

Distinguishing Coherent and Incoherent Errors in Multiround Time-Reversed Dynamics via Scramblons

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Despite the rapid development of quantum science and technology, errors are inevitable and play a crucial role in quantum simulation and quantum computation. In quantum chaotic systems, coherent errors arising from imperfect Hamiltonian control and incoherent errors induced by coupling to the environment are both exponentially amplified during time evolution due to information scrambling. A fundamental question is how these two classes of errors imprint distinct signatures on the emergent irreversibility of many-body dynamics. In this Letter, we address this question by investigating multiround time-reversed dynamics in the presence of both coherent and incoherent errors. By applying scramblon theory, we obtain closed-form expressions for the Loschmidt echo over different rounds of time-reversed evolution. For incoherent errors, the error accumulates linearly with the number of rounds, whereas coherent errors exhibit a crossover from quadratic to linear accumulation. These predictions are explicitly verified using the solvable Sachdev-Ye-Kitaev model. Our results provide a theoretical foundation for characterizing and calibrating coherent and incoherent errors in reversed dynamics, with particular relevance to nuclear magnetic resonance systems.

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Introduction—The rapid development of quantum science and technology poses new challenges for the precise control of quantum many-body dynamics. In particular, the ability to reverse quantum many-body evolution is crucial both for experimentally probing quantum information dynamics and for implementing practical quantum algorithms [1–26]. An intrinsic obstacle to realizing such time-reversed dynamics with high precision is the inevitable presence of errors during evolution. There are two distinct sources of such errors. First, weak coupling of the system to its environment leads to decoherence, which is referred to as an incoherent error. Second, the Hamiltonian governing the backward evolution may exhibit small deviations from that of the forward evolution, resulting in coherent errors. Despite their different physical origins, both types of errors lead to similar phenomenology: their effects are exponentially amplified in time in chaotic systems, a universal feature of quantum butterfly effect [27–32]. This makes it difficult to directly distinguish coherent and incoherent errors in these time-reversal protocols.

Therefore, to explore the distinct signatures of coherent and incoherent errors in time-reversed dynamics, we consider a multiround time-reversed protocol, as illustrated in Fig. 1. Starting from a high-temperature initial state, the

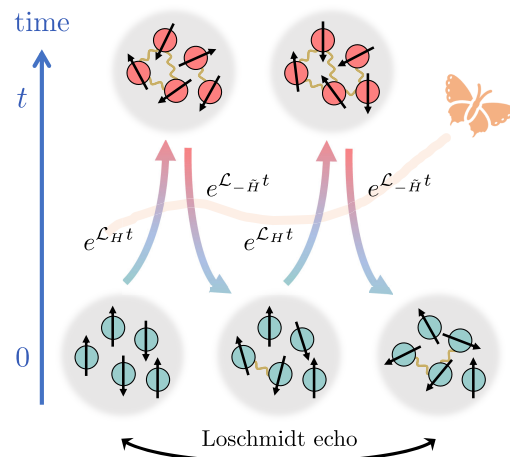


FIG. 1. Schematic of the multiround time-reversed dynamics, illustrated using the example of two rounds. Taking decoherence effects into account, the forward evolution is described by a Lindbladian superoperator $\mathcal{L}_H$ with Hamiltonian H . The Hamiltonian in the backward evolution, denoted by $\tilde{H}$, exhibits small deviations from H . These coherent and incoherent errors prevent perfect time reversal, with their effects being amplified by the quantum butterfly effect.

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system undergoes multiple forward and backward evolutions, with imperfections arising from either coherent or incoherent errors. Similar protocols have been adopted experimentally to measure high-order out-of-time-order correlators (OTOCs) [33], which provide fine-grained measures of information scrambling [34] and thermalization [35]. To develop a general theory of the resulting dynamics for systems with all-to-all connectivity, we employ scramblon theory, which was proposed as a universal description of information scrambling in quantum chaotic systems [36–49]. The key assumption is that, for weak perturbations, out-of-time-order correlations dominate the dynamics and are mediated by collective modes known as scramblons. Recent experiments have also validated scramblon theory in realistic solid-state nuclear magnetic resonance (NMR) systems using adamantane powder [48].

