

Kapitza Pendulum Route to Supercurrent Tunnel Diodes

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Superconducting diodes that support nonreciprocal supercurrent flow in principle constitute attractive, nondissipative, circuit elements for superconducting electronics. But their realization faces fundamental challenges, as conventional Josephson tunnel junctions are inherently reciprocal. The so far existing approaches to break the reciprocity typically involve magnetism or spin-orbit coupling, which often increase the device complexity and limit the reproducibility. Here, we demonstrate an alternative dynamical route to supercurrent nonreciprocity based on parametric driving. By applying a frequency-modulated supercurrent amplitude, we show that effective higher-order, nonharmonic terms are generated in the current-phase relation. Leveraging strong mathematical similarities with the Kapitza pendulum, we show that these terms dynamically break the reciprocity. This establishes the concept of a Kapitza supercurrent diode and demonstrates that nonreciprocal superconducting transport can be engineered by a non-equilibrium driving the conventional Josephson tunnel junctions. We propose two implementations of such a Kapitza supercurrent diode—via gate-controlled superconducting interferometers or flux-driven double-loop SQUIDs—to achieve nonreciprocal supercurrent transport within experimentally accessible frequencies $\omega/2\pi \sim 1$ to 10 GHz.

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The search for superconducting diodes, devices that allow supercurrent to flow predominantly in one direction, is an active focal point of both fundamental and technological research [1–8]. Nonreciprocal supercurrents, or supercurrent diode effects, have been observed across a wide range of superconducting systems, including artificial superlattice [1,9], elemental [10], noncentrosymmetric [11], and polar superconductors [12], as well as twisted graphene [13,14] and high- T_c junctions [7,15–17], highlighting diverse strategies to the break time-reversal symmetry and control directional transport [1,9,11–13,18–21]. The Josephson diode effect (JDE) has been proposed and experimentally observed on different Josephson platforms, including junctions with spin-orbit-coupled weak links [3,22–25], Dayem bridges [10,26,27], and interferometric devices with higher-harmonic contributions [28–30] and

inhomogeneities [31]. We note that the possibility of field-biased nonreciprocal supercurrents via inductive and junction asymmetry in interferometric devices was already recognized in the early literature [32,33]. The JDE has further been explored in systems featuring screening currents [34,35], self-field effects [36,37], and Josephson-current-induced phase reconfiguration mediated by kinetic inductance [38]. It has also been studied in multiterminal configurations [39,40], gate-tunable devices [39,41], as well as in vortex-driven junctions [37,42–48] and through backaction mechanisms [49].

Despite these numerous proposals for the different types and mechanisms behind superconducting diodes, a major challenge stems from the intrinsic characteristics of Josephson junctions. At the heart of this challenge lies the reciprocal nature of the conventional Josephson effect, where the supercurrent depends sinusoidally on the phase difference φ between the superconductors forming the junction, following the relation $I = I_c \sin(\varphi)$. The parity symmetry of this relation with respect to the phase bias ensures that, without additional modifications, the supercurrent amplitude is identical in both directions, precluding any rectification or nonreciprocal behavior.

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Achieving nonreciprocal supercurrents in Josephson junctions requires introducing mechanisms that induce asymmetry in the current-phase relation, such as an anomalous phase shift or higher-order harmonic components. These modifications typically involve breaking the time-reversal symmetry, for example, through magnetic fields or magnetic elements assisted by spin-orbit coupling, and typically require complex interface engineering [50–53]. Interestingly, even when the time-reversal and inversion symmetries are broken, resulting in an anomalous phase shift, denoted ϕ , that alters the current-phase relation to $I = I_c \sin(\varphi + \phi)$, the supercurrent amplitude remains reciprocal. The inclusion of a second-harmonic term in the current-phase relation, leading to $I = I_1 \sin(\varphi + \phi) + I_2 \sin(2\varphi)$, is a minimal necessary ingredient to achieve a nonreciprocal supercurrent flow. However, controlling and reliably reproducing these harmonic components in conventional junctions is challenging. In particular, beyond managing the sources of time-reversal symmetry, the high-harmonic current-phase components required for diode functionality are sensitive to junction quality, interface properties, and material-specific effects, making consistent implementation intricate.

This raises a fundamental question: can one realize nonreciprocal supercurrents using Josephson tunnel junctions with a standard current-phase relation? Addressing this question is crucial for advancing superconducting diode technology and understanding the fundamental physics that enable nonreciprocal superconducting transport.

To this end, we introduce a general method for inducing supercurrent nonreciprocity in Josephson tunnel junctions via parametric driving [54]. This approach draws inspiration from the Kapitza pendulum [55–57]—a classical pendulum with a rapidly oscillating pivot point. As high-frequency oscillations can stabilize an inverted pendulum, our work demonstrates that parametric driving of coupled Josephson junctions can generate nonharmonic components and nonreciprocal supercurrent behavior. We propose a minimal setup for realizing a supercurrent diode based on superconducting interferometers composed exclusively of Josephson tunnel junctions, in which the supercurrent is parametrically driven by either electric or magnetic fields, establishing viable routes to realizing a Kapitza supercurrent diode.

