbol-level bootstrap construction of the planar, two-loop six-gluon scattering amplitude for the $--++++$ helicity configuration in quantum chromodynamics (QCD), focusing on the maximal weight pieces—the "most complicated terms" in the sense of Lipatov *et al.* Building on recent advances in the understanding of the relevant function space, we incorporate as a crucial new ingredient the complete set of leading singularities, obtained from an analysis of on-shell diagrams. The resulting expressions are manifestly conformally invariant and clarify the structure of previous five-particle results. Combining this with the symbol bootstrap, we show that constraints from physical limits are sufficient to determine the answer. We thus obtain the first concrete characterization of two-loop six-gluon amplitudes at symbol level and at highest weight. Remarkably, we find that the effective function space involves only 137 symbol letters—significantly fewer than the full set of 167 possible letters, suggesting a yet-unexplained underlying structure akin to that seen in maximally supersymmetric Yang-Mills theory. From the novel amplitude results we extract previously unknown symbol-level results describing two-loop triple collinear and double soft limits.

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Introduction—Scattering amplitudes are fundamental objects describing particle interactions in quantum field theory. Beyond their role in high-energy phenomenology, they have revealed hidden structures of gauge theories and deep connections to mathematics, leading to radically new conceptual approaches to amplitude computation. Notable examples include generalized unitarity, on-shell methods, the Grassmannian formulation of leading singularities, and advances in understanding the special functions arising from Feynman integrals. For recent reviews, see Refs. [1–3].

The amplitude bootstrap [4,5] combines modern S-matrix ideas with these developments. In maximally supersymmetric Yang-Mills theory (sYM), bootstrap methods based on the symbol of iterated integrals [6,7] have enabled the determination of six-gluon scattering amplitudes through high loop orders, leading to a detailed understanding of their analytic structure in kinematic limits such as collinear and Regge regimes, and uncovering connections to integrability, cluster algebras, and leading singularities.

Extending bootstrap methods to QCD scattering amplitudes beyond one loop remains an open challenge. In this Letter, we take a first step in this direction by applying the symbol bootstrap, for the first time, to a six-particle scattering amplitude in massless QCD. This goes beyond the current state of the art, which is limited to two loops and five particles [8]. We parametrize the amplitude as

$$\mathcal{A} = \sum_{ij} c_{ij} R_i f_j, \tag{1}$$

where the special functions f_j and rational prefactors R_i depend on the scattering kinematics, while c_{ij} are the *Ansatz* coefficients. The relevant function space is known up to two loops [9,10] in the planar case, consisting of iterated integrals in seven kinematic variables with up to 167 symbol letters and maximal weight four. However, this knowledge alone does not determine the amplitude: the central challenge is to identify the specific linear combination selected by QCD.

A major bottleneck in this program concerns the rational prefactors R_i. Even at five points, their explicit form is highly nontrivial [11,12]. In sYM, these prefactors are

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governed by four-dimensional leading singularities [1], but no comparable organizing principle has been established for QCD.

In this Letter, we address this problem by focusing on the maximally transcendental contributions to Eq. (1), in the sense of Lipatov *et al.* [13,14]. Although the maximal-weight parts of QCD and sYM scattering amplitudes do not generally coincide [15], we show that the prefactors multiplying the highest-weight functions in QCD are remarkably simple. They are entirely captured by four-dimensional leading singularities, exhibit a conformal symmetry, and admit compact spinor-helicity representations. A relation between QCD prefactors and four-dimensional on-shell structures was previously observed in Ref. [16], but its broader implications for bootstrapping multiparticle amplitudes were not explored.

Building on this structure, we conjecture that these leading singularities form a complete basis for the rational prefactors at maximal transcendental weight. By imposing physical constraints within this bootstrap framework, we uniquely determine the weight-four symbol of the leading-color two-loop $--+++$ planar six-gluon amplitude. This constitutes the first successful application of the symbol bootstrap to gluon scattering amplitudes in massless QCD beyond one loop. As a byproduct, we extract the symbols of previously unknown triple-collinear splitting and double-soft functions. A companion paper provides more details and extends this proof of principle to all other maximally-helicity-violating (MHV) helicity configurations [17].

