

Particle-Soliton Degeneracy in 2D Quantum Chromodynamics

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Quantum chromodynamics in two spacetime dimensions admits a finite noninvertible symmetry described mathematically by a fusion category. This symmetry is spontaneously broken at long distances, leading to distinct vacua. When the theory has a mass gap, the spectrum is therefore characterized by particle excitations above a single vacuum and soliton sectors interpolating between vacua. We use anyon condensation and the representation theory of fusion categories to obtain exact results about this spectrum, exhibiting the allowed multiplets. Often, particles and solitons are in the same representation and therefore must have equal masses. Furthermore, the fusion category symmetry frequently implies the existence of certain stable states in the spectrum. The resulting degeneracies are encoded in quiver diagrams where nodes are vacua and arrows are excited states.

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Introduction—Gauge theory is a central paradigm in particle physics. Among its most fascinating properties is its strongly coupled nature: ultraviolet degrees of freedom consisting of manifest quarks and gauge fields give way in the infrared (IR) to a surprising particle spectrum of mesons and baryons. Analytically understanding this phenomenon has served as a driving goal of theoretical physics for the past half-century.

Two-dimensional (2D) theories have long served as a playground for testing new techniques to attack strongly coupled physics. In particular, in quantum chromodynamics (QCD₂) 't Hooft famously solved the theory in the large number of colors limit [1]. More recently, novel noninvertible symmetries have been applied to understand confinement [2] and to characterize the vacuum structure of QCD₂ where the distinct states are parameterized by expectation values of gauge-invariant composite operators [3–5].

In this Letter, we continue this line of development, building on the recent results [6,7]. Focusing on the case of gapped QCD₂, we aim to directly characterize the spectrum. We realize QCD₂ via an interval compactification from a three-dimensional (3D) topological quantum field theory (TQFT) and following Ref. [3] identify the fusion category $\mathcal{C}$ which describes the mathematical structure of

the noninvertible symmetry [8–11]. Physically, such symmetries are realized by topological line operators that commute with the Hamiltonian [12] but are not in general represented by unitary operators acting on the Hilbert space. Moreover, as noted in Ref. [3], the fusion category $\mathcal{C}$ is fully spontaneously broken in the IR and hence characterizes the vacua of the gauge theory.

We harness the representation theory of this fusion category to determine the allowed multiplets of particles, excitations above a single vacuum, and solitons interpolating between distinct vacua. Strikingly, we find that the fusion category symmetry $\mathcal{C}$ often implies that particle and solitons are in the same representation and hence necessarily have equal masses. This feature is unique to spontaneously broken noninvertible symmetries and illustrates the intrinsically quantum (strongly coupled) nature of these symmetries.

We exhibit our results in several explicit gauge theories. (These predictions have been recently verified numerically in Ref. [13] using Hamiltonian lattice methods.) The implied degeneracies are elegantly encoded in a quiver diagram where the nodes are vacua and the arrows are excited particle and soliton states. Such diagrams echo those discussed in many contexts such as the lattice integrable models of Ref. [14], the relation between graphs, nondiagonal modular invariants, and conformal boundary conditions of Refs. [15,16] (see also Refs. [17,18] for the relationship between anyon condensation and generalized ADE diagrams), and in the direct analysis of supersymmetric solitons [19].

Background on QCD₂—We are interested in QCD in two spacetime dimensions with massless fermions. The gauge group is denoted by G , and the fermions transform in a representation $\mathbf{R}$ of G . We take the fermions to have vanishing bare mass. The action is

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$$S_f(g_{\text{YM}}) = \int_{\Sigma} d^2x \left[\text{Tr}(\Psi^T i \not{D}_{\rho} \Psi) - \frac{1}{4g_{\text{YM}}^2} \text{Tr}(F^2) \right]. \quad (1)$$

For simplicity in the following, we consider only vectorlike theories where the left-moving and right-moving fermions transform in the same representation $\mathbf{R}$ of the gauge group G and further restrict to the case where $\mathbf{R}$ is irreducible.