We recall the fundamental principle of the Kapitza pendulum [55,56], a rigid pendulum with a vertically oscillating pivot. If the suspension vibrates with frequency ω and amplitude a , the angle φ obeys an equation $\ddot{\varphi} = -(\omega_0^2 + \alpha\omega^2 \cos \omega t) \sin \varphi$, where $\omega_0^2 = g/l$ and $\alpha = a/l$. In the regime of small amplitude ($\alpha \ll 1$) and high frequency ($\omega \gg \omega_0$), the motion separates into a slow component ξ_0 and a fast oscillation ξ , with $\varphi = \xi_0 + \xi$. A perturbative expansion and time averaging yield an effective equation $\ddot{\xi}_0 = -\partial U_{\text{eff}}(\xi_0)/\partial \xi_0$, with $U_{\text{eff}}(\xi_0) = -\omega_0^2 \cos \xi_0 + \frac{\alpha^2 \omega^2}{4} \sin^2 \xi_0$. The periodic drive thus generates a second-harmonic contribution to the potential [55,56]. The Kapitza route

is therefore not limited to the stabilization of the $\xi_0 = \pi$ state; more broadly, it provides a mechanism for dynamically generating higher harmonics in the effective free energy of a nonlinear oscillator, with inverted-pendulum stabilization representing one specific manifestation. Our work demonstrates a physical realization and extension of this mechanism in the context of superconducting transport.

It is well known that the dynamics of a Josephson junction can be effectively described by analogy with the motion of a classical pendulum. Specifically, in the resistively (R) and capacitively (C) shunted junction (RCSJ) model, the phase difference φ across the junction obeys the nonlinear differential equation: $C(\Phi_0/2\pi)\ddot{\varphi} + (1/R)[\Phi_0/2\pi]\dot{\varphi} + I_c \sin \varphi = I_b$, which mirrors the equation of motion for a damped pendulum subject to an external torque I_b (externally applied bias current). Now, suppose that the total current-phase relation of the system consists of two contributions $I = I_1 + I_2$. The first term, $I_1 = I_{c,1} \sin(\varphi + \phi)$, incorporates a phase shift ϕ arising from a source of time-reversal symmetry breaking. The second term, $I_2 = I_{c,2} \cos(\omega t) \sin(\varphi)$, indicates a periodically modulated critical current at frequency ω . Taking into account the analogy with the classical Kapitza pendulum, one expects that the resulting supercurrent has a nonreciprocal behavior when the amplitude of one of the Josephson components is parametrically driven in time.

Identifying the Josephson phase x with the pendulum angle φ , the equation can be expressed in a dimensionless form as

$$\ddot{x} + \beta \dot{x} + [i_{\text{dc}} + i_{\text{ac}} \cos(\omega t)] \sin x + i_{c,t} \sin(x + \phi) = \frac{1}{2} i_b, \quad (1)$$

where currents are normalized to a reference scale I_0 chosen as the characteristic Josephson critical current of the junction, and time is measured in units of plasma frequency $\omega_{p0} = \sqrt{2\pi I_0 / (C\Phi_0)}$ and dimensionless damping parameter $\beta = 1/(RC\omega_{p0})$. Equation (1) can be read as the equation of motion of a damped rotor (pendulum) with angular coordinate $x(t)$ under an external torque [see Secs. I–III of [59] for the detailed reduction of the Josephson dynamics to Eq. (1)]. The damping term $\beta \dot{x}$ is the usual viscous friction, while the bias i_b plays the role of a constant tilting torque that favors rotation in one direction. The term $[i_{\text{dc}} + i_{\text{ac}} \cos(\omega t)] \sin x$ is a restoring torque whose strength is modulated over time, providing the parametric driving force. The additional component $i_{c,t} \sin(x + \phi)$ is a second restoring torque that, however, is shifted in phase by ϕ . Mechanically, this corresponds to adding a second periodic potential whose minimum is displaced relative to the first one.

More broadly, Eq. (1) represents a generic class of parametrically driven nonlinear phase oscillators, suggesting that the same Kapitza-type rectification mechanism may extend well beyond superconducting tunnel junctions [60–62]. In particular, related implementations may be

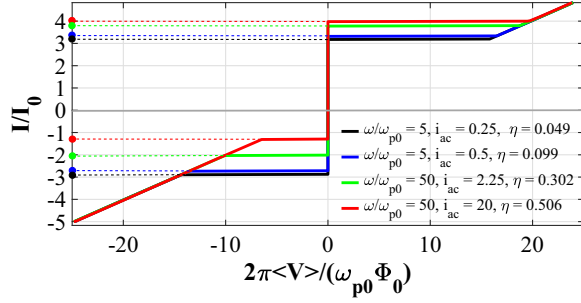


FIG. 1. Current-voltage characteristics of a parametrically driven Josephson junction with anomalous phase $\phi = \pi/2$ at $i_{dc} = 0$ for $i_{ac} = 0.25, 0.5$ ($\omega/\omega_{p0} = 5$, black and blue curves), and $i_{ac} = 2.25, 20$ ($\omega/\omega_{p0} = 50$, green and red curves). Inset: diode efficiency η extracted from each curve. Filled markers indicate $I_c^\pm$, and color-matched dotted lines guide the eye. The dimensionless damping is $\beta = 0.1$, and the amplitude of the additional phase-shifted component is $i_{c,t} = 2$.

envisioned in bosonic Josephson junctions, driven polariton weak links, spin-torque oscillators [63], and other nonlinear rotators with phase-shifted restoring components.

