

Resummation of Universal Tails in Gravitational Waveforms

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We present a formula for the universal anomalous scaling of the multipole moments of a generic gravitating source in classical general relativity. We derive this formula in two independent ways using effective field theory methods. First, we use the absorption of low-frequency gravitational waves by a black hole to identify the total multipole scaling dimension as the renormalized angular momentum of black hole perturbation theory. More generally, we show that the anomalous dimension is determined by phase shifts of gravitational waves elastically scattering off generic source multipole moments, which reproduces the renormalized angular momentum in the particular case of black holes. The effective field theory approach thus clarifies the role of the renormalized angular momentum in the multipole expansion. The universality of the point particle effective description of compact gravitating systems further allows us to extract the universal part of the anomalous dimension, which is the same for any object, including black holes, neutron stars, and binary systems. As an application, we propose a novel resummation of the universal short-distance logarithms (“tails”) in the gravitational waveform of binary systems, which may improve the modeling of signals from current and future gravitational wave experiments.

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Introduction and executive summary—The detection of gravitational waves (GWs) from inspiraling black holes (BHs) by the LIGO-Virgo-KAGRA experiment has brought about the era of precision strong-gravity science [1–8]. The two-body problem cannot be solved exactly in full general relativity (GR), so a number of approximate techniques have been utilized for precision calculations of gravitational waveforms to model the observed GW signals [9–31]. One such technique is the gravitational effective field theory of inspiraling binaries (EFT), which applies quantum field theory tools to this problem [12–14,32–35]. The EFT approach makes the relevant degrees of freedom and underlying symmetries of the problem manifest and allows for a systematic treatment of large- and small-scale divergences that appear in the perturbative description of the GWs emission. The effective action describing the GWs emitted by the binary is given by the Einstein-Hilbert term

together with the worldline effective action,

$$- \int d\tau \left[\mathcal{E} + \frac{1}{2} \omega_{ij} J^{ij} + \frac{1}{2} Q_{ij}^E E^{ij} + \frac{1}{2} Q_{ij}^B B^{ij} + \dots \right], \quad (1)$$

where τ is proper time; $\mathcal{E}$, J^{ij} are the total conserved energy and the angular momentum of the binary; ω_{ij} is its angular velocity; $Q_{ij}^{E/B}$ are its gravitational electric and magnetic quadrupole moments; while E^{ij} , B^{ij} are the electric and magnetic parts of the Weyl curvature tensor describing the emitted radiation. The dots stand for the coupling to higher multipole moments as well as tidal operators, which depend on higher powers of the curvature. We note that the multipoles generically include spin-induced contributions [36–45].

The above action simply encodes the fact that the binary, seen from large distances, can be approximated as a point source with given energy and angular momentum and with a collection of multipole moments attached to it. This approximation is adequate for GWs emitted during the inspiraling phase as their wavelength λ is much greater than the size of the binary r or has frequency $\omega \ll r^{-1}$.

Relativistic corrections to the physical waveforms arising from the nonlinear structure of GR are interpreted as classical “loop corrections” in EFT. Among these corrections, particularly interesting ones are logarithmic terms that arise from small-scale “ultraviolet” (UV) parts of EFT

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integrals, which are referred to as (dissipative) tails [46]. Such tails, when combined with a near-zone description of the binary (valid for frequencies $\omega \sim r^{-1}$), yield finite logarithmic contributions to the waveform [9,10]. However, the binary's near zone is not described by the EFT in Eq. (1), and hence, the tails manifest as UV divergences in the effective description. This is, in fact, not a “bug” but a feature of the EFT description, which allows us to interpret these divergences as the (classical) renormalization group running of the radiative multipoles [49], thus enabling the resummation of the associated logarithms in the waveform or other observable quantities.

