Dihadron Fragmentation Framework for Near-Side Energy-Energy Correlators

Zhong-Bo Kang,[1,2,3,*] Andreas Metz,[4,†] Daniel Pitonyak,[5,‡] and Congyue Zhang[1,2,§]
[1]Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA
[2]Mani L. Bhaumik Institute for Theoretical Physics, University of California, Los Angeles, California 90095, USA
[3]Center for Frontiers in Nuclear Science, Stony Brook University, Stony Brook, New York 11794, USA
[4]Department of Physics, SERC, Temple University, Philadelphia, Pennsylvania 19122, USA
[5]Department of Physics, Lebanon Valley College, Annville, Pennsylvania 17003, USA

(Received 31 July 2025; revised 5 November 2025; accepted 2 January 2026; published 23 February 2026)

We establish an approach to analyze the free hadron and transition (nonperturbative) regions of near-side energy-energy correlators (EECs) based on dihadron fragmentation functions (DiFFs). We introduce a (nonperturbative) function we call the "EEC DiFF" and explicitly show that expanding it for large relative transverse momentum between the two hadrons gives the $\mathcal{O}(\alpha_s)$ expression for the "EEC jet" function used in the quark-gluon (perturbative) region. This connection indicates that a formal theoretical matching will be able to bridge the free-hadron region, transition, and quark-gluon regions and allow all of them to be analyzed simultaneously. We further derive a result valid for near-side EECs in the free hadron and transition regions of e^+e^- annihilation in terms of the EEC DiFF. Using a simple model for the function, we perform the first fit within the dihadron framework to experimental data in this regime. We find reasonable agreement with the measurements and reproduce the salient features of near-side EECs in the free hadron and transition regions.

DOI: 10.1103/jnl4-x77t

Introduction and motivation—The theory of the strong interaction, quantum chromodynamics (QCD), is characterized by two defining features: asymptotic freedom [1,2] and confinement [3]. The former allows for collisions with a hard energy scale to be described by quarks and gluons (partons), whereas the latter conveys the fact that after such collisions, partons will hadronize into colorless bound states. Measurements of energy-energy correlators (EECs) as a function of the opening angle χ between the two hadrons, an observable first proposed in the 1970s [4–7], display clear signatures for these quark-gluon (perturbative) and free hadron (nonperturbative) regions with a continuous transition between them [8–24]. (For an extensive review on energy correlators, see Ref. [25].)

Therefore, a theoretical framework that simultaneously captures the different characteristics of EECs in these regions can offer great insight into our understanding of QCD. In particular, two angular regimes of EECs have been of interest: near side (or collinear) $\chi \approx 0$ [26,27] and away side (or back to back) $\chi \approx \pi$ [28,29]. In both cases, perturbative calculations have been performed to high orders [26,28–33] and insights have also been gained from connections to conformal field theories [26,27,30,34–39]. However, only recently has there been a more intense focus on the nonperturbative region of EECs [40–48].

In this Letter, we establish how the free hadron and transition regions of the near-side EEC can be described using the language of dihadron fragmentation functions (DiFFs). We explicitly show how at large relative transverse momentum between the two hadrons a nonperturbative function we call the "EEC DiFF" reproduces the results of the "EEC jet" function in the quark-gluon region. Therefore, this theoretical framework will serve as a bridge to allow the free hadron, transition, and quark-gluon regions to be analyzed simultaneously. In addition, we develop a simple model for our EEC DiFF based on the properties of the underlying DiFF, which allows one to understand salient features of near-side EECs in a natural way. We then perform the first fit within the dihadron framework of e^+e^- experimental measurements in the free hadron and transition regions, finding a reasonable description of the data.

EEC dihadron fragmentation function—Our starting point is the DiFF $D_1^{h_1h_2/q}(\xi_1, \xi_2, \vec{R}_T)$ defined in Ref. [49] (see also Ref. [50]):

*Contact author: zkang@physics.ucla.edu
†Contact author: metza@temple.edu
‡Contact author: pitonyak@lvc.edu
§Contact author: maxzhang2002@g.ucla.edu

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

$$D_1^{h_1h_2/q}(\xi_1,\xi_2,\vec{R}_T)=\frac{\xi^2}{64\pi^3\xi_1\xi_2}\int d^2\vec{k}_T\sum_X\int\frac{dx^+d^2\vec{x}_\perp}{(2\pi)^3}e^{ik\cdot x}\times\mathrm{Tr}\langle 0|\gamma^-\psi(x)|P_1,P_2;X\rangle\times\langle P_1,P_2;X|\bar{\psi}(0)|0\rangle|_{x^-=0},\quad(1)$$

where ξ_1, ξ_2 are the fractions carried by the hadrons h_1, h_2 of the light cone momentum of the fragmenting quark q (with momentum k), and $\vec{R}_T\equiv\frac{1}{2}(\vec{P}_{1T}-\vec{P}_{2T})$ is (half of) their relative transverse momentum. (We have suppressed gauge links for brevity and a color average is understood.) An analogous DiFF can be written down for gluon fragmentation [49]. The function (1) is a density in $(\xi_1,\xi_2,\vec{R}_T)$ for the number of hadron pairs (h_1h_2) produced when a quark q fragments [49]. The total momentum of the dihadron is denoted by P_h, with $M_h^2=P_h^2$ its invariant mass, and $\xi\equiv\xi_1+\xi_2$. We work in the "dihadron frame" where P_h has no transverse momentum and has a large light-cone-minus component.