To illustrate the emergence of nonreciprocal supercurrent behavior, we start performing numerical simulations of the driven Josephson junction [Eq. (1)]. Here, we concentrate on a single time-dependent harmonic component (i_{ac}) in the time-reversal symmetric contribution to the supercurrent amplitude, while the time-reversal broken contribution $i_{c,t}$ is kept constant over time for given values of the anomalous phase ϕ . Modifying the time-dependent profile of the driving term does not alter the qualitative features of the resulting nonreciprocal behavior. The profile of the current-voltage characteristic is reported in Fig. 1, assuming a representative case for the anomalous phase, $\phi = \pi/2$. We find that the forward (I_c^+) and backward (I_c^-) critical currents of the zero-voltage state differ in magnitude, leading to a nonvanishing rectification amplitude defined as $\eta = (I_c^+ - |I_c^-|)/(I_c^+ + |I_c^-|)$. The rectification amplitude is found to depend on both the frequency and the amplitude of the driving mode, reaching 50% supercurrent rectification when the driving frequency significantly exceeds the plasma frequency ω_{p0} . The critical currents $I_c^\pm$ are extracted from a fixed phase-locked initialization protocol, which yields a single-valued current-voltage characteristic (see Sec. VI of [59] for a detailed description of the numerical integration scheme). Hysteresis on the running branch (switching-retrapping asymmetry) is a separate phenomenon that does not affect the definition of η and is left for future study.

The frequency dependence of I_c^+ and I_c^- indicates that, in the slow-driving regime, the two quantities are nearly identical and that there exists a threshold at $\omega/\omega_{p0} \sim 2$, above which the rectification increases significantly (Fig. 2). This trend is qualitatively reproduced when the anomalous phase ϕ is varied. Indeed, a sizable rectification

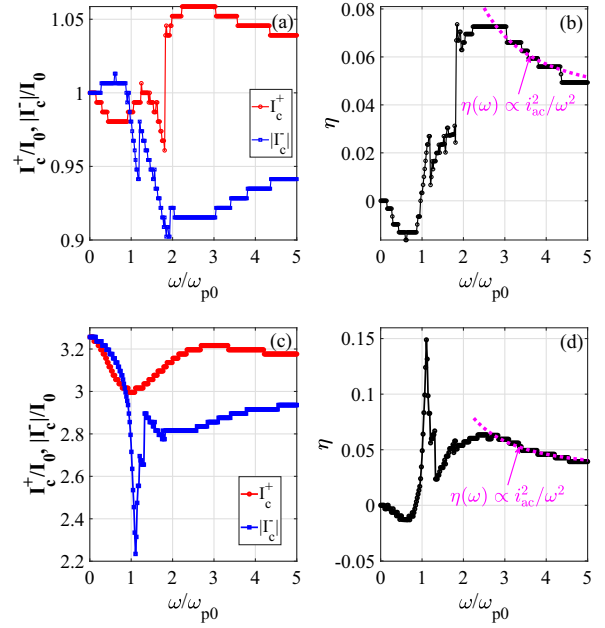


FIG. 2. (a),(c) Critical currents $I_c^\pm$ and (b),(d) diode rectification η of a driven Josephson junction versus ω/ω_{p0} for $\phi = \pi/2$ (a),(b) and $\phi = \pi/3$ (c),(d) with $i_{dc} = 0$, $i_{ac} = 0.25$. Magenta dotted lines show high-frequency asymptotics from the weak-amplitude expansion (see main text). Dimensionless damping $\beta = 0.1$, $i_{c,t} = 2$.

amplitude of the order of 15% is obtained close to the resonance at $\omega/\omega_{p0} \sim 1$, accompanied by a decrease in amplitude when moving to higher frequency [Figs. 2(c) and 2(d)].

It is both instructive and useful to analyze the emergence of nonreciprocal behavior by examining the analytical expression for the frequency-dependent rectification amplitude of the driven Josephson junction within the Kapitza framework (see Sec. IV of [59] for the detailed derivation). One can proceed by decomposing the phase as $x(t) = X(t) + \xi(t)$, where $X(t)$ is a slow variable and $\xi(t)$ is a small, rapidly oscillating correction induced by the driving field. Substituting into Eq. (1) and expanding to the second order in ξ yields

$$\begin{aligned} & \ddot{X} + \ddot{\xi} + \beta(\dot{X} + \dot{\xi}) \\ & + [i_{dc} + i_{ac} \cos(\omega t)] \left(\sin X + \xi \cos X - \frac{1}{2} \xi^2 \sin X \right) \\ & + i_{c,t} \left(\sin(X + \phi) + \xi \cos(X + \phi) \right. \\ & \left. - \frac{1}{2} \xi^2 \sin(X + \phi) \right) = \frac{1}{2} i_b. \end{aligned}$$

In the spirit of the Kapitza method, the equation can be separated for the fast variable $\xi(t)$, neglecting the slow velocity and acceleration terms, $\ddot{X}, \ddot{\xi}$, on the fast timescale. Then, by keeping only the terms linear in ξ and proportional

to the oscillatory part of the prefactor, $i_{ac} \cos(\omega t)$, we obtain

$$\ddot{\xi} + \beta \dot{\xi} + \omega_0^2 \xi \simeq -i_{ac} \cos(\omega t) \sin X, \quad (2)$$

where the local curvature (stiffness) of the effective potential is

$$\omega_0^2(X) \equiv i_{dc} \cos X + i_{c,t} \cos(X + \phi). \quad (3)$$

On the fast timescale, X can be considered to be approximately constant, so $\omega_0^2(X)$ is treated as a parameter. Equation (2) describes a damped harmonic oscillator driven at frequency ω . The steady periodic solution can be written as

$$\xi(t) = \text{Re} \left[\tilde{\xi} e^{i\omega t} \right], \quad \tilde{\xi} = -\frac{i_{ac} \sin X}{\omega_0^2(X) - \omega^2 + i\beta\omega}. \quad (4)$$

We now return to Eq. (2) and perform the time average over the fast oscillations, keeping terms up to order ξ^2 . The averaged equation for the slow variable thus becomes

$$\begin{aligned} \ddot{X} + \beta \dot{X} + i_{dc} \sin X + i_{c,t} \sin(X + \phi) \\ + i_{ac} \cos X \langle \cos(\omega t) \xi(t) \rangle_t \\ - \frac{1}{2} \langle \xi^2 \rangle_t \left[i_{dc} \sin X + i_{c,t} \sin(X + \phi) \right] = \frac{1}{2} i_b. \end{aligned}$$

