

Optical Excitation and Stabilization of Ultracold Field-Linked Tetratonic Molecules

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We propose a coherent optical population transfer of weakly bound field-linked (FL) tetratonic molecules (tetramers) to deeper FL bound states using stimulated Raman adiabatic passage. We consider static-electric-field shielded polar alkali-metal diatomic molecules and corresponding FL tetramers in their $X^1\Sigma^+ + X^1\Sigma^+$ ground electronic state. We show that the excited metastable $X^1\Sigma^+ + b^3\Pi$ electronic manifold supports FL tetramers in a broader range of electric fields with collisional shielding extended to zero field. We calculate the Franck-Condon factors between the ground and excited FL tetramers and show that they are highly tunable with the electric field. We also predict photoassociation of ground-state shielded molecules to the excited FL states in free-bound optical transitions. We propose proof-of-principle experiments to implement stimulated Raman adiabatic passage and photoassociation using FL tetramers, paving the way for the formation of deeply bound ultracold polyatomic molecules.

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Introduction—Shielding of polar molecules against collisional loss has been instrumental in the realization of stable ultracold molecular gases. Using external static-electric [1–4], near-resonant microwave [5–8] or laser [9] fields, long-range repulsion between a pair of polar molecules can be engineered by controlling their dipole-dipole interaction. Static-field and microwave shielding have been successfully implemented [10–14], where the latter paved the way for achieving Fermi degeneracy [15] and Bose-Einstein condensation of dipolar molecules [16].

The external field responsible for generating the shielding repulsion at long range also creates an attractive dipolar interaction at longer range. The resulting potential well can be deep enough to support one or more quasibound states, also known as “field-linked” (FL) states. Existence of FL states was predicted 20 years ago [17], and they have been studied for static-electric- and microwave-field scenarios [18–20]. Only recently, they have been observed in the collisions of microwave-shielded Na^{40}K molecules [21] as scattering resonances. Using such resonances, weakly bound ultracold tetratonic molecules (hereafter “tetramers”) have been associated in the long range from pairs of diatomic Na^{40}K molecules [22].

At the same time, there is a growing interest in creating ultracold polyatomic molecules. They provide unique

opportunities in studies of cold chemistry [23,24], precision measurements [25,26], exotic quantum phases [27], and quantum information processing [28]. A few triatomic species have been successfully laser cooled [29–33]; however, they need favorable rovibronic structure to establish closed optical cycles. This limits the number of polyatomic molecules that can be cooled directly to ultracold temperatures. FL tetramers, on the other hand, may provide alternate pathways for obtaining ground-state stable ultracold polyatomic molecules [22].

An extremely successful method for creating ultracold diatomic molecules (hereafter “dimers”) from pairs of ultracold atoms is magnetoassociation followed by stimulated Raman adiabatic passage (STIRAP) [34,35]. STIRAP involves two steps where the population from an initial weakly bound state $|i\rangle$ is optically transferred to a target deeply bound state $|f\rangle$ via an excited intermediate state $|e\rangle$. The $|i\rangle \rightarrow |e\rangle$ is the excitation transition, whereas the $|e\rangle \rightarrow |f\rangle$ is the stabilization step. Here, we ask whether it is possible to realize a STIRAP transfer of weakly bound FL tetramer states ($|i\rangle$) at the long range to deeper bound states ($|f\rangle$) at the short range, and thus extend the tool from dimers to tetramers. There are however twofold challenges. First, to locate an isolated $|e\rangle$ state from a very high density of states for the tetramers. Second, to determine the full sequence of STIRAP transfer of the FL tetramers to their absolute ground state, which requires a full knowledge of their interaction potential and transition dipole moment surfaces.