For simplicity, let us use the harmonic space counterparts to the quadrupole moments $Q_{2m}^{E/B}$ (where m is the azimuthal harmonic number) [50–52], which satisfy the following renormalization group (RG) equation [49],

$$\mu \frac{d}{d\mu} Q_{2m}^{E/B} = \gamma_{2m}^{E/B} Q_{2m}^{E/B}, \quad (2)$$

where μ is the matching scale and γ_{2m} is the multipole anomalous dimension, which is defined by this equation and captures the coefficient of the dissipative tail logarithms in the waveform. Previous calculations found $\gamma_{2m}^{E/B} = -(214/105)(G\mathcal{E}\omega)^2$ at the lowest order [49,53], where G is Newton's constant. The physical interpretation of this RG equation is that the relativistic gravitational potential of the binary effectively adds up to the quadrupole moment, leading to more radiation emitted. Separating the potential modes from the source quadrupole, i.e., “integrating the potential modes out,” by lowering μ (or, equivalently, by increasing the size of the near zone) increases the effective quadrupole moment, giving rise to its scale dependence akin to the scale dependence of the coupling constant in quantum chromodynamics. The multipole of a generic gravitating system satisfies such RG equation, as it stems from the interaction of the radiation with relativistic potential fields sourced by the binding energy and angular momentum in Eq. (1), which is fixed by the nonlinear structure of GR.

While some higher order results for the anomalous dimension of the quadrupole and higher multipole moments have been derived in the literature [54–57], in this Letter, we present *an exact formula* for the anomalous dimensions of all multipoles. Namely, we show that the anomalous dimension of the multipoles with angular and azimuthal numbers (ℓ, m) is determined by the partial wave phase shift $\delta_{\ell m}$ of gravitational waves elastically scattering off the system [see, e.g., Eq. (17) in Ref. [58]],

$$\gamma_{\ell m}^{E/B} = -\frac{1}{\pi} \left(\delta_{\ell m}^{E/B}(\omega) + \delta_{\ell m}^{E/B}(-\omega) \right), \quad (3)$$

where $\delta_{\ell m}^{E/B}(-\omega)$ is the phase shift describing the time reversal of the same scattering process.

The equivalence principle dictates that Eq. (1) describes any system interacting with the long-wavelength GWs. This implies that the universal part of the anomalous dimension in Eq. (3) can be extracted from any gravitational scattering process. This is true for both the emission and absorption of gravitational waves. The equivalence between the two is akin to relations between Einstein coefficients for absorption and stimulated emission in atomic physics [59]. In particular, we can use a simple problem with a known result: the Raman (i.e., inelastic) scattering of long-wavelength GWs off a solitary BH to read off the universal part of the anomalous dimension. The scattering amplitudes of this process can be computed *exactly* using black hole perturbation theory (BHPT) [60–70].

It is known from BHPT that the scale dependence of BH multipole moments in the near zone is captured by the so-called “renormalized angular momentum” ν [63–66], which depends on the BH mass M , spin $\chi = S/(GM^2)$, and the GW frequency ω (see, e.g., Refs. [51,71,72]). Our relation between the anomalous dimension and scattering phase shows that this quantity precisely determines the anomalous dimension of BH multipole moments,

$$\gamma_{\ell m}^{\text{BH}} = \nu(GM\omega, \chi) - \ell, \quad (4)$$

which can be computed analytically for generic ℓ to any given order in $GM\omega$. Below, we provide an independent argument for this based directly on the absorptive scattering of gravitational waves by BHs.

Note that the multipole moments in Eq. (1) depend on the system's spin. The rotating (Kerr) BHs obey the “no-hair” theorem [73], dictating that all of their multipole moments are uniquely fixed by their spin and mass. This is not true for a generic gravitating system, for which spin-induced multipole moments [40] and tidal effects (Love numbers) [32,51,58,74–84] break the universality starting at $\mathcal{O}(J^2)$ and $\mathcal{O}(G^{2\ell+1})$, respectively. This means that the multipole anomalous dimension of a BH is actually universal through $\mathcal{O}(JG^{2\ell+1})$. Such universal anomalous dimension, valid for a general system, is then obtained simply by expanding the BH anomalous dimension to the appropriate order and replacing $M \rightarrow \mathcal{E}$ and $\chi \rightarrow \mathcal{J} \equiv J/(G\mathcal{E}^2)$, which yields the universal anomalous dimension

$$\gamma_{\ell m}^{\text{univ}} = \left[\gamma_{\ell m}^{\text{BH}}(G\mathcal{E}\omega, 0) + \partial_{\chi} \gamma_{\ell m}^{\text{BH}}(G\mathcal{E}\omega, 0) \mathcal{J} \right]_{G^{2\ell+1}}, \quad (5)$$

where $[\cdots]_{G^n}$ denotes an expansion through order n . The knowledge of this anomalous dimension allows for a resummation of the universal short-distance logarithms (tails) by means of the RG Eq. (2). We claim that this result holds true for *any* gravitating system: a BH, a neutron star, or an inspiraling binary system.