As defined above, QCD_2 is a fermionic theory, i.e., there are fermionic local operators, and the partition functions depend on a choice of spacetime spin structure. To simplify our discussion, we can bosonize the fermions [20], and recast these degrees of freedom as a theory of currents, a Wess-Zumino-Witten (WZW) model $\text{Spin}[\dim(\mathbf{R})]_1$. Coupling to G gauge fields then results in a bosonic version of QCD_2 as a gauged WZW model with a G gauge field kinetic term:

$$S_b(g_{\text{YM}}) = S_{\text{WZW}}[g, A] - \frac{1}{4g_{\text{YM}}^2} \int_{\Sigma} d^2x \text{Tr}(F^2). \quad (2)$$

We present our results below for this bosonized theory. Since bosonization-fermionization is an invertible operation [21,22], this involves no loss in generality.

Our goal is to constrain particle and soliton states. This is cleanest when the theory in question has a mass gap so that we can separate the spectrum from any residual gapless modes. In QCD_2 , there is a simple criterion that controls the gap [4]. Denote by $I(\mathbf{R})$ the Dynkin index of the representation $\mathbf{R}$ and by c_{G_k} the central charge of the WZW model based on a Lie group G at level k . Then, the theory is gapped if and only if the coset,

$$\frac{\text{Spin}[\dim(\mathbf{R})]_1}{G_{I(\mathbf{R})}}, \quad (3)$$

has a vanishing central charge:

$$c_{\text{Spin}[\dim(\mathbf{R})]_1/G_{I(\mathbf{R})}} = c_{\text{Spin}[\dim(\mathbf{R})]_1} - c_{G_{I(\mathbf{R})}} = 0. \quad (4)$$

When this is the case, we refer to (3) as a *topological coset*. This condition for a gap is valid irrespective of the global form of the gauge group G , provided that $\mathbf{R}$ is an allowed representation of G .

Next we turn to the effective infrared (IR) description of the QCD_2 theory (2). A natural candidate arises if we examine (2) and assume that the IR is described by the corresponding $g_{\text{YM}} \rightarrow \infty$ limit:

$$S_b(g_{\text{YM}}) \xrightarrow{g_{\text{YM}} \rightarrow \infty} S_{\text{WZW}}[g, A]. \quad (5)$$

We find that the IR is described by the aforementioned gauged WZW model consisting of $\text{Spin}[\dim(\mathbf{R})]_1$ matter content coupled to G gauge fields. We assume this description below. Algebraically, this corresponds to the quotient of chiral algebras in (3); however, one must

FIG. 1. Three-dimensional construction of QCD_2 .

carefully keep track of all topological sectors, i.e., distinct vacua. (See, e.g., Refs. [3,4,23].)

One potential source of vacua of the theory can be seen directly in the ultraviolet from the presence of topological local operators, or equivalently exact (in contrast to emergent) one-form symmetries. Such symmetries arise in particular from the subgroup of the center of the gauge group G which acts trivially on the matter representation $\mathbf{R}$. As discussed in Ref. [2], exact one-form symmetries split the theory into distinct *universes*, with no finite energy configurations interpolating between them. A given universe contains, however, its own set of vacua, and it remains meaningful to discuss finite-energy configurations in between such vacua. For simplicity below, we avoid this phenomenon by focusing on examples with trivial one-form symmetry whose unique exact topological local operator is the identity.