In this Letter, we focus on the first challenge and show that FL states exist in a vibronically excited manifold that can serve as $|e\rangle$. Such states have the advantage that they are well isolated and can be controlled via the external field. To this end, we consider FL tetramers formed from shielded

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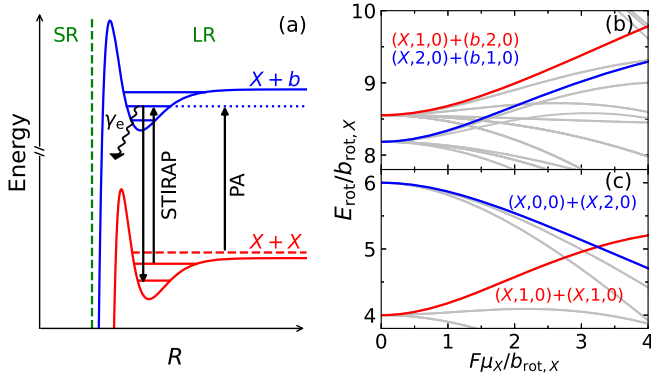


FIG. 1. (a) Schematic showing the envisaged STIRAP using FL tetramers (solid horizontal lines). Molecules with more than one FL state in $X+X$ may be used for demonstrating STIRAP via FL states of $X+b$. Additionally, the PA of shielded dimers colliding in $X+X$ (red dashed line) to FL states in $X+b$ may be envisaged. The natural linewidth γ_e of the b state determines the lifetime of the FL states in $X+b$. A representative vertical green dashed line separates the short (SR) and long (LR) ranges. (b),(c) Energies E_{rot} of the rotor pair levels of LiCs for thresholds (b) $X+b$ and (c) $X+X$ as a function of F . Panel (c) is universal for any polar diatomic molecule in state X . However, energies in panel (b) depend on the differences in μ and b_{rot} for states X and b , so they are molecule-dependent.

polar alkali-metal dimers (and similar molecules) colliding in their ground vibronic state ($X^1\Sigma^+, v=0$) (hereafter “ X ”). We use an external static-electric field F for shielding and show that FL tetramers coexist in the ground $X+X$ and the excited electronic manifold $X+b$ [Fig. 1(a)], where b denotes the long-lived excited vibronic state ($b^3\Pi_{0^+}, v'=0$). We choose such excited FL tetramer states as $|e\rangle$, and study Franck-Condon factors (FCFs) between the FL states of $X+X$ and $X+b$ as functions of $F = |F|$, with NaCs and LiCs as representative examples.

Modeling the stabilization step to the absolute ground state of tetramers remains a challenge. However, in the following, we show that FL states of $X+b$ are potential candidates for achieving this goal. For this, we propose proof-of-principle experiments for implementing STIRAP to transfer population from a shallower FL state in $X+X$ to a deeper one within the same long-range potential well via an excited FL state of $X+b$ [Fig. 1(a)]. We demonstrate this with LiCs, which supports two FL states in $X+X$ in the presence of F . Additionally, we study photoassociation (PA) prospects for converting shielded dimers of $X+X$ to FL tetramers in $X+b$. We consider RbCs and LiCs to demonstrate PA to FL states in the excited $X+b$ manifold. We choose bosonic molecules, but our treatment is equally applicable to the fermionic ones.

The metastable b state—The electronic spin-forbidden transitions between $X^1\Sigma_{0^+}^+$ and $b^3\Pi_{0^+}$ of an alkali dimer are weakly allowed only through the spin-orbit mixing of the latter with $A^1\Sigma_{0^+}^+$. The subscript 0^+ is the value of the

projection ω of the total electronic angular momentum along the dimer axis. Owing to a large FCF between the vibronic states b and X , the optical transition $X \leftrightarrow b$ is observable, but has a very narrow linewidth. The radiative decay of b to $a^3\Sigma^+$ manifold is also very slow, making b a metastable state. The natural linewidth γ_e of b for a few alkali dimers has been measured [36–39], and except for NaRb, γ_e is on the order of tens of kHz [see Supplemental Material (SM) at [40]]. As shown below, the fact that γ_e is much smaller than the binding energies of the FL states in $X+b$ enables their optical addressing and manipulating. Moreover, for an imperfect STIRAP transfer, it is an advantage to have a metastable intermediate state to mitigate any population loss.