An important aspect of $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$ (i is a generic parton) is that it has been integrated over the transverse momentum $\vec{k}_T$ of the fragmenting parton. This allows for *collinear* factorization theorems and evolution equations to be used whose partonic cross sections and splitting functions are the same as for single-hadron fragmentation functions (FFs) [50,51]. Such results do not hold for the fully unintegrated function that depends on both $\vec{k}_T$ and $\vec{R}_T$ (or $\vec{P}_{1\perp}$ and $\vec{P}_{2\perp}$ if one works in a frame where the parton has no transverse momentum).

We also introduce the variable $z_{12}\equiv\frac{1}{2}[1-(\vec{P}_1\cdot\vec{P}_2/|\vec{P}_1||\vec{P}_2|)]=\frac{1}{2}(1-\cos\theta_{12})$, where θ_{12} is the angle between h_1 and h_2. Up to power suppressed (p.s.) corrections $\sim 1/Q^2$, one can show for $e^+e^-\to h_1h_2X$,

$$z_{12}=\frac{R_T^2}{Q^2}\frac{\tau^2}{\tau_1^2\tau_2^2},\quad(2)$$

with $R_T^2\equiv\vec{R}_T^2$, $\tau_{1(2)}=2P_{1(2)}\cdot q/Q^2=2E_{1(2)}/Q$, and $\tau\equiv\tau_1+\tau_2$, where $E_{1(2)}$ is the energy of hadron $h_{1(2)}$, and $Q=\sqrt{q^2}$ is the center-of-mass (c.m.) energy of the collision.

We then define a nonperturbative function that we call the EEC DiFF as

$$\mathcal{D}^i(z_\chi,Q^2)\equiv\sum_{h_1,h_2}\int d\xi_1\int d\xi_2\int d^2\vec{R}_T\;\delta\left(z_\chi-\frac{R_T^2}{Q^2}\frac{\xi^2}{\xi_1^2\xi_2^2}\right)\times\xi_1\xi_2D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T),\quad(3)$$

where $z_\chi\equiv\frac{1}{2}(1-\cos\chi)$. (We are using z_χ instead of the standard z as to not create confusion with the momentum fraction z commonly used with FFs.) We assume that $\mathcal{D}^q$, while different than $\mathcal{D}^g$ for gluons, is flavor independent. We focus on the terms at finite z_χ and omit contributions proportional to $\delta(z_\chi)$, which arise when both EEC detectors register the same particle [26]. As we will show, the EEC DiFF enters naturally into our theoretical calculation of EEC(χ) valid for the near-side free hadron and transition regions and plays a crucial phenomenological role in explaining their behaviors.

In order to study experiments across various c.m. energies, an evolution equation for $\mathcal{D}^i(z_\chi,Q^2)$ is also needed. This can be obtained from the evolution equation for $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$, which is known to $\mathcal{O}(\alpha_s)$ [49,52]. The splitting functions for the "homogeneous" terms in the evolution of $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$ that involve the DiFFs themselves are the same as for the single-hadron FFs $D_1^{h/i}(\xi)$ [50,51] and are known to $\mathcal{O}(\alpha_s^3)$ [53–56]. However, starting at $\mathcal{O}(\alpha_s^2)$, the evolution of $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$ is expected to contain "inhomogeneous" terms that involve single-hadron FFs [49,57–61], which have not been calculated yet. Therefore, we will approximate the evolution of $\mathcal{D}^i(z_\chi,Q^2)$ by neglecting this mixing with single-hadron FFs and focus only on the homogeneous part involving DiFFs. As we derive in Supplemental Material [62], the evolution equation reads

$$\frac{\partial\vec{\mathcal{D}}(z_\chi,Q^2;\mu)}{\partial\ln\mu^2}\overset{\mathrm{DiFF}}{=}\int_0^1 dw\,w^2\,\vec{\mathcal{D}}(z_\chi,w^2Q^2;\mu)\cdot\hat{P}_T(w;\mu),\quad(4)$$

where $\vec{\mathcal{D}}\equiv(\mathcal{D}^q,\mathcal{D}^g)$ is a row vector that contains the quark and gluon EEC DiFFs.