Our maximal-weight strategy is conceptually straightforward to extend to arbitrary multiplicity and to nonplanar contributions, although its practical implementation becomes more computationally demanding and is not pursued here. The extension to lower-weight terms is more subtle, and we comment on this issue and outline concrete avenues for future investigation in the outlook. More broadly, when combined with Landau analysis to constrain the relevant singularity structure, our approach has the potential to bypass the explicit computation of Feynman integrals altogether.

Leading singularities and prefactors at maximal weight—In this section, we discuss the prefactors multiplying maximal-weight terms in scattering amplitudes, following Ref. [16]. Generalized unitarity suggests that such prefactors are governed by leading singularities. An important subtlety arises, however, in dimensionally regulated amplitudes: even at maximal weight, additional kinematic prefactors can appear beyond the four-dimensional leading singularities. This phenomenon was observed in sYM and supergravity amplitudes at two loops, [18–21]. There, the $\mathcal{O}(\epsilon^0)$ term at two loops contains additional prefactors, which originate from leading singularities evaluated in $4-2\epsilon$ dimensions. However, those prefactors cancel for appropriate infrared-subtracted hard functions. (The definition will be given in the next section.) We will therefore adopt the conjecture that the four-dimensional leading singularities form a complete basis for prefactors in our *Ansatz* for the hard function.

The leading singularities can be computed from on-shell diagrams built from three-gluon on-shell vertices [1]. It can be shown that this on-shell "gluing" procedure preserves conformal invariance. This is in line with the classical conformal invariance of massless QCD theory. Therefore, we are guaranteed to obtain formulas that are annihilated by the operator [22],

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^{n} \frac{\partial}{\partial\lambda_i^{\alpha}} \frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\alpha}}}, \tag{2}$$

where we use the usual spinor-helicty parametrization of momenta, $p_i^{\alpha\dot{\alpha}} = \lambda_i^{\alpha}\tilde{\lambda}_i^{\dot{\alpha}}$, and $\langle ij\rangle = \lambda_i^{\alpha}\lambda_{j\alpha}$. This conformal invariance is trivially seen for the tree-level amplitude, which is given by the following Parke-Taylor factor,

$$\mathrm{PT}_{1,2} = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}, \tag{3}$$

and it also holds for all known expressions for one-loop six-gluon and six-photon amplitudes [23].

Let us focus on the $--+++$ six-gluon case from now on. Our goal is to predict the relevant leading singularities at two loops.

The one-loop $--+++$ helicity amplitudes were first calculated in Ref. [4]. We can see from the explicit formulas that the coefficients of the highest weight terms are simply given by Eq. (3). Indeed, this is consistent with evaluating the relevant on-shell box diagrams.

At two loops, the Parke-Taylor prefactor $R_1 = \mathrm{PT}_{1,2}$ is supplemented by six new leading singularities. These can be calculated from two-loop generalized unitarity, cf. Fig. 1 for the relevant on-shell diagrams. We evaluate these using techniques explained in Ref. [1]. For instance, the two-loop diagram for $R_{5,6}$ evaluates to the following one-form,

$$\mathrm{PT}_{1,2} \times \frac{z^3(\langle 12\rangle + z\langle 26\rangle)^3\langle 56\rangle^4}{\langle 12\rangle^4(\langle 15\rangle + z\langle 56\rangle)^4}\, dz. \tag{4}$$

The nontrivial two-loop leading singularity $R_{5,6}$ is then obtained by computing the value at one of the residues of Eq. (4). It turns out that same formula applies to all the leading singularities obtained from Fig. 1, and that the result can be written as follows,

$$R_{i,j}/R_1 = -1 + 12u_{i,j} - 30u_{i,j}^2 + 20u_{i,j}^3, \tag{5}$$

with $u_{i,j} := \langle 1i\rangle\langle 2j\rangle/(\langle 12\rangle\langle ij\rangle)$ Note that Eq. (5) is expressed in terms of λ spinors only, and hence conformal invariance is manifest.