We obtain closed-form results for the multiround Loschmidt echo in time-reversed dynamics, which clearly elucidate the qualitative distinction between coherent and incoherent error scenarios. The effects of incoherent errors accumulate linearly with the number of rounds, reflecting the absence of inter-round correlations. In contrast, in the short-time regime, coherent errors exhibit a quadratic dependence on the number of rounds due to constructive inter-round interference, followed by a crossover to linear accumulation in the late-time regime. Our predictions are explicitly demonstrated using the solvable Sachdev-Ye-Kitaev (SYK) model [31,36,50–52] and are of direct experimental relevance to NMR systems [1–5,17,48,53–59].

Setup—We now describe the details of the multiround time-reversed dynamics for chaotic quantum many-body systems illustrated in Fig. 1. We initialize the system in a high-temperature state $\rho_0 = e^{-\beta O} / \text{tr}[e^{-\beta O}] \propto (1 - \beta O)$, where the inverse temperature $\beta \ll 1$. For nuclear magnetic resonance systems, the operator O corresponds to the total spin along the magnetic field [48]. The system evolves under a chaotic Hamiltonian H with all-to-all interactions, together with decoherence arising from coupling to the environment. The resulting dynamics is described by the Lindblad master equation $\partial_t \rho = \mathcal{L}_H[\rho]$, with the Lindbladian superoperator

$$\mathcal{L}_H[\rho] = -i[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right). \quad (1)$$

Here, L_k are usually referred to as jump operators. The density matrix at time t is therefore given by $\rho(t) = e^{\mathcal{L}_H t}[\rho_0]$. Next, we attempt to reverse the chaotic dynamics through control of the evolution Hamiltonian using techniques such as Floquet engineering [4,58,60,61]. In practice, this enables evolution under an effective Hamiltonian $-\tilde{H} = -H - \delta H$, where δH denotes a small imperfection, corresponding to the coherent error. After backward evolution for a time t , the resulting density

matrix becomes $\rho_1 = e^{\mathcal{L}_{-\tilde{H}} t} e^{\mathcal{L}_H t}[\rho_0]$, completing one round of time-reversed dynamics. In the multiround protocol, this procedure is repeated for n rounds, leading to $\rho_n = (e^{\mathcal{L}_{-\tilde{H}} t} e^{\mathcal{L}_H t})^n[\rho_0]$. Finally, the measurement of operator O is performed, which gives

$$\langle O \rangle \propto \text{tr}(O(e^{\mathcal{L}_{-\tilde{H}} t} e^{\mathcal{L}_H t})^n[O]) \equiv \text{tr}[O^2]F_n(t). \quad (2)$$

Here, we assume that $\text{tr}(O) = 0$ and $\mathcal{L}_H[\mathbb{1}] = 0$. $F_n(t)$ is known as the Loschmidt echo [62–65]. This quantity probes imperfections in the reversed dynamics: in the absence of errors, $L_k = 0$ and $\delta H = 0$, the forward and backward evolutions cancel exactly, yielding $F_n = 1$. Our aim is to understand how coherent and incoherent errors manifest themselves in the decay of $F_n(t)$. For later convenience, we fix the normalization $\text{tr}[O^2]/\mathcal{D} = 1$, where $\mathcal{D}$ is the Hilbert space dimension.