Molecules in X and b states—In the presence of a static-electric field F , the effective Hamiltonian for a spin-free diatomic molecule A in vibronic state X is

$$\hat{h}_A = \hat{h}_{\text{rot},A} + \hat{h}_{\text{Stark},A}. \quad (1)$$

Here, $\hat{h}_{\text{rot},A} = b_{\text{rot},A} \hat{j}^2$ is the rotational Hamiltonian, where $b_{\text{rot},A}$ is the rotational constant and $\hat{j}$ is the molecular rotation. $\hat{h}_{\text{Stark},A} = -\boldsymbol{\mu}_A \cdot \mathbf{F}$, where $\boldsymbol{\mu}_A$ is the dipole moment along the dimer axis. Alkali dimers have nuclear spins which give rise to an additional hyperfine Hamiltonian $\hat{h}_{\text{hf}}$ [77]. However, the effects of $\hat{h}_{\text{hf}}$ on their scattering properties under effective shielding are very small [78,79]; hence we ignore them.

Molecules in $b^3\Pi$ have additional terms in their effective Hamiltonian in Eq. (1). The dominant one is the spin-orbit coupling Hamiltonian $\hat{h}_{\text{so}}$, which splits a $^3\Pi$ term into the fine-structure components $^3\Pi_\omega$, with $|\omega| = 0^\pm, 1, 2$. The $\pm$ of $\omega = 0$ represents the symmetry under reflection through a plane containing the molecular axis. Around the minimum of the potential, the $|\omega| > 0$ levels are typically separated from $\omega = 0^\pm$ by tens of $\text{cm}^{-1} \times hc$ [40], where h and c are, respectively, the Planck constant and the speed of light. The degeneracy of $\omega = 0^\pm$ is lifted due to the interactions with other electronic states of the same symmetry. Their energy gap is typically tens of $\text{GHz} \times h$ for most molecules [40]. The remaining terms of $b^3\Pi$ Hamiltonian are namely, the orbital and spin contributions in the rotational Hamiltonian denoted by $\hat{h}_{\text{rso}}$, the spin-spin $\hat{h}_{\text{ss}}$, the spin-rotation $\hat{h}_{\text{sr}}$, the Λ -doubling $\hat{h}_{\text{LD}}$, and hyperfine $\hat{h}_{\text{hf}}$ terms. We denote these additional terms collectively as $\hat{h}_{\text{add}} = \hat{h}_{\text{rso}} + \hat{h}_{\text{ss}} + \hat{h}_{\text{sr}} + \hat{h}_{\text{LD}} + \hat{h}_{\text{hf}}$. In the Hund’s case (a) basis, $\hat{h}_{\text{add}}$ has both diagonal and off-diagonal matrix elements in ω [77].

Our methodology involves only the lowest few rotational levels of the vibronic state b . Since $|\omega| > 0$ states are energetically far away compared to the rotational spacings, $\hat{h}_{\text{add}}$ contributes small first- and second-order perturbative corrections to the rotational energy levels in b . To a first

approximation, we ignore these additional contributions as they do not affect our methodology qualitatively. Moreover, we ignore the terms in $\hat{h}_{\text{Stark}}$ that are off-diagonal in $\omega = 0^+$ and 0^- . This is again a justified approximation for most molecules where the energy gap between $\omega = 0^+$ and 0^- components is large compared to b_{rot} but it is not appropriate for KRb, NaK, and LiRb. Therefore, we consider the same effective Hamiltonian $\hat{h}$, Eq. (1), for both X and b molecules, which simplifies our treatment enormously. We will use the subscripts X and b for b_{rot} and μ to denote their values obtained after averaging over the respective vibronic states. We calculate these parameters using *ab initio* methods [41] as implemented in MOLPRO [42] for alkali dimers and similar molecules of current experimental interest [40]. We find that the ratio b_{rot}/μ is comparable for X and b molecules, allowing them to be polarizable to a similar extent in the presence of F . We label the dimer levels by $(\beta, \tilde{j}, m)$, where β represents the vibronic state (X or b), $\tilde{j}$ correlates with the free-rotor quantum number j at zero field, and m is its conserved projection along F .