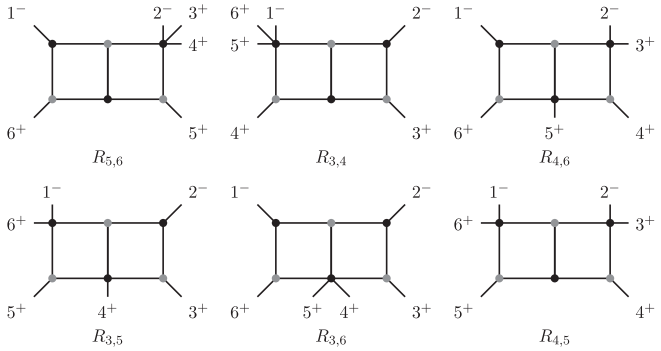


FIG. 1. On-shell diagrams giving rise to leading singularities $R_{i,j}$, for the $--++++$ helicity configuration.

We remark that Eq. (5) with $2 < i < j \leq n$ also captures all two-loop leading singularities with $--+\cdots+$ helicity configurations for arbitrary n. As a consistency check, one may verify that the known maximal-weight coefficients in the five-particle two-loop amplitudes, cf. Refs. [24,25], can be simplified to the expressions given here.

Amplitude symbol bootstrap—We now compute the symbol of the leading-color two-loop $--++++$ six-gluon amplitude in pure YM theory. We consider an infrared-finite hard function [26],

$$\mathcal{H}^{(2)} = \lim_{\epsilon\to 0}[\mathcal{A}^{(2)} - I^{(1)}\mathcal{A}^{(1)} - I^{(2)}\mathcal{A}^{(0)}]. \tag{6}$$

Here $\mathcal{A}^{(L)}(\epsilon)$ are planar color-stripped L-loop amplitudes. $I^{(1)}(\epsilon)$ is given by [27],

$$I_n^{(1)} = -\frac{1}{\epsilon^2}\sum_{i=1}^{n}\left(\frac{-s_{i,i+1}}{\mu^2}\right)^{-\epsilon}, \tag{7}$$

and, keeping only terms relevant at symbol level and at highest weight, $I^{(2)} = -\frac{1}{2}(I^{(1)})^2$, cf. [28]. Ultraviolet divergences are known to affect lower weight terms only (at least, up to the loop order relevant here), hence they do not influence the highest-weight approximation.

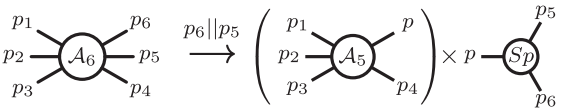
Our strategy to find the hard function $\mathcal{H}^{(2)}$ involves, 1. Choosing an appropriate *Ansatz*; 2. Imposing discrete symmetries on the *Ansatz*; 3. Requiring the physical singularity structure and consistency with the (multi)collinear limits. Remarkably, we find that the bootstrap assumptions, together with these physical conditions, not only fix the *Ansatz* uniquely, but also, as a bonus, determine the previously unknown triple collinear splitting and double soft functions.