Perturbation theory—To understand the emergence of out-of-time-order (OTO) correlations in the Loschmidt echo, we first examine the single-round case. A graphical representation of the Loschmidt echo $F_1(t)$ is shown in Fig. 2(a), with details provided in the figure caption. The special case involving only coherent errors was investigated in Refs. [48,66], whereas the case with only incoherent errors is closely related to the problem of operator growth in open systems [41,45,67–77]. To the second order in jump operator L_k and δH , the Loschmidt echo receives the contribution from

$$F_1(t) \approx 1 - 2 \int_0^t dt' \sum_k \langle O L_k^\dagger(t') [L_k(t'), O] \rangle - \int_0^t dt' dt'' \frac{1}{2} \langle [O, \delta H(t')] [\delta H(t''), O] \rangle. \quad (3)$$

Here, the expectation value is taken over the infinite-temperature ensemble $\mathbb{1}/\mathcal{D}$, and operators are evolved under the errorless Hamiltonian, $M(t) = e^{iHt} M e^{-iHt}$. The factor of 2 arises because incoherent errors L_k can occur in both the forward and backward evolutions, whereas the coherent error δH appears only in the backward evolution. Equation (3) shows that both types of errors contribute through an OTOC [27–31], which grows exponentially in time as $e^{\kappa t}$ with the quantum Lyapunov exponent κ . This leads to $F_1(t) \approx 1 - \#e^{\kappa t}$.

We proceed to the multiround Loschmidt echo. A graphical representation, using the two-round case as an example, is shown in Fig. 2(b). For incoherent errors, the insertions of L_k and $L_k^\dagger$ must occur within the same evolution superoperator, either $e^{\mathcal{L}_H t}$ or $e^{\mathcal{L}_{-\tilde{H}} t}$. In Fig. 2(b), this corresponds to pairs of branches with the same color, namely, (1,8), (2,7), (3,6), and (4,5). Consequently, within perturbation theory, the contribution from incoherent errors is amplified by a factor of n . In contrast, two insertions of the coherent error δH can occur independently on the

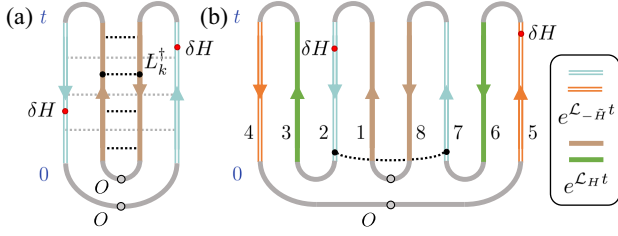


FIG. 2. Graphical representation of the Loschmidt echo $F_n(t)$ for (a) $n = 1$ and (b) $n = 2$. Branches with the same color originate from the same Lindbladian evolution. Solid lines correspond to evolution governed by $\mathcal{L}_H$, while dotted lines indicate the Lindblad term. Double lines denote evolution in the presence of coherent errors, where insertions of δH , marked by red dots, may occur. In panel (b), all branches are labeled from 1 to 8.

backward-evolution branches 2, 4, 5, and 7. In particular, an OTOC between δH and O arises when one δH insertion is on branch 2 or 4, and the other is on branch 5 or 7. Generalizing this discussion to arbitrary n immediately yields an n^2 scaling for the accumulation of coherent errors. Putting all ingredients together, the result reads

$$F_n(t) \approx 1 - 2n \int_0^t dt' \sum_k \langle OL_k^\dagger(t') [L_k(t'), O] \rangle - n^2 \int_0^t dt' dt'' \frac{1}{2} \langle [O, \delta H(t')] [\delta H(t''), O] \rangle. \quad (4)$$

In particular, the quadratic scaling of the coherent error arises because inter-round contributions are identical to intraround ones, reflecting the full coherence of the evolution in perturbation theory.

Does this distinction persist in the longtime limit? As the evolution time t increases, higher-order corrections must be taken into account. These corrections diminish correlations between inter-round branches and therefore suppress the corresponding $n(n-1)$ contributions that involve inter-round operator pairs. To illustrate this, we analyze the correlation between branches (2,5) by considering the two-point function of δH near time t , as indicated by the red dots in Fig. 2(b). This two-point function acquires corrections from both coherent and incoherent errors, which can form an OTOC. An example is an incoherent error occurring at (2,7) near $t \approx 0$, shown as black dots in Fig. 2(b). Consequently, the inter-round correlations are expected to decay with increasing time. In contrast, all contributions from OTOCs cancel in the perturbative expansion of intraround correlators due to unitarity. As a result, at sufficiently long times, we expect that the accumulation of coherent errors is dominated by intraround contributions and therefore scales linearly with the number of rounds, similar to the incoherent case. This physical intuition will be justified by explicit calculations in the next section.