The time average of the product between the parametric modulation and the induced fast oscillation, $i_{ac} \cos X \langle \cos(\omega t) \xi(t) \rangle_t$, generates the leading effective second harmonic, producing a static contribution to the current-phase relation of the form

$$I_s^{\text{eff}}(X) = A_1 \sin X + B_1 \cos X + A_2(\omega) \sin 2X, \quad (5)$$

with the first-harmonic coefficients $A_1 = i_{dc} + i_{c,t} \cos \phi$, and $B_1 = i_{c,t} \sin \phi$. The second-harmonic coefficient, instead, can be expressed as

$$A_2(\omega) = -\frac{i_{ac}^2}{4} \frac{\omega_0^2(x_0) - \omega^2}{(\omega_0^2(x_0) - \omega^2)^2 + (\beta\omega)^2}, \quad (6)$$

where X has been evaluated at the static equilibrium $X = x_0$ with $x_0 = -\arctan[\frac{i_{c,t} \sin \phi}{i_{dc} + i_{c,t} \cos \phi}]$. We neglect higher-order terms in i_{ac} and nonlinear corrections. Introducing the first-harmonic amplitude and phase shift $Q = \sqrt{A_1^2 + B_1^2}$, $\delta = \arctan(B_1/A_1)$, we can rewrite Eq. (5) as $I_s^{\text{eff}}(X) = Q \sin(X + \delta) + A_2(\omega) \sin 2X$. For $|A_2| \ll Q$, the extrema of $I_s^{\text{eff}}(X) = Q \sin(X + \delta) + A_2 \sin 2X$ give $I_c^+ \approx Q + A_2(\omega) \sin(2\delta)$, $I_c^- \approx -Q + A_2(\omega) \sin(2\delta)$, and therefore the diode efficiency

$$\eta(\omega) \approx -\frac{i_{ac}^2}{4Q} \frac{[\omega_0^2(x_0) - \omega^2] \sin 2\delta}{[\omega_0^2(x_0) - \omega^2]^2 + (\beta\omega)^2}, \quad (7)$$

with $\omega_0(x_0) \sim \omega_{p0}$. From Eq. (7), one observes that, in agreement with the numerical analysis, it attains a maximum

when $\omega \sim \omega_0(x_0)$. Although not vanishing, the rectification is suppressed at high frequencies and decays as $\eta(\omega) \propto i_{ac}^2/\omega^2$ for $|\omega| \rightarrow \infty$ within the weak-amplitude expansion of the driving field, as one can see in Figs. 2(b) and 2(d) (magenta dotted lines). Moreover, the rectification amplitude scales with the strength of the driving field and changes sign in the vicinity of the resonance at $\omega \sim \omega_0(x_0)$. It is important to recall that Eq. (7) is valid in the regime ($|\xi| \ll 1$), i.e., to the leading order in i_{ac}^2 ; a strong-driving field with large i_{ac} can lead to sizable quantitative deviations due to the occurrence of higher harmonics beyond $\sin 2X$. More precisely, Eq. (7) requires the separation of timescales $\omega \gg \omega_0(x_0)$ (good agreement is found already for $\omega/\omega_{p0} \gtrsim 2-3$) and a small fast-oscillation amplitude $|\xi| = i_{ac} |\sin x_0| / \sqrt{(\omega_0^2(x_0) - \omega^2)^2 + (\beta\omega)^2} \ll 1$, which away from resonance reduces to $i_{ac} \ll \omega^2$ and near resonance to $i_{ac} \ll \beta\omega$. No specific condition on β is required: the damping enters Eq. (7) only through the Lorentzian denominator, controlling the resonance width and the peak height of $A_2(\omega)$.

The physical (dimensional) amplitudes A_1^{phys} , B_1^{phys} , and A_2^{phys} follow from Eq. (5) by multiplying by I_0 , with $\omega_{p0} = \sqrt{2\pi I_0/\Phi_0 C}$. For $\phi = \pi/3$, $i_{dc} = 0$, $i_{c,t} = 2$, $i_{ac} = 0.25$, $\beta = 0.1$, $I_0 = 1 \mu\text{A}$, and $\omega_{p0}/2\pi = 10 \text{ GHz}$, one obtains $A_1^{\text{phys}} = 1 \mu\text{A}$, $B_1^{\text{phys}} \approx 1.73 \mu\text{A}$, and $|A_2^{\text{phys}}|_{\text{max}} \approx 156 \text{ nA}$, yielding $\eta_{\text{max}} \approx 0.068$, in agreement with Fig. 2(d). These expressions are valid within the weak-drive approximation $i_{ac} \ll 1$; all three amplitudes scale linearly with I_0 , while ω_{p0} sets only the resonance frequency at which $|A_2^{\text{phys}}|$ is maximized.

The classical RCSJ description adopted here is valid when the Josephson energy $E_J = I_0 \Phi_0/2\pi$ greatly exceeds the charging energy $E_C = e^2/2C$, i.e., $E_J/E_C \gg 1$, a condition readily satisfied for μA -scale critical currents and fF-scale capacitances where $E_J/E_C \sim 10^2-10^4$. In this regime, quantum-phase fluctuations produce only quantitative corrections (e.g., broadening of switching statistics). In the complementary regime $E_J/E_C \sim 1$, a quantum treatment is required. The Kapitza mechanism can be formulated at the Hamiltonian level via the Floquet-Magnus expansion: to leading order in $1/\Omega^2$, the effective potential acquires a $\cos 2\phi$ term with an amplitude $E_{J2} \propto (\delta E_J)^2 E_C/(\hbar^2 \Omega^2)$, in direct analogy with the classical result (see Sec. V of [59]). The drive-induced generation of higher harmonics is therefore expected to persist in the quantum regime, while the critical currents become probabilistic.