In the rest of this Letter, we provide an explicit proof to the above statements and produce waveform predictions that

Black hole anomalous dimension from the renormalized angular momentum—Let us start by showing that the anomalous dimension of a generic BH multipole moment is given by the BHPT renormalized angular momentum. To that end, we use the EFT action (1) with the full collection of internal multipole moments. These multipole moments describe the absorption of waves by the BH horizon, producing inelasticity in the Raman scattering amplitude [35,58,85]. Let us focus now on the electric moment only. The computation for the magnetic part is identical. The inclusive absorption cross section in the EFT at the tree level is given by the sum of the on-shell single-graviton absorption amplitudes over different internal excited states X of the black hole [13,35]:

$$i\mathcal{M}_{\text{abs}}^{G^2}(\omega) = \begin{array}{c} Q^{\text{abs}} \bigcirc \\ \diagup \quad \diagdown \\ M(S) \bigotimes \quad \bigotimes M \\ \diagdown \quad \diagup \\ M \bigotimes \quad \bigotimes M \end{array} + \begin{array}{c} \bigcirc \\ \diagup \quad \diagdown \\ \bigotimes \quad \bigotimes \\ \diagdown \quad \diagup \\ \bigotimes \quad \bigotimes \end{array},$$

where $\epsilon = GM\omega$ while a_k, b_k are numerical coefficients. For clarity, we omitted the unobservable IR logs, so that all the logs above stem from UV divergences. The coefficients a_n above are, in general, divergent, and they are to be renormalized via a multiplicative wave function renormalization [49]. The renormalized EFT expression can be compared with the BHPT result [63–65,85],

Focusing on the single-multipole term in Eq. (10), factoring out the tree-level part, and formally inserting the matching scale μ , we split the above formula into the EFT (or far-zone) and UV (or near-zone) parts:

$$\begin{aligned}\Gamma_{\ell m} &= (GM\omega)^{2\ell+1} (GM\omega)^{2(\nu-\ell)} \mathcal{A}_\omega \\ &= \left(\frac{\omega}{\mu}\right)^{2(\nu-\ell)} (GM\mu)^{2(\nu-\ell)} \omega^{2\ell+1} \text{Im} G_{\ell m}^R(\omega) \left(1 + \sum c_n \epsilon^n\right) \\ &= \left(\frac{\omega}{\mu}\right)^{2(\nu-\ell)} \left(1 + \sum \tilde{c}_n \epsilon^n\right) \omega^{2\ell+1} \langle Q_{\ell m}^{\text{ren}} Q_{\ell m}^{\text{ren}} \rangle(\omega, \mu),\end{aligned}\quad (12)$$

where $(\omega/\mu)^{2(\nu-\ell)}$ represents the sum of UV tails from the logarithmically divergent loop integrals, $\sum c_n \epsilon^n$ denotes the finite loop corrections [86], while $Q_{\ell m}^{\text{ren}}$ is the renormalized scale-dependent multipole,

$$Q_{\ell m}^{\text{ren}}(\omega; \mu) = \left(\frac{\mu}{\mu_0}\right)^{\nu-\ell} Q_{\ell m}^{\text{ren}}(\omega; \mu_0), \quad (13)$$

with the matching scale comparable to the inverse size of the BH, $\mu_0 \sim (GM)^{-1}$. The above implies the following RG flow of the absorptive multipole moment operator, cf. Eqs. (2), (4),

$$\frac{dQ_{\ell m}^{\text{ren}}(\omega; \mu)}{d \log \mu} = (\nu - \ell) Q_{\ell m}^{\text{ren}}(\omega; \mu), \quad (14)$$

which resums tail logarithms associated with UV divergences in diagrams such as those in Eq. (8). There are additional logarithms that are produced when the series expansion of ν in Eq. (10) hits poles in $\mathcal{B}_\omega$. These correspond to the nonuniversal pieces described by EFT diagrams that feature the insertion of dynamical tidal Love number operators; see, e.g., Refs. [58,87]. This issue manifests itself as poles for integer values of ℓ in the perturbative expansion of ν at starting order $G^{2\ell+3}$.