Scramblon theory—To obtain a Loschmidt echo that remains well behaved and saturates to zero at long times, higher-order contributions must be resummed. Scramblon theory provides an efficient framework for this resummation [36–49]. The central assumption is that, for longtime observables with $\chi t \gg 1$, OTO correlations dominate the dynamics and are mediated by collective modes known as scramblons. The scramblon propagator takes the form $-\lambda_t \equiv -e^{\chi t}/C$, which serves as a signature of quantum many-body chaos. Here, $C \propto N$ and N denotes the number of qubits or fermionic modes. Within scramblon theory, a pair of operators V and $V^\dagger$ in the past can emit scramblons, which are subsequently absorbed by another pair of operators W and $W^\dagger$ in the future, provided that these four operators form an OTOC. The scattering vertex involving $V(t_1)$, $V^\dagger(t_2)$, and m scramblons is denoted by $\Upsilon_V^m(t_{12})$, where we assume time-reversal symmetry [78] and define $t_{12} = t_1 - t_2$.

We first investigate scenarios with either incoherent or coherent errors separately. We begin with the incoherent error case, where each insertion of a jump operator can form an OTOC with the operators O , as discussed in the previous section. Within scramblon theory, this amounts to summing diagrams with an arbitrary number of jump-operator insertions ($L_k, L_k^\dagger$). Each pair of jump operators may originate from an arbitrary Lindbladian superoperator (indicated by different colors in Fig. 2) and can be inserted at arbitrary times. Since the scramblon propagator carries a factor of $1/N$, in the thermodynamic limit $N \rightarrow \infty$, each pair of jump operators emits only a single scramblon. All scramblons are subsequently absorbed by the pair of O operators at $t = 0$. An example for $n = 2$ is shown in Fig. 3(a). Summing over all possible scramblon diagrams from incoherent errors, we find

$$F_n(t)_I = \sum_{m=0}^{\infty} \frac{\Upsilon_O^m}{m!} \prod_{j=1}^m \left[\int_0^t dt_j \left(-2n\lambda_{t_j} \sum_k \Upsilon_{L_k}^1 \right) \right]. \quad (5)$$

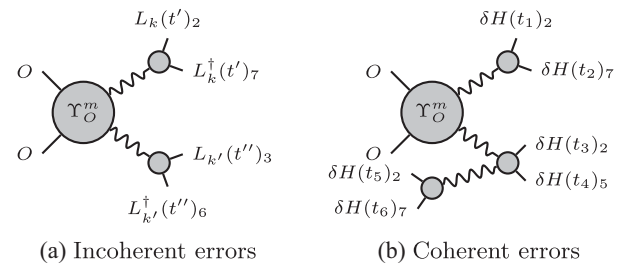


FIG. 3. Typical scramblon diagrams contributing to the Loschmidt echo with $n = 2$ for (a) incoherent errors and (b) coherent errors. Wavy lines represent scramblon propagators, while solid lines denote microscopic operators. Subscripts on the error operators indicate their branch indices (see Fig. 2). In panel (a), all scramblons emitted by jump operators are absorbed by the operator O . In panel (b), scramblons emitted by inter-round errors with $t_1, t_2, t_3, t_4 \approx t$ can also be absorbed by errors with $t_5, t_6 \approx 0$.