Having established the nonequilibrium supercurrent diode effect both numerically and analytically, we now discuss possible implementations of the Kapitza supercurrent diode. Two distinct design strategies can be pursued, both relying on interferometric schemes to break the time-reversal symmetry. The first approach is based on

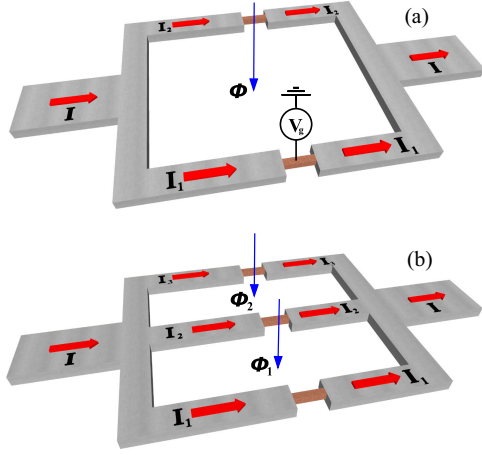


FIG. 3. Schematic implementations of the Kapitza supercurrent diode using superconducting interferometers. (a) Gate-controlled SQUID, with time-dependent gate modulation (V_g) and a static flux (Φ)-induced phase shift. (b) Double-loop interferometer with parametric flux drive and static flux breaking time-reversal symmetry. In both cases, parametric driving combined with symmetry breaking yields nonreciprocal supercurrent transport.

gate-controlled supercurrents [64–69], where the supercurrent is driven by a time-dependent gate voltage. The second relies entirely on magnetic control using superconducting interferometers, in which the parametric modulation of the supercurrent is induced by a periodically time-dependent magnetic flux. In the first implementation [Fig. 3(a)], time-reversal symmetry is broken using an electrically controlled superconducting interferometer in a SQUID-like geometry. The device consists of a superconducting loop interrupted by Josephson tunnel junctions whose critical currents can be tuned via electrostatic gates. A time-dependent gate voltage is applied to one of the junctions, inducing a periodic modulation of the supercurrent, while an externally applied magnetic flux is used to introduce an anomalous phase shift in the other Josephson junction [59].

We note that, compared to flux-biased multiarm interferometric designs such as SNAIL-based diodes [70], which achieve passive nonreciprocity at the expense of increased circuit complexity, the Kapitza approach offers a complementary route based on the dynamical generation of higher harmonics, enabling *in situ* tunability and reconfigurability within a minimal architecture, albeit requiring an external drive.

The second implementation exploits time-dependent magnetic control in a superconducting interferometer with double-loop geometry. One loop is threaded by a periodically time-dependent magnetic flux, providing the parametric drive, while the other loop is biased by a static flux and acts as a source of time-reversal symmetry breaking. By expressing current-phase relations in terms of average and relative phases, the dynamics of the system can be mapped to that of a driven Josephson junction with an effective static time-reversal symmetry-breaking term in the supercurrent, leading to nonreciprocal transport [59].

The relevant frequency window for the time-periodic modulation is set by the Josephson plasma frequency, which defines the intrinsic dynamical scale of the junction. For typical parameters ($I_c \sim 1 \text{ nA} - 5 \text{ }\mu\text{A}$ and $C \sim 1-100 \text{ fF}$) [71], one obtains $\omega_{p0}/2\pi \sim 0.2-100 \text{ GHz}$. To realize the proposed nonreciprocal dynamical regime, the drive frequency ω should satisfy $\omega_J \ll \omega \lesssim 5\omega_{p0}$, where ω_J is the characteristic Josephson frequency associated with slow phase dynamics (typically in the MHz to sub-GHz range). This condition defines an experimentally accessible window $\omega/2\pi \sim 1-10 \text{ GHz}$, well within the range achievable by using either microwave gate modulation or on-chip flux control in superconducting interferometers.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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- [1] F. Ando, Y. Miyasaka, T. Li, J. Ishizuka, T. Arakawa, Y. Shiota, T. Moriyama, Y. Yanase, and T. Ono, Observation of superconducting diode effect, *Nature (London)* **584**, 373 (2020).
 - [2] L. Bauriedl, C. Bäuml, L. Fuchs, C. Baumgartner, N. Paulik, J. M. Bauer, K.-Q. Lin, J. M. Lupton, T. Taniguchi, K. Watanabe, C. Strunk, and N. Paradiso, Supercurrent diode effect and magnetochiral anisotropy in few-layer NbSe₂, *Nat. Commun.* **13**, 4266 (2022).
 - [3] C. Baumgartner, L. Fuchs, A. Costa, S. Reinhardt, S. Gronin, G. C. Gardner, T. Lindemann, M. J. Manfra, P. E. Faria Junior, D. Kochan, J. Fabian, N. Paradiso, and C. Strunk, Supercurrent rectification and magnetochiral effects in symmetric Josephson junctions, *Nat. Nanotechnol.* **17**, 39 (2022).
 - [4] H. Wu, Y. Wang, Y. Xu, P. K. Sivakumar, C. Pasco, U. Filippozzi, S. S. P. Parkin, Y.-J. Zeng, T. McQueen, and M. N. Ali, The field-free Josephson diode in a van der Waals heterostructure, *Nature (London)* **604**, 653 (2022).
 - [5] K.-R. Jeon, J.-K. Kim, J. Yoon, J.-C. Jeon, H. Han, A. Cottet, T. Kontos, and S. S. P. Parkin, Zero-field polarity-reversible Josephson supercurrent diodes enabled by a proximity-magnetized Pt barrier, *Nat. Mater.* **21**, 1008 (2022).