Next, we discuss how to characterize the zero-form symmetries of gapped QCD_2 . These symmetries play a crucial role in our treatment of particle-soliton degeneracy. An insightful approach is to construct QCD_2 via compactification from three spacetime dimensions on a transverse interval [2,3,5,24]. In this viewpoint, QCD_2 with a simply connected gauge group can be constructed by taking a Chern-Simons theory in bulk:

$$\mathcal{Z}(\mathcal{C}) := \text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}, \quad (6)$$

and setting coset boundary conditions on the left and right of the interval. Notice that, since we are considering the gapped case, these are *topological boundary conditions* for $\mathcal{Z}(\mathcal{C})$. See Fig. 1. An interface $\mathcal{I}_{\text{phys}}$ controlling the RG flow at finite values of the coupling is set in the middle of the interval. The assumption that the IR is described by the topological coset (5) is then equivalent to the statement that in the far IR the surface $\mathcal{I}_{\text{phys}}$ becomes the transparent identity interface in $\mathcal{Z}(\mathcal{C})$.

The construction of QCD_2 sketched above manifests the symmetries. Indeed, given any 3D TQFT, a fusion category $\mathcal{C}$ of line operators is always supported at any topological boundary [25]. Viewed purely from two dimensions, $\mathcal{C}$ are topological lines and hence yield the desired symmetries. Moreover, the 3D construction of QCD_2 implies that such a fusion category of line operators persists along the whole RG flow and hence can be used to analyze the spectrum. (One can also see that the quotient of chiral algebras is preserved along the whole RG directly in 2D [4,5,26].) In the IR, the surface $\mathcal{I}_{\text{phys}}$ becomes transparent, and this symmetry is fully spontaneously broken:

$$\mathcal{C} \longrightarrow \mathbf{1}. \quad (7)$$

In particular, the vacua are in one-to-one correspondence with simple objects in $\mathcal{C}$ [3]. Finally, we remark that, in general, $\mathcal{C}$ may not be the complete finite symmetry of a given (bosonized) gapped QCD_2 . However, because of its manifest nature from 3D, $\mathcal{C}$ can be readily determined in practice, and hence its implications for spectra are especially computable. [We also remark that, following Ref. [3], our methods are in principle generalizable to any gauged WZW model described by a conformal embedding in (2) with IR description (5).]

Anyon condensation and topological cosets—Our previous analysis has indicated the central importance of $\mathcal{C}$, the fusion category of lines at a topological boundary of a 3D TQFT. To determine $\mathcal{C}$ directly, we use anyon condensation. We refer to Ref. [25] for a detailed presentation and in the following provide a simplified presentation sufficient for our examples. For an introduction to the algebraic theory of anyons, see Refs. [27,28]. Our conventions follow Ref. [3].

In general, an arbitrary topological boundary condition of a 3D TQFT $\mathcal{T}$ can be described in terms of a nonsimple anyon called a *Lagrangian algebra* [25,29]:

$$\mathcal{L} = \bigoplus_{a \in \mathcal{T}} n_a a, \quad n_a \in \mathbb{N}, \quad (8)$$

where a are simple anyons and n_a are positive integers. For simplicity, we consider the multiplicity-free case $n_a = 0, 1$. A useful interpretation of the Lagrangian algebra is that it dictates the anyons that are allowed to end perpendicularly in a topological junction at the topological boundary described by $\mathcal{L}$. See Fig. 2. Lagrangian algebras can only be composed of bosons, that is, if the topological spin $\theta_a \neq 1 \Rightarrow n_a = 0$. Moreover, Lagrangian algebras satisfy the quantum dimension constraints $\dim(\mathcal{T}) = [\dim(\mathcal{L})]^2$, where $\dim(\mathcal{T}) = \sum_{a \in \mathcal{T}} d_a^2$ and $\dim(\mathcal{L}) = \sum_{a \in \mathcal{T}} n_a d_a$. In general, it is a nontrivial requirement for a 3D TQFT to have a set of anyons fulfilling these properties, so this already provides a nontrivial tool to characterize topological boundaries. For a complete characterization of Lagrangian algebras, we refer to the aforementioned references.