Renormalization of radiative multipoles from scattering—Let us now give a more general argument that will confirm the equivalence between the renormalized angular momentum and the multipole anomalous dimensions. The emission of gravitational waves in the far zone of a generic system is controlled by the radiative multipoles $Q_{\ell m}^{\text{rad}}$ [12,32,49]. For instance, the leading order is described by the Einstein quadrupole formula. The universality of the worldline EFT suggests that the tail effects in the inspiral binary waveforms originate from short-distance (near-zone) corrections to the radiative multipoles [49]. A more general formula for the anomalous dimension of multipoles in terms of scattering phase shifts is provided by Eq. (3). We now derive this formula.

Let us consider how gravitational waves emitted from the radiative multipoles travel through the gravitational background of the binary out to infinity. This process is

described by the following local worldline EFT operators: $\mathbf{O}_{\ell m}^E(\tau) = Q_{\ell m}^{\text{E,rad}} E_{\ell m}$ and $\mathbf{O}_{\ell m}^B(\tau) = Q_{\ell m}^{\text{B,rad}} B_{\ell m}$. It is useful to consider the symmetric (Keldysh) correlator, which is manifestly time-reversal invariant,

$$G_S^P(\omega) = \frac{1}{2} \langle \{ \mathbf{O}_{\ell m}^P(-\omega), \mathbf{O}_{\ell m}^P(\omega) \} \rangle, \quad (15)$$

with parity $P = \pm 1$ for E/B . This correlator captures the intrinsic fluctuations of compact objects in a gravitational wave background across time and energy scales. Since the classical tidal fields do not experience classical RG running, the dilatation operator, $D \equiv \omega \partial_\omega = -\mu \partial_\mu$, acts trivially on the gravitational field, which implies that this correlator satisfies the RG evolution equation dictated by the multipole moments:

$$\frac{dG_S^P(\omega; \mu)}{d \log \mu} = 2\gamma_{\ell m}^P(\omega) G_S^P(\omega; \mu). \quad (16)$$

To relate the anomalous dimension for the radiative multipoles to scattering, we follow the ideas of [88] and study the analytic dependence of this correlator as a function of the frequency, ω . In particular, we consider the analytic continuation to negative frequencies,

$$\omega \rightarrow e^{i\pi} \omega. \quad (17)$$

On the one hand, by making use of the dilatation operator, such analytic continuation extracts the anomalous dimension as a phase,

$$G_S^P(e^{i\pi} \omega) = e^{i\pi D} G_S^P(\omega) = e^{i\pi 2\gamma_{\ell m}^P} G_S^P(\omega). \quad (18)$$

On the other hand, the analytically continued correlator is simply related to its complex conjugation,

$$G_S^P(e^{i\pi} \omega) = G_S^{*P}(\omega) = \frac{1}{2} \langle \{ \mathbf{O}_{\ell m}^{P\dagger}(-\omega), \mathbf{O}_{\ell m}^{P\dagger}(\omega) \} \rangle. \quad (19)$$

Considering the insertion of operators $\mathbf{O}^P$ as a perturbation to the S matrix describing the scattering of gravitational waves by the system, $\delta S = i\mathbf{O}^{P\dagger}$, unitarity then implies

$$\mathbf{O}_{\ell m}^{P\dagger} = S^\dagger \mathbf{O}_{\ell m}^P S^\dagger. \quad (20)$$

Inserting this relation into Eq. (19) and using the partial wave basis, we find

$$G_S^P(e^{i\pi} \omega) = e^{-2i[\delta_{\ell m}^P(\omega) + \delta_{\ell m}^P(-\omega)]} G_S^P(\omega), \quad (21)$$

where $\delta_{\ell m}^P(-\omega)$ is the analytic continuation of the phase shift performed with fixed $\mathcal{J}$, that is, the time-reversed phase shift. Comparing Eqs. (19) and (21), we find that the anomalous dimension is directly related to the phase shift,

as advanced in Eq. (3). This is an exact relation for the anomalous dimension of radiative multipoles, valid for a generic system.

For the specific case of BHs, using the known formulas for Raman scattering from BHPT [see, e.g., Eq. (4.3) of Ref. [70] and Eq. (3.13) in [89]], we recover the claimed result for the BH anomalous dimension in Eq. (4) for generic ℓ .