For conciseness, we define $\Upsilon_V^m \equiv \Upsilon_V^m(0)$. The subscript indicates only incoherent errors present. The result involves integrations over the insertion times t_j of each pair of jump operators, with a total of m insertions. The factor of $2n$ arises from the n copies of forward and backward evolutions. Introducing the auxiliary function $f_V(x) \equiv \sum_m [(-x)^m/m!] \Upsilon_V^m$, we find

$$F_n(t)_I = f_O(n\gamma_I e^{\chi t}), \quad \text{with } \gamma_I = \frac{2}{C\chi} \sum_k \Upsilon_{L_k}^1. \quad (6)$$

Here, we assume that $\sum_k \Upsilon_{L_k}^1 \propto N$ is extensive in the system size, and consequently $\gamma_I \sim O(1)$. This assumption is valid when the system is coupled homogeneously to the environment. Equation (6) clearly shows the linear accumulation of incoherent errors for arbitrary time t . For all solvable models in which analytical expressions for $f(x)$ are available, including the large- q SYK model, the Brownian SYK model, and Brownian circuits, we have $f_O(x) = 1/(1+x)^{2\Delta_O}$, with an effective scaling dimension Δ_O [39,43]. A similar ansatz has also been demonstrated in solid-state NMR systems [48]. Using this ansatz, the result for incoherent errors is

$$F_n(t)_I = \frac{1}{(1+n\gamma_I e^{\chi t})^{2\Delta_O}}. \quad (7)$$

Next, we study the dynamics with coherent errors only. We first focus on the $n = 2$ case as an illustration. Similar to the incoherent case, the decay of the Loschmidt echo is driven by the exchange of scramblons between a pair of coherent errors δH and the operators O . As discussed in the previous section, there are four distinct ways of pairing errors on different branches, namely, (2,5), (2,7), (4,5), and (4,7), that contribute to this process. However, additional diagrams appear in the coherent error case. As noted earlier, a pair of δH operators inserted on branches (2,5) near time t can also form an OTOC with another pair of δH operators inserted on branches (2,7) near $t \approx 0$. Consequently, scramblons emitted by δH near time t can be absorbed either by the operators O or by the δH operators near $t = 0$. A concrete example is shown in Fig. 3(b).

As detailed in Supplemental Material [79], summing the contributions from operators near $t = 0$ effectively renormalizes the scattering vertices for the inter-round terms near time t . This leads to

$$F_2(t)_c = \sum_{m=0}^{\infty} \frac{\Upsilon_O^m}{m!} \prod_{j=1}^m \left[\int_0^t dt_j \left(-2\lambda_{t_j} (\tilde{\Upsilon}_{\delta H}^1 + \tilde{\Upsilon}_{\delta H, t_j}^1) \right) \right]. \quad (8)$$

Here, we have made the Markovian approximation $\Upsilon_{\delta H}^1(t) \approx \delta(t) \tilde{\Upsilon}_{\delta H}^1$, which is justified by the fact that the Loschmidt echo decays over a parametrically long

timescale in the limit of weak errors. The term $\tilde{\Upsilon}_{\delta H}^1$ represents the intraround perturbations at (2,7) and (4,5), contributing in a manner similar to the incoherent case (5). $\tilde{\Upsilon}_{\delta H, t}^1$ denotes the renormalized scattering vertex, describing additional inter-round contributions that are absent in the incoherent case. The detailed expression for $\tilde{\Upsilon}_{\delta H, t}^1$ is presented in Supplemental Material [79], which leads to

$$F_2(t)_c = f_O \left(2\gamma_c e^{\chi t} + \frac{2}{\tilde{\Upsilon}_{\delta H}^1} [\tilde{f}_{\delta H}(0) - \tilde{f}_{\delta H}(\gamma_c e^{\chi t})] \right). \quad (9)$$

Here, we have introduced the strength of the coherent error as $\gamma_c = (1/\chi C) \tilde{\Upsilon}_{\delta H}^1$ [80]. Equation (9) clearly demonstrates the quadratic-to-linear crossover. In the short-time limit, we can expand $\gamma_c e^{\chi t} \ll 1$, giving $F_2(t) \approx f_O(4\gamma_c e^{\chi t})$. In contrast, in the late-time regime, the first term in the argument dominates since $\tilde{f}_{\delta H}(x)$ is bounded for arbitrary $x > 0$ [39]. As a consequence, we have $F_2(t) \approx f_O(2\gamma_c e^{\chi t})$, which exhibits linear scaling. Closed-form expressions can be obtained by further assuming $\tilde{f}_{\delta H}(x) = C_0/(1+bx)^{2\Delta_\delta}$, which yields the results for coherent errors

$$F_2(t)_c = \frac{1}{\left(1 + 2\gamma_c e^{\chi t} + \frac{1}{b\Delta_\delta} \left(1 - \frac{1}{(1+b\gamma_c e^{\chi t})^{2\Delta_\delta}} \right) \right)^{2\Delta_O}}. \quad (10)$$