Let us discuss the bootstrap procedure in more detail. 1. Our bootstrap *Ansatz* for the weight-four symbol terms of the two-loop hard function takes into account the seven leading singularities determined above,

$$\mathcal{H}^{(2)}_{\rm YM} = R_1 G_1 + \sum_{2<i<j\leq 6} R_{i,j} G_{i,j}. \tag{8}$$

The *Ansätze* for the G_1, $G_{i,j}$ are weight-four symbols of the two-loop six-leg Feynman integrals closed under cyclic permutations. In order to have the relevant space of functions G, it is important to take into account Eq. (6). Since the tree-level and one-loop amplitudes involve only the R_1 prefactor, it follows that only G_1 is affected by the infrared subtractions. We take this into account by adding appropriate product terms of $I^{(L)}$ and lower-loop amplitudes to our *Ansatz*. 2. The flip symmetry $(123456) \leftrightarrow (216543)$ of the $--++++$ helicity configuration reduces the *Ansatz* size for G. Requiring the functions G_1 and $G_{i,j}$ to have the same transformation law as R_1 and $R_{i,j}$ leads to an *Ansatz* (8) that contains 2412 unknowns. 3. The rational prefactors $R_{i,j}$ contain spurious poles $1/\langle ij\rangle^3$ for nonadjacent i, j, and higher-order poles $1/\langle ij\rangle^4$ for adjacent i, j, which have to be suppressed by the vanishing of the symbol $G_{i,j}$ at the locus $\langle ij\rangle = 0$. We also take into account that symbol entries of G have to be dimensionless.

If a pair of adjacent momenta become collinear, e.g., $p_5^{h_5} \to z p^h$ and $p_6^{h_6} \to (1-z)p^h$, an amplitude may become singular, in which case its leading terms factorize [29–31]. This can be schematically represented as follows,



. (9)

For the two-loop hard function $\mathcal{H}_6^{(2)} \equiv \mathcal{H}_{\rm YM}^{(2)}$, this turns into the following equation at maximal transcendental weight level,

$$\mathcal{H}_6^{(2)}/\mathcal{A}_6^{(0)} \overset{p_5\|p_6}{\longrightarrow} \mathcal{H}_5^{(2)}/\mathcal{A}_5^{(0)} + \mathcal{C}^{h,(1)}_{h_5h_6}\mathcal{H}_5^{(1)}/\mathcal{A}_5^{(0)} + \frac{1}{2}\left(\mathcal{C}^{h,(1)}_{h_5h_6}\right)^2, \tag{10}$$

where $\mathcal{A}_n^{(0)}$ are tree-level color-stripped amplitudes, $\mathcal{H}_5^{(L)}$ are five-particle L-loop MHV hard functions, and $\mathcal{C}^{h,(L)}_{h_5h_6}$ are infrared-subtracted L-loop corrections to the splitting functions for helicity $h_5h_6 \to h$. The $++ \to -$ splitting functions vanish at tree level, and their loop functions are of subleading transcendental weight, so we can neglect them in our Letter. The remaining splitting functions are (regardless of helicity)

$$\begin{aligned}\mathcal{C}^{h,(1)}_{h_5h_6} &= \log(1-z)\log\frac{p_1\cdot p}{p_5\cdot p_6} \\ &\quad + \log(z)\log(1-z) + \log(z) \\ &\quad \times \log\frac{p_4\cdot p}{p_5\cdot p_6}.\end{aligned} \tag{11}$$

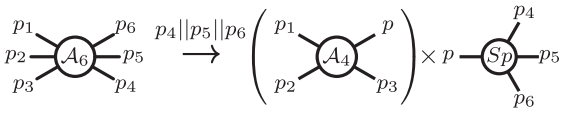
Likewise, for $\mathcal{H}_5^{(L)}$, calculated in Refs. [24,25], we need the $--+++$ helicity configuration only. The number of constraints obtained from collinear limits for adjacent pairs

TABLE I. Two-loop bootstrap constraints for $\mathcal{H}^{(2)}_{\rm YM}$.

Condition on $\mathcal{H}^{(2)}_{\rm YM}$	No. of constraints	
Dimensionless symbols G	996	996
No spurious/high-order poles		
$\langle 36\rangle = 0$	333	1189
$\langle 35\rangle = 0$	614	1546
$\langle 34\rangle = 0$	629	1916
$\langle 45\rangle = 0$	343	2116
Collinear limit $p_5 \| p_6$		
$--++++$	1785	2389
$+--+++$	1307	2394
$++--++$	1785	2394
$+++--+$	1646	2403
$-+++-+$	1646	2403
Triple collinear limit $p_4 \| p_5 \| p_6$		
$--++++$	1836	2412
$++--++$	724	2412
Total	2412	2412

of momenta (or, equivalently, taking $p_5 \| p_6$ for cyclic permutations of $\mathcal{H}_6$) are shown Table I.