The line operators at the topological boundary can be found by taking a simple anyon a of the bulk TQFT and moving it to the boundary. Generically, such a simple anyon becomes a nonsimple line operator at the boundary. This is encapsulated in the following “splitting” rule:

$$a = \sum_{\alpha} z_{\alpha}^a \alpha, \quad z_{\alpha}^a \in \mathbb{N}, \quad (9)$$

where α stands for the distinct simple line operators of the boundary fusion category. Generically, a simple line operator α may appear with multiplicity in the splitting rule, a fact that is encoded in the non-negative integers z_{α}^a .

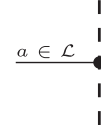


FIG. 2. Anyon $a \in \mathcal{L}$ can end at a topological junction at the topological boundary defined by $\mathcal{L}$.

Simple anyons in the Lagrangian algebra $\mathcal{L} = \bigoplus_a n_a a$ always have a component of the identity line of the boundary theory: $a \rightarrow 1 + \dots$.

The boundary fusion category $\mathcal{C}$ is constrained to satisfy the following set of consistency conditions: (i) $a = \sum_{\alpha} z_{\alpha}^a \alpha \Rightarrow d_a = \sum_b z_b^a d_b$, (ii) $a = \sum_{\alpha} z_{\alpha}^a \alpha \Rightarrow a = \sum_{\alpha} z_{\alpha}^a \bar{\alpha}$, and (iii) $a \otimes b = \bigoplus_c N_{a,b}^c c \Rightarrow \left(\sum_{\alpha} z_{\alpha}^a \alpha \right) \times \left(\sum_{\beta} z_{\beta}^b \beta \right) = \sum_{c,\gamma} N_{a,b}^c z_{\gamma}^c \gamma$. The boundary fusion category must also satisfy the standard conditions of associativity, existence of a unique identity line, and existence of unique conjugates with a unique way to fuse to the identity line. Often, one can exploit the previous constraints and find the fusion ring of the fusion category exactly. This is our main practical tool below.

The relation between topological cosets and topological boundary conditions has recently been studied in detail in Ref. [3]. In short, when we have an embedding of chiral algebras $G_k \hookrightarrow \mathcal{Q}_1$ with an associated gauged WZW model with vanishing central charge and simply connected G , then the Chern-Simons theory $\mathcal{Q}_1 \times G_{-k}$ admits a topological boundary. (This is the statement that topological cosets belong to the trivial Witt class [30].) More specifically, the branching rules of the conformal embedding,

$$\chi_{\Lambda}^{\mathcal{Q}_1}(q) = \sum_{\lambda} b_{(\Lambda,\lambda)} \chi_{\lambda}^{G_k}(q), \quad b_{(\Lambda,\lambda)} \in \mathbb{N}, \quad (10)$$

with q being the modular parameter, induce the existence of a Lagrangian algebra $\mathcal{L} = \bigoplus_{(\Lambda,\lambda)} b_{(\Lambda,\lambda)} (\Lambda, \lambda^{\text{op}})$ defining the corresponding topological boundary of $\mathcal{Q}_1 \times G_{-k}$. Thus, we can in particular apply this observation to the topological cosets (3) pertinent to QCD_2 and apply the rules of anyon condensation to obtain the fusion categories that are present throughout the RG flow.

As discussed above, in order to avoid the phenomenon of universe splitting in QCD_2 , we may consider global forms of the gauge group that have a trivial center. To address this in the 3D construction, we follow Refs. [23,31,32] and gauge the “common center” one-form symmetry in $\text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}$. After this is done, no exact one-form symmetry given by the center of the gauge group remains, and all of the topological local operators in the IR correspond to vacua of a single universe. Notice that a gapped boundary for the gauged theory $\{\text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}\}/Z$ (with Z being the “common center”) is guaranteed to exist given the gapped boundary determined by (10) since the gauging by Z is just a topological

manipulation. In practice, a Lagrangian algebra in $\{\text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}\}/Z$ is easily found by direct examination of the TQFT.