Finally, we combine Eqs. (7) and (10) to obtain the expression for $F_2(t)$ in the presence of both coherent and incoherent errors. The intraround contributions simply add, yielding a total error strength $\gamma = \gamma_I + \gamma_c$. These intraround error pairs also contribute to the renormalization of inter-round coherent error pairs. Generalizing these calculations to arbitrary n [79], we obtain

$$F_n(t) = \left(1 + n\gamma e^{\chi t} + \frac{1}{b\Delta_\delta} \sum_{a=1}^{n-1} \frac{(n-a)\gamma_c}{a^2\gamma_c + a\gamma_I} \left[1 - \left(1 + [a^2\gamma_c + a\gamma_I] b e^{\chi t} \right)^{-2\Delta_\delta} \right] \right)^{-2\Delta_O}. \quad (11)$$

In the regime where both $n^2\gamma_c e^{\chi t} \ll 1$ and $n\gamma_I e^{\chi t} \ll 1$, Eq. (11) can be expanded as $F_n(t) \approx (1 + (n^2\gamma_c + n\gamma_I) e^{\chi t})^{-2\Delta_O}$. This agrees with the perturbative result in Eq. (4), where coherent and incoherent errors exhibit quadratic and linear scaling with n , respectively, at short times. In the longtime regime, this yields $F_n(t) \approx (1 + n\gamma e^{\chi t})^{-2\Delta_O}$, demonstrating the linear scaling for both errors. This result provides a concrete framework for the characterization and calibration of coherent and incoherent errors in realistic quantum platforms. In particular, single-round time-reversed dynamics have been extensively explored in NMR experiments [48,81–84]. Extending these

protocols to multiround dynamics is straightforward in state-of-the-art NMR systems.

Example: SYK model—To further support our theoretical analysis, we provide an explicit verification using the solvable SYK model [31,36,50–52]. The SYK model describes N randomly interacting Majorana fermions χ_a , with $a \in \{1, 2, \dots, N\}$. We take the canonical commutation relation $\{\chi_a, \chi_b\} = \delta_{ab}$. The Hamiltonian reads

$$H = \sum_{a < b < c < d} J_{abcd} \chi_a \chi_b \chi_c \chi_d, \quad (12)$$

with independent Gaussian variables $\overline{J_{abcd}} = 0$ and $\overline{J_{abcd}^2} = 3!J^2/N^3$. We consider coherent errors that originate from time-dependent fluctuations of the couplings, taking the form of the Brownian SYK model [85,86]

$$\delta H(t) = \sum_{a < b < c < d} V_{abcd}(t) \chi_a \chi_b \chi_c \chi_d, \quad (13)$$

where $V_{abcd}(t)$ are Brownian variables with zero expectation and $\overline{V_{abcd}(t)V_{abcd}(t')} = 3!V\delta(t-t')/N^3$. To ensure the same structure for coherent and incoherent errors, we choose the jump operator $L_{abcd} = \sqrt{3V/N^3} \chi_a \chi_b \chi_c \chi_d$, identifying $k = abcd$ with $a < b < c < d$. This choice guarantees $\gamma_c = \gamma_I$ by construction.