Connection to the EEC quark-gluon region—When $R_T\gtrsim\Lambda_{\mathrm{QCD}}$, one can assume the two hadrons emerged from a single parton fragmentation encoded by a DiFF like $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$ [50,51,63–65]. Once $R_T\gg\Lambda_{\mathrm{QCD}}$, though, one must also consider contributions where the two hadrons were produced from two different parton fragmentations. In this region, the DiFF $D_1^{h_1h_2/i}(\xi_1,\xi_2,\vec{R}_T)$ can be expanded for large R_T into single-hadron FFs [49,66]. A straightforward calculation gives

$$D_1^{h_1h_2/q}(\xi_1,\xi_2,\vec{R}_T)\overset{R_T\gg\Lambda_{\mathrm{QCD}}}{=}\frac{1}{R_T^2}\frac{C_F\alpha_s}{2\pi^2}\times\int_{\xi_1}^{1-\xi_2}\frac{dw}{w(1-w)}\left[\frac{1+w^2}{1-w}D_1^{h_1/q}\left(\frac{\xi_1}{w}\right)D_1^{h_2/g}\left(\frac{\xi_2}{1-w}\right)\right],\quad(5)$$

where we also implicitly include a term with $(h_1\leftrightarrow h_2)$ and omit a term proportional to $\delta^2(\vec{R}_T)$, in accordance with our earlier treatment, as it contributes only to $\delta(z_\chi)$. Using Eq. (3) along with the momentum sum rule $\sum_h\int dz\,z\,D_1^{h/i}(z)=1$, we find

$$\mathcal{D}^q(z_\chi,Q^2)\overset{R_T\gg\Lambda_{\mathrm{QCD}}}{=}\frac{\alpha_s}{4\pi}\frac{3C_F}{z_\chi}.\quad(6)$$

This agrees exactly with the $\mathcal{O}(\alpha_s)$ term for the EEC jet function $dJ_q(z_\chi)/dz_\chi$ in Ref. [26] that underlies EEC(χ) in the quark-gluon (perturbative) region. We verified that the same occurs with the gluon EEC DiFF $\mathcal{D}^g(z_\chi)$ and gluon EEC jet function $dJ_g(z_\chi)/dz_\chi$. Thus, within the DiFF framework, an analytical matching will be able to bridge the near-side free hadron, transition, and quark-gluon regions and allow all of them to be analyzed simultaneously. The connection between the EEC DiFF and the EEC jet functions also provides theoretical support for the universality of EEC jet functions and the analyses presented in Refs. [44,67].

Theoretical result for EECs in e^+e^- annihilation—The energy-energy correlator in $e^+e^- \to h_1h_2X$ for two hadrons separated by an angle χ is defined as

$$\mathrm{EEC}(\chi) \equiv \frac{1}{\sigma_t}\frac{d\Sigma}{d\chi} = \frac{\sin\chi}{2}\frac{1}{\sigma_t}\sum_{h_1,h_2}\int d\sigma \frac{E_1E_2}{Q^2}\delta(z_\chi - z_{12}), \quad (7)$$

where $d\sigma$ is the differential cross section for $e^+e^- \to h_1h_2X$, and σ_t is the total cross section for $e^+e^- \to$ hadrons. As we derive in Supplemental Material [62], in a region where the cross section is only due to dihadron fragmentation, EEC(χ) is given by

$$\mathrm{EEC}(\chi) \overset{\mathrm{DiFF}}{=} \frac{\sin\chi}{2}\frac{\sigma_0}{\sigma_t}\int_0^1 dw\, w^2\, \vec{\mathcal{D}}(z_\chi, w^2Q^2;\mu)\cdot\vec{H}\left(w;\frac{\mu}{Q}\right), \quad (8)$$

where $\vec{H} \equiv (H^q, H^g)$ are hard factors for $e^+e^- \to hX$, which are known to $\mathcal{O}(\alpha_s^3)$ [54–56,68], and σ_0 is the Born-level cross section for $e^+e^- \to$ hadrons.

Model for the EEC DiFF—In order to analyze experimental measurements on near-side EECs in e^+e^- annihilation, we need a model for the EEC DiFF $\mathcal{D}^i(z_\chi, Q^2)$. To do so, we utilize properties of the DiFF $D_1^{h_1h_2/i}(\xi_1, \xi_2, \vec{R}_T)$ that enters Eq. (3) and underlies $\mathcal{D}^i(z_\chi, Q^2)$. Evaluating the R_T integration in Eq. (3) with the delta function generates a prefactor proportional to Q^2 and fixes $R_T^2 = z_\chi Q^2(\xi_1^2\xi_2^2/\xi^2)$. We also assume $D_1^{h_1h_2/i}(\xi_1, \xi_2, \vec{R}_T)$ to have a Gaussian dependence on $R_T \sim \sqrt{z_\chi}Q$, analogous to the simple ansatz used in a parton model framework for the transverse momentum dependence of transverse momentum dependent parton distribution and fragmentation functions (TMDs) [69–71]. Thus, the observed increase in the peak value of the near-side EEC with Q, its location at roughly the same $\sqrt{z_\chi}Q \approx \chi Q/2$ for all Q, as well as the scaling of EEC(χ) with Q^2 as $\chi \to 0$ [23,24,44,45,72] can all naturally be understood on theoretical grounds alone within the DiFF framework.