Shielding and FL states in $X + X$ —The interaction between a pair of polar molecules A and B involved in shielding, occurring at long range, is given by their dipole-dipole interaction $\hat{H}_{\text{dd}} = [\mu_A \cdot \mu_B - 3(\mu_A \cdot \hat{R})(\mu_B \cdot \hat{R})]/(4\pi\epsilon_0 R^3)$, where R is the intermolecular distance and $\hat{R}$ is a unit vector along the intermolecular axis. Shielding may occur when two pair levels that are connected by $\hat{H}_{\text{dd}}$ are close enough in energy such that they are strongly mixed. For static-field shielding, molecules colliding in the state $(\beta, \tilde{j}, m) = (X, 1, 0)$ experience repulsion due to mixing with the lower lying pair level $(X, 0, 0) + (X, 2, 0)$ at fields above $F = 3.244b_{\text{rot},X}/\mu_X$ [Fig. 1(c)]. The repulsion, however, becomes weak for $F > 3.8b_{\text{rot},X}/\mu_X$.

Shielding may be understood by adiabats that are the R -dependent eigenvalues of $\hat{h}_A + \hat{h}_B + \hbar^2 \hat{L}^2/(2\mu_{\text{red}} R^2) + \hat{H}_{\text{dd}}$. Here, $\hat{L}$ is the angular momentum operator for relative rotation of the two molecules and μ_{red} is the corresponding reduced mass. Figure 2(b) shows the adiabats for LiCs for different partial waves L correlating with the initial pair level $(X, 1, 0) + (X, 1, 0)$ at $F = 3.57b_{\text{rot},X}/\mu_X$ (7.5 kV/cm). It demonstrates that the adiabats are repulsive for $R < 400$ bohr. In addition, $\hat{H}_{\text{dd}}$ couples the incoming $L = 0$ with $L = 2$ channel to produce an attraction at larger distances that is asymptotically proportional to $-d^4/R^4$, where d is the induced dipole moment along F . The resulting potential well is shown in the inset of Fig. 2(b). This potential well for molecules with larger values of μ_X can be deep enough to support FL states. For example, NaRb and NaCs support one FL state each, LiCs supports two, and KAg and CsAg [43] support more than two FL states [20]. Here, we perform scattering calculations with MOLSCAT [44] to determine the rate coefficients for elastic scattering k_{el} and two-body loss $k_{2,\text{loss}}$ for various molecules

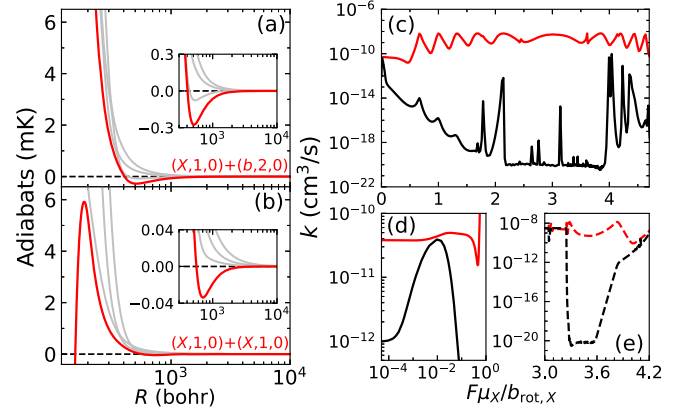


FIG. 2. (a),(b) Adiabats for LiCs correlating to partial waves $L = 0, 2, 4, 6$ for the initial threshold (a) $(X, 1, 0) + (b, 2, 0)$, and (b) $(X, 1, 0) + (X, 1, 0)$ at $F = 3.57b_{\text{rot},X}/\mu_X$. The adiabats are shown relative to their initial thresholds. The incoming s-wave ($L = 0$) channel is shown in red. Insets show expanded views of the long-range potential wells that support FL tetramer states. (c)–(e) Rate coefficients k for elastic scattering (red) and total two-body loss (black) for collisions in $X + b$ [solid curves in (c) and (d)] and $X + X$ [dashed curves in (e)].

using their *ab initio* determined μ and b_{rot} . We solve coupled-channel equations using a fully absorbing boundary condition at the short range [45,46]. We follow the methodology of Ref. [4], but with a slightly different basis set [40]. Figure 2(e) shows k_{el} and $k_{2,\text{loss}}$ for LiCs at a collision energy $E_{\text{coll}} = 10 \text{ nK} \times k_B$, where k_B is the Boltzmann constant. The peaks in k_{el} indicate the fields where FL states cross the threshold.