The model with both errors can be analyzed using the large- N expansion. Leaving the details to Supplemental Material [79], we present the numerical results for $F_n(t)$ in Fig. 4(a) for the operator $O = i\chi_1\chi_2$. We first focus on $F_n(t)_I$ with incoherent errors for $n \in \{1, 2, 4\}$. The results exhibit the same line shape, with relative time shifts that are approximately equally spaced, consistent with the theoretical prediction (7). We further fit the numerical results to (7), treating γ_I , κ , and Δ_O as fitting parameters. The resulting fits, plotted as solid lines in Fig. 4(a), match the numerical data with high accuracy. Next, we consider the two-round Loschmidt echo $F_2(t)_c$ for systems with coherent errors. The numerical results show that the short-time behavior matches $F_4(t)_I$, while the late-time behavior matches $F_2(t)_I$, consistent with our theoretical prediction. Finally, we plot Eq. (10) with $n = 2$ in the dashed line using the fitted values $\tilde{\gamma} = \gamma_c = \gamma_I$, κ , and Δ_O , together with exact relations $\Delta_\delta = 2\Delta_O$ and $b = 1$ (see Supplemental Material [79]). The resulting curves match the numerical data with good accuracy. To further quantify the scaling with respect to n , we extract the effective scaling coefficient $g(t)$, which characterizes the accumulation of error for given time t . It is defined by the formula

$$F_n(t) = \frac{1}{(1 + g(t)\tilde{\gamma}e^{\kappa t})^{2\Delta_O}}, \quad (14)$$

using the numerical data alongside the parameters determined from the preceding fits. We present the results in

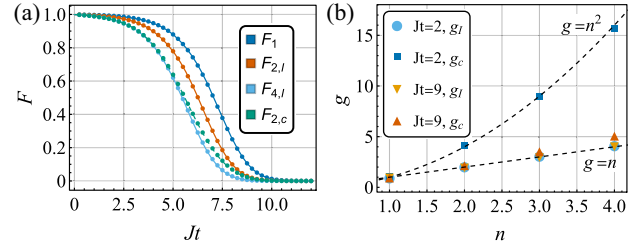


FIG. 4. Numerical demonstration of our predictions using the solvable SYK model with either coherent or incoherent errors at $V/J = 0.01$. (a) n round Loschmidt echo. The data points represent results from numerical simulations, while the solid lines are fits based on Eq. (7), and the dashed lines are plotted using Eq. (10) with fitted parameters $\tilde{\gamma} = \gamma_c = \gamma_I = 6.02 \times 10^{-4}$, $\kappa/J = 0.859$, $\Delta_O = 1.50$. (b) Effective scaling coefficient $g(t)$ extracted from the numerical data at each time t using Eq. (14). The two black dashed lines in the figure correspond to $g = n^2$ and $g = n$, respectively.

Fig. 4(b). For incoherent errors, we find $g_I = n$ at both early and late times. In contrast, for coherent errors, the extracted coefficient evolves from $g_c = n^2$ at early times to $g_c = n$ at late times, providing direct evidence for the predicted quadratic-to-linear crossover.

Discussions—In this Letter, we investigate the signatures of coherent and incoherent errors in multiround time-reversed dynamics. We derive concrete results for the Loschmidt echo using scramblon theory [36–49], which clearly demonstrate distinct scalings for error accumulation. Incoherent errors accumulate linearly with the number of rounds, whereas coherent errors exhibit a crossover from quadratic scaling at short times to linear scaling at late times. We explicitly test these theoretical predictions using a concrete solvable SYK model. Our results can be readily explored in NMR experiments.

We conclude with several remarks. First, while the scramblon formalism relies on all-to-all interactions, our early-time perturbative analysis in Eq. (4) does not, and therefore remains valid for arbitrary interacting systems. Second, it may be possible to derive self-consistent relations between Loschmidt echoes with different n , providing a way to test scramblon theory without assuming the form of $f_V(x)$ [43]. Finally, while we focus on the Loschmidt echo, it would be interesting to generalize our analysis to other observables, such as the fidelity of many-body teleportation [87–98], in the multiround setting and investigate their signatures of errors. We leave these directions to future work.

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Data availability—The codes and data that support the findings of this article are openly available [99]; embargo periods may apply.

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