Representation theory—Having identified a collection of finite noninvertible symmetry in gapped QCD₂ theories, we now consider their implications for particle spectra. Recent work [6,7] has shown that noninvertible symmetry in a QFT can enforce mass degeneracies. An especially novel feature of such degeneracies is that they can be between stable particle and soliton excitations. In this section we briefly review the representation theory that governs such degeneracies, following and deferring detailed discussion to Ref. [7].

We first consider a general 2D bosonic QFT before returning to the specific context of QCD₂. In general, the finite symmetry is described by a fusion category $\mathcal{C}$ [9–11,33], and the phase of the symmetry is described by a $\mathcal{C}$ -module category $\mathcal{M}$ [10,34,35]. In a gapped theory quantized on $\mathbb{R}$, the simple objects of $\mathcal{M}$ correspond to the clustering grounding states of the theory [2,6].

In a gapped 2D QFT quantized on $\mathbb{R}$, the action of the finite noninvertible symmetry on the state space is through an algebra called the “strip algebra.” This algebra depends both on the symmetry ($\mathcal{C}$) and its phase ($\mathcal{M}$) and is denoted $\text{Str}_{\mathcal{C}}(\mathcal{M})$. It is the representation theory of the strip algebra that governs symmetry enforced degeneracies of the theory. $\text{Str}_{\mathcal{C}}(\mathcal{M})$ is a C^* -weak Hopf algebra, which means that its representations can act on multiparticle states (tensor products), have charge conjugates (duals), and are compatible with the unitary structure on the Hilbert space of states. The representation theory of such algebras is well studied and admits techniques for analysis similar to those used in the unitary representation theory of finite groups. In fact, in the special case of a symmetry described by a finite group H in its unbroken phase, $\text{Str}_{\mathcal{C}}(\mathcal{M}) = \mathbb{C}[H]$, the group algebra of H , and hence recovers the familiar representation theory of finite groups.

In analyzing the degeneracies of gapped QCD₂ theories, one can leverage that the representation category $\text{Rep}[\text{Str}_{\mathcal{C}}(\mathcal{M})]$ also admits a more abstract description,

$$\text{Rep}[\text{Str}_{\mathcal{C}}(\mathcal{M})] \simeq \mathcal{C}_{\mathcal{M}}^*. \quad (11)$$

Here $\mathcal{C}_{\mathcal{M}}^*$ is called the “dual category” of $\mathcal{C}$ with respect to $\mathcal{M}$ and is physically the dual symmetry obtained by performing a generalized gauging of $\mathcal{C}$ associated to $\mathcal{M}$ [10,36]. This category naturally acts on $\mathcal{M}$. Taking α to be a simple line, its corresponding irreducible representation is the boundary junction vector space,

$$V_{\alpha} = \bigoplus_{m,n} \text{Hom}(m \otimes \alpha, n), \quad (12)$$

where m, n are the clustering ground states of the theory. This presentation of the representation is especially useful

since it makes clear how many particles (solitons) there are above (between) each vacuum in it.

To compute the action of $\text{Str}_{\mathcal{C}}(\mathcal{M})$ on V_{α} in this approach, it is necessary to know the complete $(\mathcal{C}-\mathcal{C}_{\mathcal{M}}^*)$ -bimodule category structure on $\mathcal{M}$. This data is generically difficult to compute. Fortunately, Eq. (12) contains a significant amount of information about the representation and is determined by far less data. The dimensions of each summand is given by the module category fusion coefficients,

$$m \otimes \alpha = \bigoplus_n \tilde{N}_{m,\alpha}^n n, \quad \tilde{N}_{m,\alpha}^n \in \mathbb{N}. \quad (13)$$

The fusion coefficients can be conveniently encoded in a quiver (i.e., a directed graph) as follows. (1) For each clustering ground state $m \in \mathcal{M}$, draw a node of the quiver. (2) Add $\tilde{N}_{m,\alpha}^n$ directed arrows from node m to node n . This provides a convenient graphical presentation of the module fusion coefficients.