- [6] M. Nadeem, M. S. Fuhrer, and X. Wang, The superconducting diode effect, *Nat. Rev. Phys.* **5**, 558 (2023).
- [7] S. Ghosh, V. Patil, A. Basu, Kuldeep, A. Dutta, D. A. Jangade, R. Kulkarni, A. Thamizhavel, J. F. Steiner, F. von Oppen, and M. M. Deshmukh, High-temperature Josephson diode, *Nat. Mater.* **23**, 612 (2024).
- [8] N. Dirneger, P. Narang, and A. Arora, Nonreciprocal quantum information processing with superconducting diodes in circuit quantum electrodynamics, *arXiv:2511.20758*.
- [9] H. Narita, J. Ishizuka, R. Kawarazaki, D. Kan, Y. Shiota, T. Moriyama, Y. Shimakawa, A. V. Ognev, A. S. Samardak, Y. Yanase, and T. Ono, Field-free superconducting diode effect in noncentrosymmetric superconductor/ferromagnet multilayers, *Nat. Nanotechnol.* **17**, 823 (2022).
- [10] D. Margineda, A. Crippa, E. Strambini, Y. Fukaya, M. T. Mercaldo, M. Cuoco, and F. Giazotto, Sign reversal diode effect in superconducting Dayem nanobridges, *Commun. Phys.* **6**, 343 (2023).
- [11] R. Wakatsuki, Y. Saito, S. Hoshino, Y. M. Itahashi, T. Ideue, M. Ezawa, Y. Iwasa, and N. Nagaosa, Nonreciprocal charge transport in noncentrosymmetric superconductors, *Sci. Adv.* **3**, e1602390 (2017).
- [12] Y. M. Itahashi, T. Ideue, Y. Saito, S. Shimizu, T. Ouchi, T. Nojima, and Y. Iwasa, Nonreciprocal transport in gate-induced polar superconductor SrTiO₃, *Sci. Adv.* **6**, eaay9120 (2020).
- [13] J.-X. Lin, P. Siriviboon, H. D. Scammell, S. Liu, D. Rhodes, K. Watanabe, T. Taniguchi, J. Hone, M. S. Scheurer, and J. I. A. Li, Zero-field superconducting diode effect in small-twist-angle trilayer graphene, *Nat. Phys.* **18**, 1221 (2022).
- [14] A. Rothstein, R. J. Dolleman, L. Klebl, A. Achtermann, F. Volmer, K. Watanabe, T. Taniguchi, F. Hassler, L. Banszerus, B. Beschoten, and C. Stampfer, Gate-tunable Josephson diodes in magic-angle twisted bilayer graphene, *Nano Lett.* **26**, 2119 (2026).
- [15] S. Qi, J. Ge, C. Ji, Y. Ai, G. Ma, Z. Wang, Z. Cui, Y. Liu, Z. Wang, and J. Wang, High-temperature field-free superconducting diode effect in high-T_c cuprates, *Nat. Commun.* **16**, 531 (2025).
- [16] G.-L. Guo, X.-H. Pan, and X. Liu, ϕ_0 junction and Josephson diode effect in high-temperature superconductors, *Phys. Rev. B* **112**, 014509 (2025).
- [17] P. A. Volkov, E. Lantagne-Hurtubise, T. Tummuru, S. Plugge, J. H. Pixley, and M. Franz, Josephson diode effects in twisted nodal superconductors, *Phys. Rev. B* **109**, 094518 (2024).
- [18] E. Zhang, X. Xu, Y.-C. Zou, L. Ai, X. Dong, C. Huang, P. Leng, S. Liu, Y. Zhang, Z. Jia, X. Peng, M. Zhao, Y. Yang, Z. Li, H. Guo, S. J. Haigh, N. Nagaosa, J. Shen, and F. Xiu, Nonreciprocal superconducting NbSe₂ antenna, *Nat. Commun.* **11**, 5634 (2020).
- [19] Y. Yerin, S.-L. Drechsler, A. A. Varlamov, M. Cuoco, and F. Giazotto, Supercurrent rectification with time-reversal symmetry broken multiband superconductors, *Phys. Rev. B* **110**, 054501 (2024).
- [20] Y. Yerin, S.-L. Drechsler, A. A. Varlamov, F. Giazotto, and M. Cuoco, Supercurrent diode effect in Josephson interferometers with multiband superconductors, *Commun. Phys.* **8**, 356 (2025).