Our model for $\vec{\mathcal{D}}(z_\chi, Q^2;\mu_0)$ at the initial scale $\mu_0 = m_b = 4.18$ GeV then reads

$$\mathcal{D}^q(z_\chi, Q^2;\mu_0) = NQ^2\frac{\exp(-z_\chi Q^2/a)}{1 + z_\chi Q^2/b}, \quad (9)$$

where $\{N, a, b\}$ are free parameters to be fit to the data. The denominator introduces a deviation from pure Gaussian behavior at larger values of $\sqrt{z_\chi}Q$, allowing for a more accurate description of the transition region. We set $\mathcal{D}^g(z_\chi, Q^2;\mu_0) = 0$, allowing it to become nonzero through evolution. Since the EEC DiFF is assumed to be flavor independent, we have chosen $\mu_0 = m_b$ so that the evolution starts with all five flavors active.

Data selection—An important consideration in our study is what cut to make on the data to ensure we are in a region dominated by DiFFs. In this regard, a recent extraction of DiFFs [73] from a Belle cross section measurement [74] and PYTHIA data can serve as guidance. To be specific, Ref. [73] was able to successfully describe data from $Q = 10.6$ to 91.2 GeV on $\pi^+\pi^-$ final states within a leading-order DiFF framework that involved the function $D_1^{\pi^+\pi^-/i}(\tau, M_h)$. For all c.m. energies, the data satisfied $M_h/\tau \lesssim 3.5$ GeV. Since $R_T = (\sqrt{\tau_1\tau_2}/\tau)M_h$ up to p.s. corrections, and recall $\sqrt{z_\chi} = (R_T/Q)[\tau/(\tau_1\tau_2)]$, one then has the condition $\sqrt{z_\chi}Q\sqrt{\tau_1\tau_2}/\tau \lesssim 3.5$ GeV. Assuming $\tau_1 \approx \tau_2 \approx \tau/2$ for most near-side EEC events, we arrive at a practical data cut of $\sqrt{z_\chi}Q < 7$ GeV. For the range of Q values in Table I, this condition corresponds to angular separations approximately within $1.5° \lesssim \chi \lesssim 34°$. Therefore, one should not interpret "large angles" as necessarily corresponding to the perturbative (quark-gluon) region. Rather, as also emphasized in a factorization-based analysis of EECs [26], it is the size of the scale $\sqrt{z_\chi}Q$ (or, equivalently, R_T or M_h) that determines the physical regime being probed.

In fitting our model parameters, we used datasets with $Q \geq 22$ GeV that included collectively both charged and neutral particles, provided statistical and systematic uncertainties or their quadrature sum, and had more than one bin after the cut $\sqrt{z_\chi}Q < 7$ GeV. This left us with the following datasets for our study: TASSO [13], MAC [9],

TABLE I. Summary of the χ^2/N_{pts} for each dataset.

Collaboration	Q (GeV)	N_{pts}	χ^2/N_{pts}
TASSO [13]	22.0	9	0.35
MAC [9]	29.0	7	1.69
MARKII [12]	29.0	7	1.85
TASSO [13]	34.8	5	1.42
TASSO [13]	43.5	4	6.71
TOPAZ [14]	53.3	3	1.91
TOPAZ [14]	59.5	3	1.71
OPAL [17]	91.2	8	1.70
Total		46	1.88
			($Z = 3.44$)

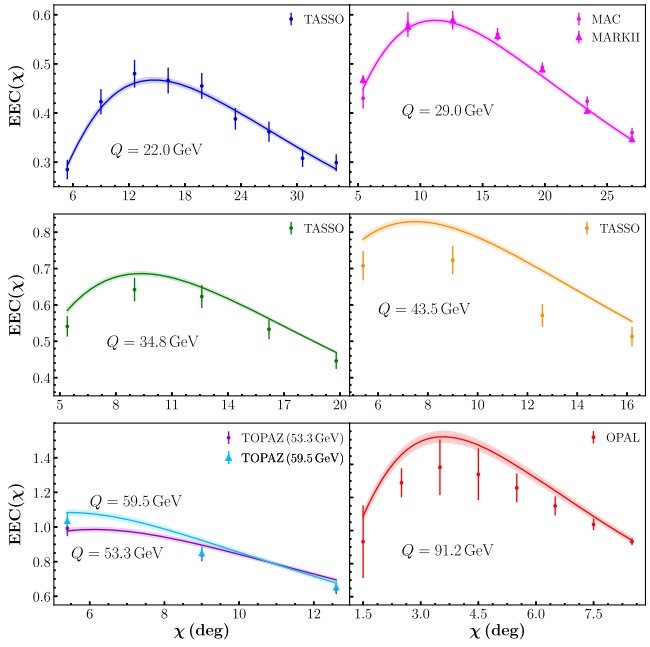


FIG. 1. EEC(χ) vs χ for our theoretical central curve and 1σ error band compared to the data used in our analysis.