For six-gluon amplitudes we can also take the triple collinear limit, which is defined by $p_i^{h_i} \to z_i p^h$, $i = 4, 5, 6$, with $z_4 + z_5 + z_6 = 1$. In this limit of three external legs becoming collinear, gauge theory amplitudes are expected to factorize into products of splitting functions, and lower point functions,



(12)

When applied to the finite hard functions of Eq. (6), this translates to

$$\mathcal{H}^{(2)}_6/\mathcal{A}^{(0)}_6 \overset{p_4\|p_5\|p_6}{\longrightarrow} \mathcal{H}^{(2)}_4/\mathcal{A}^{(0)}_4 + \mathcal{C}^{h,(1)}_{h_4h_5h_6}\mathcal{H}^{(1)}_4/\mathcal{A}^{(0)}_4 + \frac{1}{2}\left(\mathcal{C}^{h,(1)}_{h_4h_5h_6}\right)^2 + \mathcal{C}^{h,(2)}_{h_4h_5h_6}. \quad (13)$$

$\mathcal{H}^{(L)}_4$ are the four-point pure YM hard functions of the sector $--++$, and $h_4h_5h_6 \to h$ can be $+++ \to +$, $-++ \to -$, or $++- \to -$. The two-loop corrections to the triple splitting functions $\mathcal{C}^{h,(2)}_{h_4h_5h_6}$ are not yet known. However, we can demand that the kinematic dependence of $\mathcal{H}_6$ factorizes in the limit such that $\mathcal{C}^{h,(2)}_{h_4h_5h_6}$ depends solely on p_4, p_5, p_6. Moreover, we can impose that $\mathcal{C}^{+,(2)}_{+++}$ is invariant under swapping p_4 and p_6.

Table I summarizes the conditions we impose, with the second column listing the number of constraints each condition places on the initial *Ansatz*. The third column gives the cumulative number of independent constraints obtained by applying the conditions sequentially from top to bottom. At any stage of this process, an inconsistency between the *Ansatz* and the imposed conditions could in principle arise. The fact that all conditions can be satisfied simultaneously therefore provides a nontrivial cross-check of our procedure.

Including fermions—So far we have considered the pure YM part of the amplitude. As we are interested in QCD, let us include fermions circulating in the loops. In order to implement the planar limit, we take the number of flavors N_f to be large, while keeping N_f/N_c fixed. As a consequence, QCD hard functions at two loops always have the following expansion,

$$\mathcal{H}^{(2)}_{\rm QCD} = \mathcal{H}^{(2)}_{\rm YM} + \left(\frac{N_f}{N_c}\right)\mathcal{H}^{[1]} + \left(\frac{N_f}{N_c}\right)^2\mathcal{H}^{[2]}. \quad (14)$$

Compared to the pure-gluon sector, there are fewer physically relevant on-shell diagrams involving fermion loops. As a result, contributions at higher orders in N_f are relatively easier to compute.

For $\mathcal{H}^{[2]}$, the result is IR finite and receives contributions solely from "kissing box" on-shell diagrams and their associated integrals [17]. What is more, the helicity configuration $--++++$ does not receive any contribution from this cut. Consequently, the maximally transcendental component of $\mathcal{H}^{[2]}$ vanishes [32].

For $\mathcal{H}^{[1]}$, we find the following six leading singularities,

$$S_{i,j}/R_1 = -2 + 12u_{i,j} - 21u^2_{i,j} + 11u^3_{i,j}, \quad (15)$$

with $2 < i < j \le 6$. Based on this input, we bootstrap the corresponding hard function as described in the previous section. The fermionic bootstrap turns out to be simpler—this sector can be fully determined by enforcing the first three groups of conditions in Table I; the triple-collinear limit is not required.