- [21] A. Arora and P. Narang, Chiral cavity control of superconducting diode-like nonlinearities, *arXiv:2501.17924*.
- [22] A. Costa *et al.*, Sign reversal of the Josephson inductance magnetochiral anisotropy and $0 - \pi$ -like transitions in supercurrent diodes, *Nat. Nanotechnol.* **18**, 1266 (2023).
- [23] N. Lotfizadeh, W. F. Schiela, B. Pekerten, P. Yu, B. H. Elfeky, W. M. Strickland, A. Matos-Abiague, and J. Shabani, Superconducting diode effect sign change in epitaxial Al-InAs Josephson junctions, *Commun. Phys.* **7**, 120 (2024).
- [24] L. Borgogino, R. Seoane Souto, A. Paghi, G. Senesi, K. Skibinska, L. Sorba, E. Riccardi, F. Giazotto, and E. Strambini, Biharmonic-drive tunable Josephson diode, *Nano Lett.* **25**, 14451 (2025).
- [25] M. Amundsen, J. Linder, J. W. A. Robinson, I. Žutić, and N. Banerjee, Colloquium: Spin-orbit effects in superconducting hybrid structures, *Rev. Mod. Phys.* **96**, 021003 (2024).
- [26] R. S. Souto, M. Leijnse, and C. Schrade, Josephson diode effect in supercurrent interferometers, *Phys. Rev. Lett.* **129**, 267702 (2022).
- [27] A. M. Bozkurt, J. Brookman, V. Fatemi, and A. R. Akhmerov, Double-Fourier engineering of Josephson energy-phase relationships applied to diodes, *SciPost Phys.* **15**, 204 (2023).
- [28] Y. V. Fominov and D. S. Mikhailov, Asymmetric higher-harmonic SQUID as a Josephson diode, *Phys. Rev. B* **106**, 134514 (2022).
- [29] A. Greco, Q. Pichard, and F. Giazotto, Josephson diode effect in monolithic dc-SQUIDs based on 3D Dayem nanobridges, *Appl. Phys. Lett.* **123**, 092601 (2023).
- [30] A. Greco, Q. Pichard, E. Strambini, and F. Giazotto, Double loop dc-SQUID as a tunable Josephson diode, *Appl. Phys. Lett.* **125**, 072601 (2024).
- [31] L. Chirulli, A. Greco, A. Crippa, E. Strambini, M. Cuoco, L. Amico, and F. Giazotto, Diode effect in the Fraunhofer pattern of disordered planar Josephson junctions, *Commun. Phys.* **8**, 483 (2025).
- [32] T. A. Fulton, L. N. Dunkleberger, and R. C. Dynes, Quantum interference properties of double Josephson junctions, *Phys. Rev. B* **6**, 855 (1972).
- [33] A. Barone and G. Paternò, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
- [34] Y. Hou, F. Nichele, H. Chi, A. Lodesani, Y. Wu, M. F. Ritter, D. Z. Haxell, M. Davydova, S. Ilić, O. Glezakou-Elbert, A. Varambally, F. S. Bergeret, A. Kamra, L. Fu, P. A. Lee, and J. S. Moodera, Ubiquitous superconducting diode effect in superconductor thin films, *Phys. Rev. Lett.* **131**, 027001 (2023).
- [35] A. Sundaresh, J. I. Väyrynen, Y. Lyanda-Geller, and L. P. Rokhinson, Diamagnetic mechanism of critical current non-reciprocity in multilayered superconductors, *Nat. Commun.* **14**, 1628 (2023).
- [36] V. M. Krasnov, V. A. Oboznov, and N. F. Pedersen, Fluxon dynamics in long Josephson junctions in the presence of a temperature gradient or spatial nonuniformity, *Phys. Rev. B* **55**, 14486 (1997).
- [37] T. Golod and V. M. Krasnov, Demonstration of a superconducting diode-with-memory, operational at zero magnetic field with switchable nonreciprocity, *Nat. Commun.* **13**, 3658 (2022).
- [38] S. Chen, S. Park, U. Vool, N. Maksimovic, D. A. Broadway, M. Flaks, T. X. Zhou, P. Maletinsky, A. Stern, B. I. Halperin,