Universal anomalous dimension from black holes—Let us now discuss to what extent the exact result for the anomalous dimension of BH multipole moments in Eq. (4) applies to generic systems. First of all, while the nonlinear interactions with the energy term in the action (1) are the same for any system, the angular momentum (spin) dependent terms beyond the linear one are specific to a gravitating source. Furthermore, it is important to note that the scattering phase shift, as computed in the EFT, receives contributions from nonuniversal tidal Love numbers starting at $\mathcal{O}(G^{2\ell+1})$. The leading contribution from these is odd in the frequency, and hence, cancels in Eq. (3). However, starting at the next order, diagrams containing the tidal operators will generate nonuniversal corrections to the

anomalous dimension. The situation is even worse starting at $\mathcal{O}(G^{2\ell+3})$, where UV divergences in the far-zone phase shift appear, requiring the introduction of (running) dynamical Love numbers [35,58,87,89].

Hence, the universal part of the anomalous dimension can be extracted from that of BH via a formal Taylor expansion in spin and G :

$$\gamma_{\ell m}^{\text{BH}}(GM\omega, \chi) = \left[\gamma_{\ell m}^{\text{BH}}(GM\omega, 0) + \partial_\chi \gamma_{\ell m}^{\text{BH}}(GM\omega, 0) \chi \right]_{G^{2\ell+1}} + \gamma_{\ell m}^{\text{BH, nonuniversal}}. \quad (22)$$

Replacing M by $\mathcal{E}$ and χ by $\mathcal{J}$ in the first two terms then gives the universal part of the anomalous dimension for a generic system, reproducing Eq. (5).

The fact that the anomalous dimension of the BHs is given by the BHPT renormalized angular momentum (reviewed in Supplemental Material [90]) provides us with a detailed understanding of γ^{univ} . For instance, for general ℓ , its low-orders perturbative expansion is given by

$$\gamma_{\ell m}^{\text{univ}} = -\frac{2[15\ell^2(\ell+1)^2 + 13\ell(\ell+1) + 24]}{(2\ell+1)\ell(\ell+1)[4\ell(\ell+1)-3]} \epsilon^2 + \frac{8m\chi(5\ell^3(\ell+1)^3 - \ell^2(\ell+1)^2 + 18\ell(\ell+1) + 108)}{(2\ell+1)\ell^2(\ell+1)^2[\ell(\ell+1)-2][4\ell(\ell+1)-3]} \epsilon^3 + \mathcal{O}(\epsilon^4), \quad (23)$$

with $\epsilon = G\mathcal{E}\omega$, where the first term agrees with the well known tail prefactor [54,102]. The explicit form through $\mathcal{O}(\epsilon^6)$ is given in Supplemental Material.

In particular, the quadrupolar ($\ell = 2$) universal anomalous dimensions are given by

$$\gamma_{2m}^{\text{univ}} = -\frac{214}{105} \epsilon^2 + \frac{2m\mathcal{J}}{3} \epsilon^3 - \frac{3390466}{1157625} \epsilon^4 + \frac{381863m\mathcal{J}}{99225} \epsilon^5, \quad (24)$$

and the octupolar ($\ell = 3$) one is

$$\gamma_{3m}^{\text{univ}} = -\frac{26}{21} \epsilon^2 + \frac{7m\mathcal{J}}{3} \epsilon^3 - \frac{21842}{33957} \epsilon^4 + \frac{286631m\mathcal{J}}{935550} \epsilon^5 - \frac{381415329076}{481821815475} \epsilon^6 + \frac{96516668989m\mathcal{J}}{136150591500} \epsilon^7. \quad (25)$$

The first three terms of γ_{2m} and the first term in γ_{3m} agree with the known results [49,53–56,91], while the rest are new results. Our formalism thus explicitly confirms that the anomalous dimensions for electric and magnetic multipoles are the same through $\mathcal{O}(JG^{2\ell+1})$, confirming earlier leading-order results by [91]. This settles the tension in the literature between [54] and [91]. Note, however, that while the universal magnetic and electric anomalous dimensions are the same, the electric and magnetic phase shifts $\delta_{\ell m}^{E/B}$ are different, but their difference cancels in Eq. (3).