We can apply this discussion to analyze symmetry-enforced degeneracies of gapped QCD₂ theories. As discussed in the background section, the relevant fusion category symmetry is the category of lines of the coset boundary condition of the 3D coset TQFT, $\mathcal{C}$. The assumed symmetry breaking pattern (7), implies that the phase of the symmetry is given by $\mathcal{M} = \mathcal{C}$ viewed as a $\mathcal{C}$ -module category. Therefore, we must compute $\mathcal{C}_{\mathcal{C}}^*$, which is just the category itself [37],

$$\mathcal{C}_{\mathcal{C}}^* \simeq \mathcal{C}. \quad (14)$$

To compute the quivers describing the allowed symmetry enforced degeneracies one therefore only needs to know the fusion rules for $\mathcal{C}$,

$$\tilde{N} = N, \quad \alpha \otimes \beta = \bigoplus_{\gamma} N_{\alpha,\beta}^{\gamma} \gamma. \quad (15)$$

Before continuing to examples, a comment regarding one-form symmetry is in order. The mixing of particle and soliton states by noninvertible symmetry is particularly powerful because it can require the existence of particle states in a theory. This is unlike the case of grouplike symmetry where the realization of a particular representation is a dynamical rather than kinematic question. This follows from the fact that a theory has no nontrivial one-form symmetry if and only if the quiver describing the complete collection of stable solitons is connected. We provide a short argument for this in Supplemental Material [38], where we also discuss the precise implications of this on the possible realized representations. As will be demonstrated in the following examples, this additional information is often enough to require the existence of some stable particles or solitons from purely kinematic considerations.

Examples of particle-soliton degeneracy—We now consider examples of this analysis in gapped QCD₂ theories. In doing so we demonstrate the necessary existence of a collection of stable particle and/or soliton states in each theory. Details of the computations, including the data of the boundary condensation maps and allowed quivers for each of the following theories, are included in the End Matter and Supplemental Material [38].

$SO(3) + \psi_5$: Our first example is $SO(3)$ gauge theory with Majorana fermions in the five. The bosonized theory has the 3D coset TQFT,

$$\frac{Spin(5)_1 \times SU(2)_{-10}}{\mathbb{Z}_2} = \mathcal{Z}(\mathcal{C}). \quad (16)$$

Its coset boundary condition has three lines $\{1, v, A\}$ with the nontrivial fusions,

$$\begin{array}{c|cc} \times & v & A \\ \hline v & 1 & A \\ A & A & 1 + v + 2A \end{array}. \quad (17)$$

The QCD₂ theory therefore has three vacua and three possible irreducible multiplets. Because the theory has no one-form symmetry along the flow, the stable solitons and particles must realize a representation furnishing a connected quiver. All such quivers contain

$$1 \longleftrightarrow A \longleftrightarrow v, \quad (18)$$

as a subquiver. It follows that this theory must contain at least two stable particles over the $|\Omega_A\rangle$ vacuum and two soliton-antisoliton pairs. Furthermore, this quiver is realized by an irreducible representation of the strip algebra, and therefore the excitations all have equal mass.

$Spin(9) + \psi_\sigma$: Our next example is $Spin(9)$ gauge theory with Majorana fermions in the spinorial. The bosonized theory has the 3D coset TQFT,

$$Spin(16)_1 \times Spin(9)_{-2} = \mathcal{Z}(\mathcal{C}). \quad (19)$$

The coset boundary condition has 6 lines $\{1, s, v, c, A, B\}$ with nontrivial fusions,

$$\begin{array}{c|cccccc} \times & s & v & c & A & B \\ \hline s & 1 & c & v & A & B \\ v & c & 1 & s & B & A \\ c & v & s & 1 & B & A \\ A & A & B & B & 1 + s + A & v + c + B \\ B & B & A & A & v + c + B & 1 + s + A \end{array}. \quad (20)$$