MARKII [12], TOPAZ [14], OPAL [17]. For measurements whose uncertainties were rounded to one significant digit, we expand them to two significant digits assuming maximal uncertainty before rounding, following the procedure in Ref. [75].

Phenomenological methodology and results for EECs in e^+e^- annihilation—We work at leading order in the hard factors, $H^q(w) = \frac{1}{2}\delta(1-w), H^g(w) = 0$ in Eq. (8), and leading-log evolution, where w is set to 1 in the argument of $\vec{\mathcal{D}}$ on the rhs of Eq. (4). We use a Bayesian Monte Carlo method with data resampling to obtain the posterior distributions of the parameters $\{N, a, b\}$. We find a better description of the data is achieved evolving $\vec{\mathcal{D}}(z_\chi, Q^2; \mu)$ to $\mu = 0.5Q$ instead of $\mu = Q$, which can be associated with the fact that the dihadron will only carry some fraction of the energy of the parton that initiated the jet [45]. We also account for the uncertainty in the choice of scale by varying it to $\mu = 0.5Q/2$ and $\mu = 2 \times 0.5Q$. For each scale setting $(\mu = 0.5Q, 0.5Q/2, 2 \times 0.5Q)$ we gather 150 replicas. From the collective 450 replicas we compute the mean and standard deviation and use them to generate a central curve and 1σ error band for our theory calculation.

The results for EEC(χ) vs χ are shown in Fig. 1, where we find reasonable agreement with the data, with $\chi^2/N_{\text{pts}} = 86.4/46 = 1.88$ and Z score $= 3.44$, where $Z \equiv \sqrt{2}\,\text{erf}^{-1}(2p-1)$ with the p value computed using the total χ^2. Table I summarizes the χ^2/N_{pts} for each dataset. We see that the somewhat larger χ^2/N_{pts} is mostly due to the TASSO 43.5 GeV data. Given that the other TASSO datasets at 22 and 34.8 GeV, as well as measurements by other collaborations at higher energies, are well described, there could be a normalization issue with the TASSO 43.5 GeV measurement. Higher-order corrections to the evolution of the EEC DiFF and hard factors in Eq. (8) could improve the χ^2/N_{pts} [67].

The extracted EEC DiFF $\mathcal{D}^i(z_\chi, Q^2; \mu)$ is displayed in Fig. 2 at fixed $Q = 91.2$ GeV for various values of μ. As expected, the quark contribution is significantly larger than the gluon, which was initialized to zero at the starting scale. The fitted parameter values for $\mathcal{D}^q(z_\chi, Q^2; \mu_0)$ are $N = 0.035 \pm 0.003$ GeV^{-2}, $a = 150 \pm 19$ GeV^2, $b = 10.1 \pm 0.6$ GeV^2. Since $\mathcal{D}^i(z_\chi, Q^2; \mu)$ is constructed from the underlying DiFF $D_1^{h_1h_2/i}(\tau_1, \tau_2, \vec{R}_T)$ via Eq. (3), it may be possible to independently constrain or validate the EEC DiFF through a dedicated e^+e^- cross section measurement of $d\sigma/d\tau_1 d\tau_2 dR_T^2$.

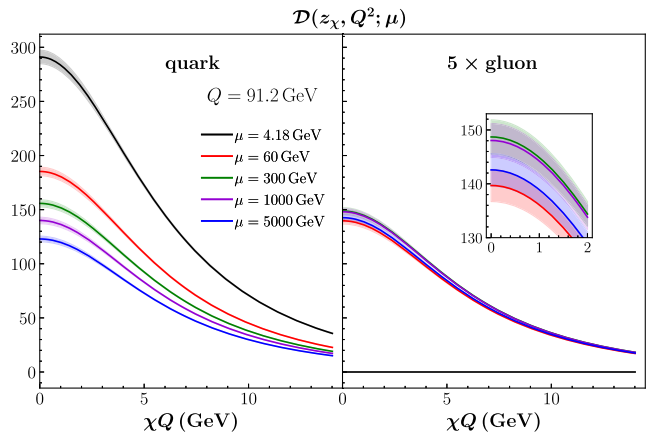


FIG. 2. $\mathcal{D}^i(z_\chi, Q^2; \mu)$ vs χQ with a 1σ error band for the quark (left) and gluon (right) at fixed $Q = 91.2$ GeV for various values of μ. Note that the gluon curves have been multiplied by a factor of 5 and the inset shows finer details of the region $0 < \chi Q < 2$ GeV.