Shielding and FL states in $X + b$ manifold—Shielding in $X + b$ originates from a different physics. Owing to the difference in b_{rot} and μ for X and b , any two pair levels $(X, \tilde{j}, m) + (b, \tilde{j} + 1, m)$ and $(X, \tilde{j} + 1, m) + (b, \tilde{j}, m)$ have distinct energies. Two such pair levels for LiCs (with $b_{\text{rot},b} > b_{\text{rot},X}$) are shown as colored lines in Fig. 1(b). This means molecules in the upper level, here $(X, 1, 0) + (b, 2, 0)$, experience repulsion due to mixing via $\hat{H}_{\text{dd}}$ with the lower lying $(X, 2, 0) + (b, 1, 0)$. This feature exists in all fields, including $F = 0$, which is an important and surprising finding in this Letter. Note that molecules with $b_{\text{rot},X} > b_{\text{rot},b}$ will be shielded in $(X, 2, 0) + (b, 1, 0)$ due to coupling to the lower lying $(X, 1, 0) + (b, 2, 0)$ level.

There are two contributions to $\hat{H}_{\text{dd}}$ for molecules interacting in two different vibronic states. The first is between the permanent dipoles in X and b , and the other is due to a resonant dipolar interaction between a pair of superposition states of X and b . The former is responsible for shielding repulsion in the upper level. The latter is proportional to μ_{Xb}^2 , where μ_{Xb} is the $X \leftrightarrow b$ transition dipole moment, and thus is usually smaller. The details of the interaction potential, the coupled-channel approach, and the basis sets are given in SM [40]. Figure 2(a) shows the adiabats for different L correlating to the pair level

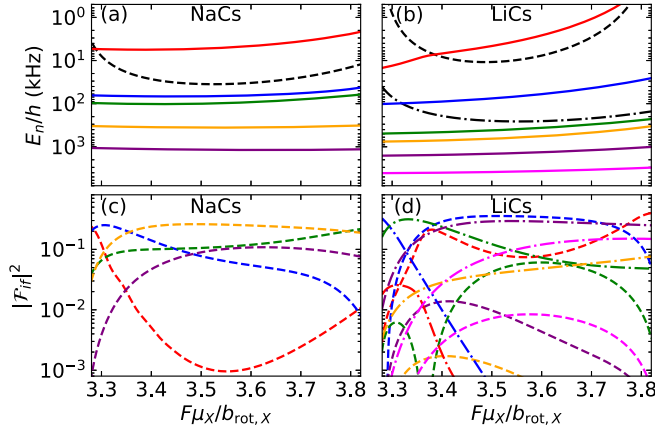


FIG. 3. Upper panels show binding energies E_n of FL tetramer states for (a) NaCs and (b) LiCs in their $X + X$ (dashed black) and $X + b$ (solid colored) thresholds as functions of F . The FCFs are shown in the lower panels. Their dash type and colors correlate to the tetramer states of $X + X$ and $X + b$, respectively, from the upper panel.

$(X, 1, 0) + (b, 2, 0)$ for LiCs at $F = 7.5$ kV/cm, a field where shielding in $(X, 1, 0) + (X, 1, 0)$ is optimal. They are repulsive at $R < 350$ bohr. Hereafter, we use $X + X$ and $X + b$ to denote the thresholds $(X, 1, 0) + (X, 1, 0)$ and $(X, 1, 0) + (b, 2, 0)$, respectively. The inset in Fig. 2(a) shows the long-range potential well for $X + b$, which is deeper than the one in $X + X$ (since $\mu_b > \mu_X$). The well can support up to six FL states. Subsequently, we determine k_{el} and $k_{2,\text{loss}}$ for LiCs for a broad range of fields at $E_{\text{coll}} = 10 \text{ nK} \times k_B$, as shown in Fig. 2(c) and 2(d). It is evident that $k_{2,\text{loss}}$ for $X + b$ collisions is highly suppressed at most fields, including $F = 0$ and the ones relevant for shielding in $X + X$. A discussion on the behavior of $k_{2,\text{loss}}$ is given in SM [40].