The theory therefore has 6 vacua and 6 possible irreducible

representations. This theory also does not have one-form symmetry along the flow, so its stable excitations must realize representations assembling into a connected quiver. All such quivers contain

$$\begin{array}{ccccc} & v & & 1 & \\ & & & & \\ c & & A \longleftrightarrow B & & s \end{array}, \quad (21)$$

as a subquiver. Therefore this theory must contain a soliton-antisoliton pair of equal mass. Moreover, this pair necessarily exists as a member of one of three possible minimal connected subquivers (meaning that every connected quiver of stable solitons must contain one of these as a subquiver):

$$\begin{array}{ccccc} & 1 & & v & \\ & \swarrow & & \searrow & \\ c & & A \longleftrightarrow B & & s \end{array}, \quad (22)$$

$$\begin{array}{ccccc} & 1 & & v & \\ & \swarrow & & \searrow & \\ c & & A \longleftrightarrow B & & s \end{array}, \quad (23)$$

$$\begin{array}{ccccc} & 1 & & v & \\ & \swarrow & & \searrow & \\ c & & A \longleftrightarrow B & & s \end{array}, \quad (24)$$

where dashes are used to distinguish the sub-quivers of irreducible representations comprising a reducible representation. One such quiver must be realized, but symmetry considerations alone do not determine which. This instead is a question of dynamics. Note that in the later two cases, since the representations are reducible, not all solitons are required to have degenerate masses. Rather, this allows for two multiplets of stable solitons, each having a different mass.

$PSU(4) + \psi_{15}$: Our final example is $PSU(4)$ gauge theory with Majorana fermions in the fifteen. The bosonized theory has the 3D coset TQFT,

$$\frac{Spin(15)_1 \times SU(4)_{-4}}{\mathbb{Z}_4} = \mathcal{Z}(C). \quad (25)$$

The coset boundary condition has 4 lines $\{1, v, A, B\}$ with nontrivial fusions,

$\times$	v	A	B
v	1	B	A
A	B	$1 + A + B$	$v + A + B$
B	A	$v + A + B$	$1 + A + B$

(26)

The theory therefore has 4 vacua and 4 possible irreducible multiplets. As in the prior examples, this theory has no one-form symmetry along the flow, so its stable particles and solitons must furnish representations that combine to give a connected quiver. All such quivers contain

$$1 \quad \begin{array}{c} \curvearrowright \\ A \end{array} \longleftrightarrow \begin{array}{c} \curvearrowright \\ B \end{array} \quad v, \quad (27)$$

as a subquiver. Therefore the theory contains at least two stable particles, one above $|\Omega_A\rangle$ and the other above $|\Omega_B\rangle$, and a soliton-antisoliton pair between these vacua.

These excitations are necessarily members of one of two possible minimal connected subquivers:

$$\begin{array}{c} 1 \\ \updownarrow \\ A \end{array} \begin{array}{c} \curvearrowright \\ B \end{array} \begin{array}{c} 1 \\ \updownarrow \\ v \end{array}, \quad \begin{array}{c} 1 \\ \longleftrightarrow \\ A \end{array} \begin{array}{c} \curvearrowright \\ B \end{array} \begin{array}{c} 1 \\ \longleftrightarrow \\ v \end{array}. \quad (28)$$

Again, which subquiver is realized by the theory is not determined by symmetry but by dynamics. Both quivers correspond to irreducible representations, and therefore all stable particles and solitons labeled by them will have degenerate mass.

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- [38] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/jckx-z293> for a derivation of the relationship between solitons and one-form symmetry, explicit quiver computations, and the derivation of vacuum condensates.