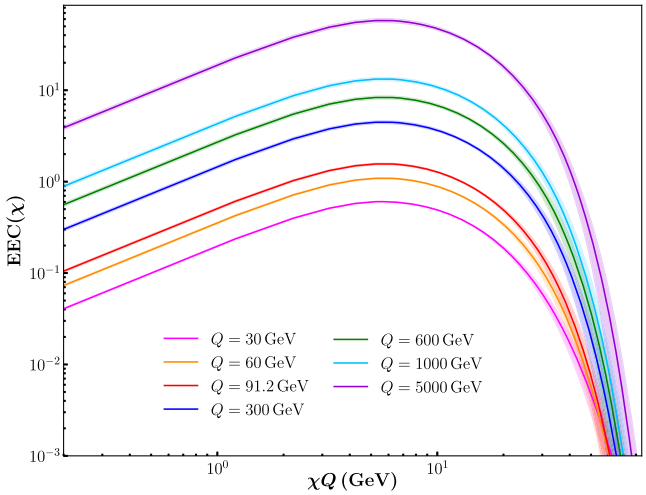


FIG. 3. EEC(χ) vs χQ with a 1σ error band for various values of Q.

In Fig. 3, we plot EEC(χ) as a function of χQ to explicitly demonstrate that our phenomenological analysis reproduces the key features of the near-side EEC anticipated from DiFF-based theoretical arguments. Specifically, we observe that the peak value of the near-side EEC increases with Q, while its position remains approximately fixed in χQ across all Q values. We also confirmed that the curves for EEC(χ)/Q^2 versus χ tend to converge as $\chi \to 0$, consistent with the expected scaling behavior [72]. Overall, our EEC DiFF framework quantitatively captures the main features of the near-side free hadron and transition regions, while simultaneously providing a formal theoretical foundation that enables matching to the quark-gluon regime.

Summary and outlook—We have established a framework, using dihadron fragmentation, for analyzing the free hadron and transition regions of near-side EECs. We explicitly showed to $\mathcal{O}(\alpha_s)$ that at large relative transverse momentum of the two hadrons an agreement occurs between the EEC DiFF in Eq. (3) and the EEC jet function used in the quark-gluon region [26]. Therefore, it will be possible in the future to match the free hadron, transition, and quark-gluon regions in this theoretical framework, allowing all of them to be analyzed simultaneously. Using a simple model for the EEC DiFF, we have been able to successfully describe for the first time within the dihadron framework near-side EEC measurements in the free hadron and transition regions across a range of center-of-mass energies. The DiFF approach also naturally explains the Q dependence of EEC(χ) in this regime and its scaling behaviors. The DiFF formalism for EECs can provide new insights into hadronization in QCD and, due to its close connection with transverse spin physics [73,76–97], will also open up an avenue to explore azimuthal or spin-dependent near-side EEC observables in e^+e^-, lepton-nucleon, and proton-proton collisions.

Note added—We were recently made aware of Refs. [98–100]. Our main results and conclusions are consistent with theirs, and, in addition, we have performed a phenomenological fit to existing e^+e^- experimental data.

Acknowledgments—Z. K. thanks the DESY theory group for their hospitality during the course of this work. This work was supported by the National Science Foundation under Grants No. PHY-2515057 (Z. K. and C. Z.), No. PHY-2110472, and No. PHY-2412792 (A. M.), and No. PHY-2308567 (D. P.).

Data availability—The data that support the findings of this article are openly available [101].