Discussion of the result—We obtained the full QCD answer for the maximal-weight symbol of the planar two-loop $--++++$ helicity amplitude. The explicit results are provided in Supplemental Material [33]. It is worth noting that the functions accompanying the leading singularities $R_{i,j}$ and $S_{i,j}$ with the same indices coincide. This can be understood by the fact that these two sectors can effectively be viewed as components of a six-particle $\mathcal{N} = 1$ sYM amplitude. Moreover, since the subtraction, cf. Eq. (7), involves two-particle poles only, the hard functions G_1 and $G_{i,j}$ satisfy the extended Steinmann relations [5,34]. Finally, we find that the symbol alphabet, together with its cyclic permutations, comprises 137 letters. The same set has already been identified in studies of six-point two-loop Wilson loops with Lagrangian insertion [35,36].

Triple-collinear and double-soft splitting functions—In the bootstrap procedure, we enforced consistency with

triple-collinear limits to fix the result. An important byproduct of our result is that we can determine the triple-collinear splitting functions. At one loop, the splitting functions were considered in Refs. [37–39]. Thanks to our bootstrap result, we can provide novel information at two loops, on $\mathcal{C}^{h,(2)}_{h_4h_5h_6}$. We find, at maximal transcendental weight level,

$$\mathcal{C}^{+,(2)}_{+++} = \mathcal{C}^{(2)}_{\mathcal{N}=4\ \mathrm{sYM}}, \tag{16}$$

$$\mathcal{C}^{-,(2)}_{-++} = \mathcal{C}^{+,(2)}_{+++} + \left(r^{-}_{-++} + \frac{N_f}{N_c} s^{-}_{-++}\right) w. \tag{17}$$

Here $\mathcal{C}^{(2)}_{\mathcal{N}=4\ \mathrm{sYM}}$ is the triple splitting function known from the $\mathcal{N}=4$ sYM six-point two-loop remainder function [40], w is a weight-four function depending on the two variables s_{45}/s_{456} and s_{56}/s_{456} only, where $s_{ij} = (p_i + p_j)^2$ and $s_{456} = (p_4 + p_5 + p_6)^2$, and finally

$$r^{-}_{-++} = -2 + 12u - 30u^2 + 20u^3, \tag{18}$$

$$s^{-}_{-++} = -2 + 12u - 21u^2 + 11u^3 \tag{19}$$

where u is given by the triple-collinear limit of $(\langle 35\rangle\langle 46\rangle)/(\langle 34\rangle\langle 56\rangle)$, depending on the details of the implementation of the limit, see, e.g., Ref. [39].

Another important byproduct of our result are the double-soft splitting functions, which are currently known up to one-loop order [41,42] only. When two external legs $p_5^{h_5}$, $p_6^{h_6}$ are taken to be soft, amplitudes are expected to factorize. At two loops, the six-point hard functions behave under double-soft limits as follows,

$$\mathcal{H}^{(2)}_6/\mathcal{A}^{(0)}_6 \overset{p_5,p_6\to 0}{\longrightarrow} \mathcal{H}^{(2)}_4/\mathcal{A}^{(0)}_4 + \mathcal{S}^{(1)}_{h_5h_6}\mathcal{H}^{(1)}_4/\mathcal{A}^{(0)}_4 + \frac{1}{2}\left(\mathcal{S}^{(1)}_{h_5h_6}\right)^2 + \mathcal{S}^{(2)}_{h_5h_6}. \tag{20}$$

The maximal-weight part of $\mathcal{S}^{(2)}_{++}$ can be extracted from our bootstrap result. We find

$$\mathcal{S}^{(2)}_{++} = \mathcal{S}^{(2)}_{\mathcal{N}=4\ \mathrm{sYM}}, \tag{21}$$

where, remarkably, $\mathcal{S}^{(2)}_{\mathcal{N}=4\ \mathrm{sYM}}$ is the double-soft splitting function from the $\mathcal{N}=4$ sYM six-point two-loop remainder function.