End Matter

We briefly explain in this End Matter how to calculate the quivers for $SO(3) + \psi_5$ QCD₂. See Supplemental Material [38] for the remaining examples. Recall that we are interested in topological cosets where for simply connected gauge group the bulk 3D TQFT is given by

$$\text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}. \quad (\text{A1})$$

The line operators in this 3D TQFT will be labeled as (Λ, λ) , with Λ being an integrable representation of $\text{Spin}[\dim(\mathbf{R})]_1$ and λ being an integrable representation of $G_{I(\mathbf{R})}$, with the negative level implicit in the notation. When $\dim(\mathbf{R})$ is even, we denote the four integrable representation of $\text{Spin}[\dim(\mathbf{R})]_1$ as $\{\mathbf{1}, \mathbf{v}, \mathbf{s}, \mathbf{c}\}$, with $\mathbf{1}$ labeling the identity, $\mathbf{v}$ being the vectorial, and $\mathbf{s}$ and $\mathbf{c}$ being the spinorials. When $\dim(\mathbf{R})$ is odd, we denote the three integrable representations of $\text{Spin}[\dim(\mathbf{R})]_1$ as $\{\mathbf{1}, \mathbf{v}, \mathbf{s}\}$, with $\mathbf{v}$, $\mathbf{s}$ being the vectorial and spinorial representations, respectively. When the gauge group is not simply connected, we denote the lines of the bulk TQFT $\{\text{Spin}[\dim(\mathbf{R})]_1 \times G_{-I(\mathbf{R})}\}/Z$ with Z being the “common center” in terms of their representatives in the simply connected case before gauging Z .

$SO(3) + \psi_5$: The 3D coset TQFT is

$$\frac{\text{Spin}(5)_1 \times SU(2)_{-10}}{\mathbb{Z}_2} = \mathcal{Z}(\mathcal{C}). \quad (\text{16})$$

This theory has 10 lines, while its coset boundary condition has 3, $\{1, v, A\}$. The condensation maps from the 3D theory to the coset boundary are

a	$p(a)$	a	$p(a)$
$(\mathbf{1}, \mathbf{1})$	1	$(\mathbf{1}, \mathbf{7})$	$1 + A$
$(\mathbf{1}, \mathbf{11})$	v	$(\mathbf{s}, \mathbf{2})$	A
$(\mathbf{1}, \mathbf{3})$	A	$(\mathbf{s}, \mathbf{4})$	$1 + v + A$
$(\mathbf{1}, \mathbf{9})$	A	$(\mathbf{s}, \mathbf{6})_1$	A
$(\mathbf{1}, \mathbf{5})$	$v + A$	$(\mathbf{s}, \mathbf{6})_2$	A

(A2)

The boundary lines have nontrivial fusions,

$$\begin{array}{c|cc} \times & v & A \\ \hline v & 1 & A \\ A & A & 1 + v + 2A \end{array}. \quad (\text{17})$$

Because the coset boundary has 3 lines, there are therefore 3 irreducible representations of the strip algebra, with one labeled by each line.

Recall from the main text that the module category fusion coefficients are simply the fusion coefficients which are computed from (17),

$$\begin{aligned} N_{1,1}^1 &= N_{v,1}^v = N_{A,1}^A = 1, \\ N_{1,v}^v &= N_{v,v}^1 = N_{A,v}^A = 1, \\ N_{1,A}^A &= N_{v,A}^A = N_{A,A}^1 = N_{A,A}^v = 1, \quad N_{A,A}^A = 2. \end{aligned} \quad (\text{A3})$$

Therefore the quivers realized by 1 , v , and A are

$$1: \quad \begin{array}{ccc} \curvearrowright & \curvearrowright & \curvearrowright \\ 1 & A & v \end{array}, \quad (\text{A4})$$

$$v: \quad \begin{array}{ccc} & \curvearrowright & \\ 1 & A & v \end{array}, \quad (\text{A5})$$

$$A: \quad \begin{array}{ccc} & \curvearrowright & \\ 1 & A & v \end{array}. \quad (\text{A6})$$

The single minimal connected subquiver is clearly (A6). Therefore this must be realized as a subquiver in the QCD₂ theory, requiring the particle-soliton degeneracies discussed in the bulk text.