[1] D. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
[2] H. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
[3] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).
[4] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. D **17**, 2298 (1978).
[5] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. Lett. **41**, 1585 (1978).
[6] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. D **19**, 2018 (1979).
[7] C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Lett. B **85**, 297 (1979).
[8] H. J. Behrend *et al.* (CELLO Collaboration), Z. Phys. C **14**, 95 (1982).
[9] E. Fernandez *et al.*, Phys. Rev. D **31**, 2724 (1985).
[10] W. Bartel *et al.* (JADE Collaboration), Z. Phys. C **25**, 231 (1984).
[11] C. Berger *et al.* (PLUTO Collaboration), Z. Phys. C **28**, 365 (1985).
[12] D. R. Wood *et al.*, Phys. Rev. D **37**, 3091 (1988).
[13] W. Braunschweig *et al.* (TASSO Collaboration), Z. Phys. C **36**, 349 (1987).
[14] I. Adachi *et al.* (TOPAZ Collaboration), Phys. Lett. B **227**, 495 (1989).
[15] D. Decamp *et al.* (ALEPH Collaboration), Phys. Lett. B **257**, 479 (1991).
[16] M. Z. Akrawy *et al.* (OPAL Collaboration), Phys. Lett. B **252**, 159 (1990).
[17] P. D. Acton *et al.* (OPAL Collaboration), Phys. Lett. B **276**, 547 (1992).
[18] B. Adeva *et al.* (L3 Collaboration), Phys. Lett. B **257**, 469 (1991).
[19] O. Adrian *et al.* (L3 Collaboration), Phys. Lett. B **284**, 471 (1992).
[20] K. Abe *et al.* (SLD Collaboration), Phys. Rev. D **51**, 962 (1995).
[21] K. Abe *et al.* (SLD Collaboration), Phys. Rev. D **50**, 5580 (1994).
[22] H. Bossi, Y.-C. Chen, Y. Chen, J. Zhang, G. M. Innocenti, A. Badea, A. Baty, M. Maggi, C. McGinn, and Y.-J. Lee, arXiv:2505.11828.
[23] A. Hayrapetyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **133**, 071903 (2024).
[24] S. Acharya *et al.* (ALICE Collaboration), arXiv:2409.12687.
[25] I. Moult and H. X. Zhu, arXiv:2506.09119.
[26] L. J. Dixon, I. Moult, and H. X. Zhu, Phys. Rev. D **100**, 014009 (2019).
[27] H. Chen, J. High Energy Phys. 01 (2024) 035.
[28] I. Moult and H. X. Zhu, J. High Energy Phys. 08 (2018) 160.
[29] C. Duhr, B. Mistlberger, and G. Vita, Phys. Rev. Lett. **129**, 162001 (2022).
[30] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Phys. Rev. Lett. **112**, 071601 (2014).
[31] L. J. Dixon, M.-X. Luo, V. Shtabovenko, T.-Z. Yang, and H. X. Zhu, Phys. Rev. Lett. **120**, 102001 (2018).
[32] M.-X. Luo, V. Shtabovenko, T.-Z. Yang, and H. X. Zhu, J. High Energy Phys. 06 (2019) 037.
[33] J. M. Henn, E. Sokatchev, K. Yan, and A. Zhiboedov, Phys. Rev. D **100**, 036010 (2019).
[34] D. M. Hofman and J. Maldacena, J. High Energy Phys. 05 (2008) 012.

[35] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Nucl. Phys. **B884**, 305 (2014).

[36] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Nucl. Phys. **B884**, 206 (2014).

[37] M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, J. High Energy Phys. 01 (2021) 128.

[38] C.-H. Chang, M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, J. High Energy Phys. 05 (2022) 059.

[39] H. Chen, I. Moult, and H. X. Zhu, Phys. Rev. Lett. **126**, 112003 (2021).

[40] S. T. Schindler, I. W. Stewart, and Z. Sun, J. High Energy Phys. 10 (2023) 187; 10 (2024) 175(E).

[41] K. Lee, A. Pathak, I. W. Stewart, and Z. Sun, Phys. Rev. Lett. **133**, 231902 (2024).

[42] H. Chen, P. F. Monni, Z. Xu, and H. X. Zhu, Phys. Rev. Lett. **133**, 231901 (2024).

[43] Z.-B. Kang, J. Penttala, and C. Zhang, arXiv:2410.21435.

[44] J. Barata, Z.-B. Kang, X. Mayo López, and J. Penttala, Phys. Rev. Lett. **134**, 251903 (2025).

[45] X. Liu, W. Vogelsang, F. Yuan, and H. X. Zhu, Phys. Rev. Lett. **134**, 151901 (2025).

[46] C. Csáki, S. Ferrante, and A. Ismail, J. High Energy Phys. 07 (2025) 117.

[47] C. Csáki and A. Ismail, J. High Energy Phys. 11 (2024) 140.

[48] K. Lee, F. Turro, and X. Yao, Phys. Rev. D **111**, 054514 (2025).

[49] D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin, and N. Sato, Phys. Rev. Lett. **132**, 011902 (2024).

[50] D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin, and N. Sato, arXiv:2502.15817.

[51] T. C. Rogers, M. Radici, A. Courtoy, and T. Rainaldi, Phys. Rev. D **111**, 056001 (2025).

[52] F. A. Ceccopieri, M. Radici, and A. Bacchetta, Phys. Lett. B **650**, 81 (2007).

[53] A. Mitov and S.-O. Moch, Nucl. Phys. **B751**, 18 (2006).

[54] A. Mitov, S. Moch, and A. Vogt, Phys. Lett. B **638**, 61 (2006).

[55] S. Moch and A. Vogt, Phys. Lett. B **659**, 290 (2008).

[56] A. A. Almasy, S. Moch, and A. Vogt, Nucl. Phys. **B854**, 133 (2012).

[57] D. de Florian and L. Vanni, Phys. Lett. B **578**, 139 (2004).

[58] A. Majumder and X.-N. Wang, Phys. Rev. D **72**, 034007 (2005).

[59] A. Majumder and X.-N. Wang, Phys. Rev. D **70**, 014007 (2004).

[60] H. Chen, M. Jaarsma, Y. Li, I. Moult, W. J. Waalewijn, and H. X. Zhu, J. High Energy Phys. 07 (2023) 185.

[61] H. Chen, M. Jaarsma, Y. Li, I. Moult, W. J. Waalewijn, and H. X. Zhu, Phys. Rev. D **111**, 076021 (2025).