The results for the splitting functions of Eqs. (16), (17), and (21) are recorded in the Supplemental Material [33].

Discussion and outlook—We bootstrapped, for the first time, the symbol of the planar two-loop $--+++$ scattering amplitude in QCD, at maximal transcendental weight. While this is the most complicated part of the amplitude in terms of special functions, we showed that it is also the simplest part in terms of coefficients. The fact that we were able to find an answer that is fully consistent with various physical requirements can be taken as evidence for the correctness of the result, and in particular of the conjecture that the prefactors at maximal weight can be obtained from four-dimensional leading singularities. We provide our results in computer-readable form in Supplemental Material [33].

Although the total two-loop alphabet comprises 245 letters, of which 167 can appear up to weight four, we find that only 137 of them appear in the final answer. We take this observation as a hint for further structure, which is yet to be understood. The first steps in this direction have been made in two recent papers that suggest connections of some of the six-particle letters to flag varieties, cf. [43,44].

The symbol of the triple collinear splitting function $g \longrightarrow ggg$ that we provide predicts a part of the leading-color two-loop amplitude $gg \to Hgg$. Similarly, we obtained a new result for the symbol of the planar double soft function, extending previous one-loop results [41,42]. In view of its phenomenological relevance, it would be very interesting to extend these results to the function level, and to include the lower-weight terms.

There are several natural extensions of our Letter. 1. Our bootstrap framework extends to other helicity sectors and to processes with external fermions. In upcoming work, we explicitly work out the result for all MHV helicity configurations [17]. The next logical step is to consider next-to-MHV helicity configurations, where we expect additional alphabet letters from Refs. [9,10] to be required, and where certain leading singularities related to three-mass cut configurations may appear. 2. In the present Letter, we benefited from knowledge of the complete two-loop function space, which had been obtained by traditional means. We anticipate that this requirement can be relaxed in future work. One possibility is to instead obtain the information for the symbol bootstrap solely from Landau analysis, see, e.g., Refs. [45–48]. Interestingly, experience from the symbol bootstrap program in sYM suggests that it may be easier to fix the *Ansatz* for higher multiplicity [49]. Another possibility, which we pursue in upcoming work [17], is to combine our approach with ideas from prescriptive unitarity [50], which effectively reduces the number of Feynman integrals that need to be considered. 3. The connection between amplitude coefficients and on-shell diagrams opens up a number of exciting new directions. These include applications to the nonplanar case, as well as to arbitrary particle multiplicities, for which closed-form expressions may be obtained—something that is entirely out of reach for conventional methods. Another promising avenue is to further investigate the effective supersymmetry relations we have identified, along with Grassmannian representations of on-shell diagrams for the QCD case, extending ideas from Ref. [1]. 4. We focused on the symbol, since the latter captures the essential structural

information. Repeating this bootstrap at the function level can be done at the level of iterated integrals, using the differential equation representation to generate the necessary expansions (cf. for example Ref. [51]). 5. Extending our results below the maximal weight is perhaps the most interesting conceptual point. What is the set of prefactors of transcendental functions needed to describe QCD scattering amplitudes? How much information beyond four dimensions, if any, is needed to describe appropriately defined hard parts of scattering amplitudes? These questions are even interesting at the one-loop order, where the rational part is known to be related to subtle "0/0" effects, cf., e.g., Ref. [52]. There are a number of interesting approaches to answer these questions, including loop-level recursion relations [53], novel integrand representations that highlight the transcendental weight structure of scattering amplitudes [54], and the novel surface integral formulation of Yang-Mills integrands [55]. We hope to revisit these questions in the near future.

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Data availability—The data that support the findings of this article are openly available [33].

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