FCFs and STIRAP prospects—Here, we study the bound-bound transitions between the FL states of $X + X$ and $X + b$. We calculate the binding energies E_n of the FL tetramers as functions of F using coupled-channel methods [20] interfaced with BOUND [47]. We choose fields where shielding in both $X + X$ and $X + b$ is effective. The number of the FL states supported in the $X + X$ and $X + b$ thresholds for various molecules is given in Table S2 in SM [40]. The binding energies of the FL states of NaCs and LiCs are shown in the upper panels of Fig. 3. Note that the FL states in both $X + X$ and $X + b$ are quasibound with a finite energy width. However, static-electric-field shielding can achieve extreme two-body loss suppression [see Fig. 2(c) and 2(e)], making the energy widths of the FL states very narrow [40]. Thus, for $X + b$ FL states, their decay is almost fully determined by the decay of the b molecules.

We calculate the FCFs ($|\mathcal{F}_{if}|^2$) between the tetramer states i of $X + X$ and states f of $X + b$ as shown in the lower panels of Fig. 3. The quantities $|\mathcal{F}_{if}|^2$ contain the radial plus the angular overlap of the tetramer wave

functions as described in SM [40]. The FCFs are highly tunable with the electric field, particularly for LiCs, which has multiple FL states in the ground $X + X$ manifold. Thus, transitions between FL tetramers are controllable with the external field via the tunability of their FCFs. This is unlike the scenario for the dimers, where FCFs cannot be tuned easily [80]. The transition dipole moments D_{if} between the FL states are calculated from the FCFs using the relation $D_{if} = \mu_{Xb} |\mathcal{F}_{if}|$ [40]. The transition wavelength λ_0 is mostly determined by the vibronic transition energy T_{Xb} between X and b . Among the alkali dimers, λ_0 ranges from 850 to 1200 nm, whereas for KAg and CsAg, $\lambda_0 \sim 500$ and 550 nm, respectively [40].

Using our calculated FCFs for the FL tetramers of LiCs, a proof-of-principle STIRAP experiment can be envisaged, in which the population from the shallower tetramer state of $X + X$ can be transferred to the deeper one. The above stabilization reduces the vibrationally averaged interdimer distance $\langle R \rangle_v$ from 2800 to 950 bohr, nearly by a factor of 3. Similar proof-of-principle transfers can be realized for KAg and CsAg, which accommodate multiple FL states in $X + X$ [20]. $\langle R \rangle_v$ for the deepest FL state in $X + b$ is about 850 and 600 bohr for NaCs and LiCs, respectively, which are much smaller than the ones in $X + X$. This might be promising in generating FCFs with deeper rovibrational states embedded inside the long-range barrier of $X + X$. An accurate determination of the intermediate and short-range interactions is needed to verify this. It should be noted that the STIRAP laser intensities should be low enough such that the Rabi couplings and the differential ac Stark shifts are smaller than the level spacings [40].

Photoassociation prospects—PA involves absorption of a photon by a pair of colliding atoms to form an excited diatomic molecule [81,82]. There have been theoretical proposals on extending PA to atom-molecule [83,84] and molecule-molecule [85] collisions, where the former have been recently observed [86].

Here, we consider PA prospects by studying free-bound transitions of shielded dimers colliding in $X + X$ to FL tetramers in $X + b$. We calculate the PA rate coefficient k_{PA} using the relation $k_{\text{PA}} = (gh/2\mu_{\text{red}}k_0) \sum_n |S_{kn}|^2$ [82,87]. Here, $g = 2$ for identical molecules colliding in $X + X$ and $k_0 = \sqrt{2\mu_{\text{red}}E_{\text{coll}}/\hbar^2}$. The PA process for the transition of scattering states $|k\rangle$ of $X + X$ to bound states $|n\rangle$ of $X + b$ is encapsulated by the energy dependent S -matrix element $S_{kn}(k_0)$, defined as

$$|S_{kn}|^2 = \frac{\Gamma_n^s \Gamma_n^d}{(E_{\text{coll}} + hc/\lambda - hc/\lambda_0)^2 + \frac{1}{4}[\Gamma_n^s + \Gamma_n^d]^2}. \quad (2)$$