In the eikonal limit, i.e., $G\mathcal{E}\omega \gg 1, \ell \gg 1$ but with $G\mathcal{E}\omega/\ell$ fixed, we are able to obtain the result

$$\gamma_{\ell m}^{\text{univ}} \Big|_{\text{eik}} = \ell \left(-1 + {}_3F_2 \left[-\frac{1}{2}, \frac{1}{6}, \frac{5}{6}; \frac{1}{2}, 1; 27x^2 \right] \right) + 5m\mathcal{J}x^3 {}_3F_2 \left[\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, \frac{5}{2}; 27x^2 \right], \quad (26)$$

where $x = G\mathcal{E}\omega/\ell = G\mathcal{E}/b$, and $b = \ell/\omega$ is the impact parameter. In this exact formula, we observe that the result has a branch cut starting at the impact parameter $b = 3\sqrt{3}G\mathcal{E}$, which intriguingly coincides with the radius of the BH shadow. See Refs. [103,104], where some of these functions also appeared recently.

Applications to waveform tail resummation—EFT allows one to compute the binary inspiral waveforms directly from the radiative multipoles [12]. The universal anomalous dimension in Eq. (5) can then be used to resum the ultraviolet tails in the waveform. This is most conveniently done in the factorized multipolar post-Minkowskian (MPM) framework [9,10,96,97,105,106]. The mode decomposition for the complex linear combination of the GW polarizations $h(t) \equiv h_+(t) - ih_\times(t)$ in terms of the spin weight $s = -2$ spherical harmonics is

$$h(t; \theta, \phi) = \sum_{\ell \geq 2} \sum_{|m| \leq \ell} -2Y_{\ell m}(\theta, \phi) h_{\ell m}(t). \quad (27)$$

The mode function $h_{\ell m}(t)$ in the inspiral phase can be factorized as [96–98,105,106]

$$h_{\ell m} = h_{\ell m}^N \hat{S}_{\text{eff}} T_{\ell m} \tilde{h}_{\ell m}, \quad (28)$$

where $h_{\ell m}^N$ is the Newtonian multipole; $\hat{S}_{\text{eff}}$ is the dimensionless effective source term given by either the effective one-body energy, E_{eff} [30,31], or the orbital angular momentum, p_ϕ ; $T_{\ell m}$ is a tail resummation factor; and $\tilde{h}_{\ell m}$ is the remainder, often further decomposed in amplitude and phase as $\tilde{h}_{\ell m} = (\rho_{\ell m})^\ell e^{i\delta_{\ell m}}$. In this Letter, we focus on improving the tail resummation $T_{\ell m}$. Physically, the tail effects capture the amplitude and the phase deflection from the wave propagation in the asymptotic background geometry. We find it convenient to further decompose the tail part as follows:

$$T_{\ell m} = \mathcal{S}_{\ell m} e^{i\delta_{\ell m}^{\text{tail}}}. \quad (29)$$

We will refer to the amplitude $\mathcal{S}_{\ell m}$ as the Sommerfeld enhancement factor by analogy with the Coulombic scattering. Damour and Nagar proposed the following tail factors [96]:

$$\mathcal{S}_{\ell m} = \frac{|\Gamma(\ell + 1 - 2iG\mathcal{E}\omega)|}{\Gamma(\ell + 1)} e^{\pi G\mathcal{E}\omega}, \quad (30)$$

$$\delta_{\ell m}^{\text{tail}} = \frac{1}{2} \text{Arg} \left[\frac{\Gamma(\ell + 1 - 2iG\mathcal{E}\omega)}{\Gamma(\ell + 1 + 2iG\mathcal{E}\omega)} \right] + (2G\mathcal{E}\omega) \log(2\omega r_{\text{orb}}), \quad (31)$$

which resum an infinite number of leading (infrared) logarithms of the form $\omega^n \log^n \omega$ and associated finite parts in the Sommerfeld factor. Indeed, this form was inspired by considering wave propagation and rescattering against the Newtonian $1/r$ potential of the binary [107] in the far zone, and Eqs. (30) and (31) correspond to the Sommerfeld factor and phase shift for Coulombic scattering.