[62] See Supplemental Material at http://link.aps.org/supplemental/10.1103/jnl4-x77t for a derivation of the dihadron fragmentation result for EEC(χ) in e^+e^- annihilation and the evolution equation for the EEC DiFF.

[63] J. C. Collins, S. F. Heppelmann, and G. A. Ladinsky, Nucl. Phys. **B420**, 565 (1994).

[64] J. C. Collins and G. A. Ladinsky, arXiv:hep-ph/9411444.

[65] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D **62**, 034008 (2000).

[66] J. Zhou and A. Metz, Phys. Rev. Lett. **106**, 172001 (2011).

[67] E. Herrmann, Z.-B. Kang, J. Penttala, and C. Zhang, arXiv:2507.17704 [Phys. Rev. Lett.] (to be published).

[68] C.-Q. He, H. Xing, T.-Z. Yang, and H. X. Zhu, arXiv:2503.20441.

[69] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, Phys. Rev. D **71**, 074006 (2005).

[70] A. Signori, A. Bacchetta, M. Radici, and G. Schnell, J. High Energy Phys. 11 (2013) 194.

[71] M. Anselmino, M. Boglione, J. O. Gonzalez Hernandez, S. Melis, and A. Prokudin, J. High Energy Phys. 04 (2014) 005.

[72] H. Chen, H. Ruan, and H. X. Zhu, J. High Energy Phys. 12 (2025) 168.

[73] C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl, Phys. Rev. D **109**, 034024 (2024).

[74] R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **96**, 032005 (2017).

[75] A. Kardos, S. Kluth, G. Somogyi, Z. Tulipánt, and A. Verbytskyi, Eur. Phys. J. C **78**, 498 (2018).

[76] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D **62**, 034009 (2000).

[77] M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D **65**, 074031 (2002).

[78] A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003).

[79] A. Bacchetta and M. Radici, Phys. Rev. D **69**, 074026 (2004).

[80] D. Boer, R. Jakob, and M. Radici, Phys. Rev. D **67**, 094003 (2003).

[81] A. Bacchetta and M. Radici, Phys. Rev. D **70**, 094032 (2004).

[82] A. Bacchetta, F. A. Ceccopieri, A. Mukherjee, and M. Radici, Phys. Rev. D **79**, 034029 (2009).

[83] A. Bacchetta, A. Courtoy, and M. Radici, Phys. Rev. Lett. **107**, 012001 (2011).

[84] A. Courtoy, A. Bacchetta, M. Radici, and A. Bianconi, Phys. Rev. D **85**, 114023 (2012).

[85] A. Bacchetta, A. Courtoy, and M. Radici, J. High Energy Phys. 03 (2013) 119.

[86] H. H. Matevosyan, A. W. Thomas, and W. Bentz, Phys. Rev. D **88**, 094022 (2013).

[87] S. Gliske, A. Bacchetta, and M. Radici, Phys. Rev. D **90**, 114027 (2014); **91**, 019902(E) (2015).

[88] M. Radici, A. Courtoy, A. Bacchetta, and M. Guagnelli, J. High Energy Phys. 05 (2015) 123.

[89] M. Radici, A. M. Ricci, A. Bacchetta, and A. Mukherjee, Phys. Rev. D **94**, 034012 (2016).

[90] H. H. Matevosyan, A. Kotzinian, and A. W. Thomas, Phys. Rev. D **96**, 074010 (2017).

[91] H. H. Matevosyan, A. Kotzinian, and A. W. Thomas, Phys. Rev. D **97**, 014019 (2018).

[92] H. H. Matevosyan, A. Kotzinian, and A. W. Thomas, Phys. Rev. Lett. **120**, 252001 (2018).

[93] M. Radici and A. Bacchetta, Phys. Rev. Lett. **120**, 192001 (2018).

[94] H. H. Matevosyan, A. Kotzinian, and A. W. Thomas, J. High Energy Phys. 10 (2018) 008.
[95] H. H. Matevosyan, A. Bacchetta, D. Boer, A. Courtoy, A. Kotzinian, M. Radici, and A. W. Thomas, Phys. Rev. D **97**, 074019 (2018).
[96] J. Benel, A. Courtoy, and R. Ferro-Hernandez, Eur. Phys. J. C **80**, 465 (2020).
[97] C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl (JAM Collaboration), Phys. Rev. Lett. **132**, 091901 (2024).
[98] K. Lee and I. Stewart, companion Letter, Phys. Rev. Lett. **136**, 081902 (2026).
[99] Y. Guo, F. Yuan, and W. Zhao, companion Letter, Phys. Rev. Lett. **136**, 081904 (2026).
[100] C.-H. Chang, H. Chen, X. Liu, D. Simmons-Duffin, F. Yuan, and H. X. Zhu, companion Letter, Phys. Rev. Lett. **136**, 081903 (2026).
[101] Z.-B. Kang, A. Metz, D. Pitonyak, and C. Zhang, https://github.com/pitonyak25/EEC-DiFF (2025).