Here, λ is the PA laser wavelength, hc/λ_0 is the energy of $|n\rangle$ relative to $X + X$, and Γ_n^s and Γ_n^d are, respectively, the energy widths for the stimulated and spontaneous decay of $|n\rangle$. The stimulated width is energy-dependent and is defined as $\Gamma_n^s(k_0) = (4\pi^2 I / \epsilon_0 c) D_{kn}^2(k_0)$, where ϵ_0 is the

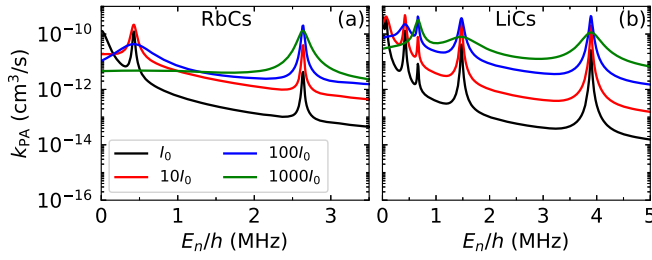


FIG. 4. Photoassociation rates k_{PA} for colliding dimers in $X + X$ with $E_{\text{coll}} = 500 \text{ nK} \times k_B$ as functions of the binding energy of $X + b$ tetramer for (a) RbCs ($F = 2.82 \text{ kV/cm}$, $\lambda_0 = 1190 \text{ nm}$) and (b) LiCs ($F = 7.30 \text{ kV/cm}$, $\lambda_0 = 1160 \text{ nm}$) for different laser intensities with $I_0 = 5 \mu\text{W/cm}^2$.

vacuum permittivity, I is the laser intensity, and we approximate Γ_n^d by γ_e of the b molecules.

Figure 4 shows the PA spectra for RbCs + RbCs and LiCs + LiCs at fixed electric fields for different laser intensities. We consider $\gamma_e = 20 \text{ kHz} \times h$ for both molecules. We observe well-resolved PA signals for both molecules. However, the intensities required are small. This is mainly due to (1) the requirement that $\Gamma_n^s \ll E_n$, and (2) the long-range nature of the scattering wave functions of $X + X$ and bound wave functions of $X + b$ produce large values of $|\mathcal{F}_{kn}|^2$, thus requiring smaller I . Figure 4 shows that $I \gtrsim 5 \text{ mW/cm}^2$ destroys the PA peaks. Also, the PA rates become smaller as E_{coll} of the colliding dimers in $X + X$ increases [40]. For RbCs, FL states do not exist in ground $X + X$. Nevertheless, our methods show that the PA of RbCs + RbCs to FL tetramers in the $X + b$ manifold is possible. Similar physics is expected for KCs molecules [40].

Outlook—The FL tetramers in $X + b$ provide an excellent tool to optically control ultracold tetramers. Our predictions can be extended to microwave (MW) shielded molecules, which offer higher tunability. One can control the ellipticity of the MW polarization [21] or can implement double MW fields [8] to make the long-range potential well deeper. This may allow at least two FL states in $X + X$ and implement our proposed STIRAP pathway with more molecules. However, this may also increase the two-body loss of the dimers [88]. Coupled-channel calculations are required to verify this. Since our methodology is based on the long-range dipolar interaction of the dimers, our predictions are not susceptible to experimental or theoretical uncertainties in determinations of the molecular potentials at the short range.

Our ultimate goal is to transfer the FL states to deeply bound tetramers near the global minimum of the ground interaction potential. There might be two possible strategies. First, the transfer to deeply bound tetramer states is envisaged via $X + b$ intermediate states. For this, one might lower and shift the repulsive barrier in $X + b$ to shorter distances by controlling the external field parameters. This will increase the FCFs with the $X + X$ states. Second, the

deepest $X + X$ FL state, which gets populated through our proposed STIRAP pathway, can be coupled to a deeply bound rovibrational state within $X + X$ via a subsequent STIRAP sequence. These directions require knowledge of shorter-range tetramer potentials in both the ground and excited electronic manifolds. We intend to address these in our future studies.

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Data availability—The data that support the findings of this article are openly available [89].

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