- and A. Yacoby, Current induced hidden states in Josephson junctions, *Nat. Commun.* **15**, 8059 (2024).
- [39] M. Gupta, G. V. Graziano, M. Pendharkar, J. T. Dong, C. P. Dempsey, C. Palmstrøm, and V. S. Pribiag, Gate-tunable superconducting diode effect in a three-terminal Josephson device, *Nat. Commun.* **14**, 3078 (2023).
- [40] F. Zhang, A. S. Rashid, M. Tanhayi Ahari, G. J. de Coster, T. Taniguchi, K. Watanabe, M. J. Gilbert, N. Samarth, and M. Kayyalha, Magnetic-field-free nonreciprocal transport in graphene multiterminal Josephson junctions, *Phys. Rev. Appl.* **21**, 034011 (2024).
- [41] C. Ciaccia, R. Haller, Asbjorn C. C. Drachmann, T. Lindemann, M. J. Manfra, C. Schrade, and C. Schönenberger, Gate-tunable Josephson diode in proximitized InAs supercurrent interferometers, *Phys. Rev. Res.* **5**, 033131 (2023).
- [42] C. C. de Souza Silva, J. Van de Vondel, M. Morelle, and V. V. Moshchalkov, Controlled multiple reversals of a ratchet effect, *Phys. Rev. Lett.* **94**, 177001 (2005).
- [43] M. Beck, E. Goldobin, M. Neuhaus, M. Siegel, R. Kleiner, and D. Koelle, High-efficiency deterministic Josephson vortex ratchet, *Phys. Rev. Lett.* **95**, 090603 (2005).
- [44] T. Golod, A. Rydh, and V. M. Krasnov, Detection of the phase shift from a single Abrikosov vortex, *Phys. Rev. Lett.* **104**, 227003 (2010).
- [45] T. Golod, A. Iovan, and V. M. Krasnov, Single Abrikosov vortices as quantized information bits, *Nat. Commun.* **6**, 8628 (2015).
- [46] A. Gutfreund, H. Matsuki, V. Plastovets, A. Noah, L. Gorzawski, N. Fridman, G. Yang, A. Buzdin, O. Millo, J. W. A. Robinson, and Y. Anahory, Direct observation of a superconducting vortex diode, *Nat. Commun.* **14**, 1630 (2023).
- [47] J. Jiang, Y.-L. Wang, M. V. Milošević, Z.-L. Xiao, F. M. Peeters, and Q.-H. Chen, Reversible ratchet effects in a narrow superconducting ring, *Phys. Rev. B* **103**, 014502 (2021).
- [48] Y. Fukaya, M. T. Mercaldo, D. Margineda, A. Crippa, E. Strambini, F. Giazotto, C. Ortix, and M. Cuoco, Supercurrent diode with high winding vortex, *Commun. Phys.* **8**, 518 (2025).
- [49] G. De Simoni and F. Giazotto, Quasi-ideal feedback-loop supercurrent diode, *Phys. Rev. Appl.* **21**, 064058 (2024).
- [50] Y. S. Yerin, A. S. Kiyko, A. N. Omelyanchouk, and E. Il'ichev, Josephson systems based on ballistic point contacts between single-band and multi-band superconductors, *Low Temp. Phys.* **41**, 885 (2015).
- [51] Y. Yerin and A. N. Omelyanchouk, Proximity and Josephson effects in microstructures based on multiband superconductors, *Low Temp. Phys.* **43**, 1013 (2017).
- [52] I. Askerzade, *Unconventional Superconductors. Anisotropy and Multiband Effects* (Springer, New York, 2012), Vol. 153.
- [53] I. N. Askerzade, Effects of anharmonicity of current-phase relation in Josephson junctions, *Low Temp. Phys.* **41**, 241 (2015).
- [54] A. Eichler and O. Zilberberg, *Classical and Quantum Parametric Phenomena* (Oxford University Press, New York, 2023).
- [55] P. L. Kapitza, Dynamic stability of a pendulum when its point of suspension vibrates, *Zh. Eksp. Teor. Fiz.* **21**, 588 (1951).
- [56] P. L. Kapitza, Pendulum with a vibrating suspension (in Russian), *Usp. Fiz. Nauk* **44**, 7 (1951).
- [57] The Kapitza pendulum is an inverted pendulum stabilized by rapid vertical oscillations of its pivot, whose slow dynamics is described by an effective time-averaged potential. The stabilization effect was first noted by Stephenson in 1908 [58], while Kapitza's 1951 work established the standard averaging-based description [55,56].
- [58] A. Stephenson, XX. On induced stability, *London Edinburgh Dublin Philos. Mag. J. Sci.* **15**, 233 (1908).
- [59] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/jn4z-j1ll> for the reduction of the single- and double-loop SQUID to an effective parametrically driven pendulum equation, the detailed Kapitza averaging with ac current, the Floquet-Magnus derivation of the effective second Josephson harmonic in the quantum limit, and the numerical protocol used to compute the current-voltage characteristics and extract the diode efficiency.
- [60] Y. M. Shukrinov, A. Mazanik, I. R. Rahmonov, A. E. Botha, and A. Buzdin, Re-orientation of the easy axis in a φ_0 -junction, *Europhys. Lett.* **122**, 37001 (2018).
- [61] K. V. Kulikov, D. V. Anghel, A. T. Preda, M. Nashaat, M. Sameh, and Y. M. Shukrinov, Kapitza pendulum effects in a Josephson junction coupled to a nanomagnet under external periodic drive, *Phys. Rev. B* **105**, 094421 (2022).
- [62] M. Nashaat, M. Sameh, A. E. Botha, K. V. Kulikov, and Y. M. Shukrinov, Bifurcation structure and chaos in dynamics of nanomagnet coupled to Josephson junction, *Chaos Interdiscip. J. Nonlinear Sci.* **32**, 093142 (2022).
- [63] C. Han, M. Wang, B. Zhang, M. I. Dykman, and H. B. Chan, Coupled parametric oscillators: From disorder-induced current to asymmetric Ising model, *Phys. Rev. Res.* **6**, 023162 (2024).
- [64] L. Ruf, C. Puglia, T. Elalaily, G. De Simoni, F. Joint, M. Berke, J. Koch, A. Iorio, S. Khorshidian, P. Makk, S. Gasparinetti, S. Csonka, W. Belzig, M. Cuoco, F. Giazotto, E. Scheer, and A. Di Bernardo, Gate control of superconducting current: Mechanisms, parameters, and technological potential, *Appl. Phys. Rev.* **11**, 041314 (2024).
- [65] G. De Simoni, F. Paolucci, C. Puglia, and F. Giazotto, Josephson field-effect transistors based on all-metallic Al/Cu/Al proximity nanojunctions, *ACS Nano* **13**, 7871 (2019).
- [66] M. T. Mercaldo, P. Solinas, F. Giazotto, and M. Cuoco, Electrically tunable superconductivity through surface orbital polarization, *Phys. Rev. Appl.* **14**, 034041 (2020).
- [67] F. Paolucci, G. De Simoni, P. Solinas, E. Strambini, C. Puglia, N. Ligato, and F. Giazotto, Field-effect control of metallic superconducting systems, *AVS Quantum Sci.* **1**, 016501 (2019).
- [68] D. Monroe, M. Alidoust, and I. Žutić, Tunable planar Josephson junctions driven by time-dependent spin-orbit coupling, *Phys. Rev. Appl.* **18**, L031001 (2022).
- [69] D. Monroe, C. Shen, D. Tringali, M. Alidoust, T. Zhou, and I. Žutić, Phase control of the Josephson diode effect via time-dependent spin-orbit coupling, *Appl. Phys. Lett.* **125**, 012601 (2024).
- [70] R. Haenel and O. Can, Superconducting diode from flux biased Josephson junction arrays, [arXiv:2212.02657](https://arxiv.org/abs/2212.02657).
- [71] A. A. Golubov, M. Y. Kupriyanov, and E. Il'ichev, The current-phase relation in Josephson junctions, *Rev. Mod. Phys.* **76**, 411 (2004).