We are now in the position to improve upon Eqs. (30) and (31) by proposing a formula that resums both the infrared tails and the universal ultraviolet tails:

$$\mathcal{S}_{\ell m} = \left| \frac{\Gamma(\hat{\nu} + 1 - 2iG\mathcal{E}\omega)}{\Gamma(\hat{\nu} + 1)} \right| e^{\pi G\mathcal{E}\omega} (r_{\text{orb}}\omega)^{\hat{\nu}-\ell}, \quad (32)$$

$$\delta_{\ell m}^{\text{tail}} = \frac{1}{2} \text{Arg} \left[\frac{\Gamma(\hat{\nu} + 1 - 2iG\mathcal{E}\omega)}{\Gamma(\hat{\nu} + 1 + 2iG\mathcal{E}\omega)} \right] + (2G\mathcal{E}\omega) \log(2\omega r_{\text{orb}}) + \frac{\ell - \hat{\nu}}{2} \pi, \quad (33)$$

where the universal anomalous dimension given by Eq. (5) enters as

$$\hat{\nu}(\omega) = \ell + \gamma_{\ell m}^{\text{univ}}(\omega). \quad (34)$$

The factor of $(r_{\text{orb}}\omega)^{\hat{\nu}-\ell}$ in the amplitude and the one proportional to π in the phase are a direct consequence of running the RG evolution of the multipoles down to the orbital scale $\mu = 1/r_{\text{orb}}$. These factors resum all universal subleading logarithms of the form $\omega^{n+k} \log^n \omega$ with $k > 0$ corresponding to dissipative tails. The rest of the dependence on $\hat{\nu}$ is a proposal inspired by the test particle limit [91] and resums additional finite terms.

This formula can be interpreted as follows: the universal tail contributions to the binary waveform are captured by the free-wave propagation in the linearized-in-spin Kerr background (i.e., the Schwarzschild-Lense-Thirring metric) sourced by the binary. The universality arises from the fact that this background is the universal part of the asymptotic metric of all compact gravitating sources.

For quasicircular orbits, $r_{\text{orb}}\omega = v_\Omega/v_\Omega^0$, where $v_\Omega = (GM\Omega)^{1/3}$, with Ω the orbital velocity and $M = m_1 + m_2$ the total static mass of the binary system; v_Ω^0 is a reference velocity (see Supplemental Material). For gravitational waves sourced by the binary, $\omega = m\Omega$. In this regime, we have verified our proposed resummation using the state-of-the-art PN waveform up to 4PN [99,100], where we find that both logarithmic and π -dependent terms are resummed. Of course, our formula also predicts an infinite number of universal logarithms in the waveform at higher PN orders. We record these checks and some of these predictions in Supplemental Material [90].

The formula in Eqs. (32) and (33), with anomalous dimension given in Eq. (3), does not resum all logarithms starting at 4PN order, because they contain the effects of tails of memory [55], which are not universal. These depend on the intrinsic and spin-induced multipole moments of the system, and hence, they cannot be simply extracted by studying the case of BHs.

Conclusions—In this Letter, we present the universal anomalous dimension of the gravitational multipole moments of a gravitating system in general relativity. Using unitarity and analyticity, we derive a formula relating the anomalous dimensions of multipole moments to the scattering phase shift of GWs by the system. When applied to BHs, the formula identifies the multipoles' anomalous dimension with the renormalized angular momentum of BHPT. Thanks to the universality of the EFT action, we were able to extract the part of the BH anomalous dimension that is universal to all compact gravitating objects regardless of their nature. This conceptual advance motivates us to propose a new factorization formula for the gravitational waveform that resums all universal tails.

Our analysis provides yet another illustration that EFT is a powerful tool that allows for a consistent interpretation of the low-frequency limit of the near- or far-zone expansion of GW sources. This adds to recent progress with the definition and extraction of the tidal effects of a black hole

from the scattering amplitudes [58,85,87,89], which allows one to resolve the tension in the literature on the dynamical Love numbers of BHs [51,71,108,109].

The results of this Letter are, however, limited to the universal tails. There are nonuniversal tail effects in the waveform that have not been addressed with our formula. For example, the tails of memory appear at 4PN order [55], which could be beyond the description of the anomalous dimension of multipoles. This may require a new framework to deal with the worldline EFT by considering the operator algebra of multipole moments. Furthermore, various finite-size effects, which one might call tails of tides, enter the description beyond the orders considered here.

Going forward, it will be important to rigorously prove the factorization formulas (28) and (33) and their possible generalizations. Additionally, our Eq. (3) strongly motivates the computation of the Raman scattering of GWs off the binary, including the nonuniversal near-zone effects, which capture the tidal deformation of the binary. We leave these and other exciting research directions, such as the application of the tail-resummed waveforms to GW data, for future exploration.

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