

Spatiotemporal Energy Cascade in Three-Dimensional Magnetohydrodynamic Turbulence

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We present a new scale decomposition method to investigate turbulence in wavenumber-frequency space. Using 3D magnetohydrodynamic turbulence simulations, we show that magnetic fluctuations with time scales longer than the nonlinear time exhibit an inverse cascade toward even smaller frequencies. Low frequency magnetic fluctuations support turbulence, acting as an energy reservoir that is converted into plasma kinetic energy, the latter cascading toward large wavenumbers and frequencies, where it is dissipated. Our results shed new light on the spatiotemporal properties of turbulence, potentially explaining the origin and role of low frequency turbulent fluctuations in the solar wind.

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Turbulence is ubiquitous in space and astrophysical plasmas, including the solar wind (SW) [1], interstellar medium [2], accretion flows [3], galaxy clusters [4]. Turbulent plasmas generate fluctuations with broadband wavenumber and frequency spectra, and are typically subject to strong background magnetic fields, inducing a wavenumber anisotropy with $k_{\perp} > k_{\parallel}$ [5–11], where $k_{\perp}$ and $k_{\parallel}$ are wavenumbers perpendicular and parallel to the magnetic field. Numerical [12–16] and observational [17,18] studies have revealed that plasma turbulence has nontrivial spatiotemporal properties, with most of the magnetic field, velocity, and density fluctuations concentrated at low frequencies ω and small $k_{\parallel}$. Low $(k_{\parallel}, \omega)$ modes are often interpreted as quasi-2D structures [19–24], since their $(k_{\perp}, k_{\parallel}, \omega)$ spectra do not follow wave dispersion relations.

The origin of low $(k_{\parallel}, \omega)$ turbulent fluctuations is still debated. Candidate theoretical explanations for the formation of $(\omega \sim 0, k_{\parallel} \sim 0)$ modes include models of nearly incompressible turbulence [25–27], Alfvén wave turbulence [28,29], frequency broadening of magnetohydrodynamic (MHD) waves [30,31], inverse cascades driven by MHD invariants [32,33]. In addition to understanding their origin, the presence of low $(k_{\parallel}, \omega)$ modes also raises the question of whether such fluctuations play a role in driving turbulence, or if they are dynamically irrelevant. Despite long-standing efforts in this context, a unifying description

for the origin and role of low $(k_{\parallel}, \omega)$ turbulent fluctuations is still missing.

Understanding the spatiotemporal properties of turbulence is fundamental in space and astrophysical problems, including the solar corona heating [34,35], the SW acceleration [36,37], the emergence of “1/f” SW spectra [38,39], cosmic rays transport [40–42], the dynamics of accretion flows [43,44] and molecular clouds [45,46], and the turbulent dynamo [47–49].

In this Letter, we present a new framework to study how turbulence reorganizes energy among fluctuations with different wavenumbers and frequencies, investigating the origin and role of low frequency fluctuations. We employ the coarse graining (CG) technique, frequently used to study turbulence [50–53]. The CG method consists of low-pass filtering the system equations of motion, cutting off small scales. The global energy balance obtained from filtered equations gives a set of quantities describing large scale energy transfers, plus cascade terms, representing energy exchanges between large and small scales. We consider the MHD equations

$$\begin{aligned}\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) &= 0, \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) &= -\nabla P + \mathbf{J} \times \mathbf{B} + \nabla \cdot \mathbf{\Pi} + \mathbf{F}_u, \\ \partial_t \mathbf{B} &= \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J}) + \mathbf{F}_B,\end{aligned}\quad (1)$$

where ρ , $\mathbf{u}$, and P are the plasma density, velocity, and pressure. $\mathbf{B}$ is the magnetic field, $\mathbf{J} = \nabla \times \mathbf{B}$, η is the magnetic diffusivity. $\mathbf{\Pi} = \rho \nu [\nabla \mathbf{u} + \nabla \mathbf{u}^T - (2/3)(\nabla \cdot \mathbf{u})\mathbf{I}]$ is the viscous stress tensor, ν is the viscosity, $\mathbf{I}$ is the identity matrix, and T the transpose operation. $\mathbf{F}_u$ and $\mathbf{F}_B$ are forcing terms.

The CG method typically involves only spatial scales, but we extend it by introducing the spatiotemporal

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low-pass filter

$$\bar{q}(\mathbf{x}, t, \mathbf{k}, \tau) = \sum_{\mathbf{k}' < \mathbf{k}} \int dt' Q(\mathbf{k}', t') e^{i\mathbf{k}' \cdot \mathbf{x}} G_\tau(t - t'), \quad (2)$$

for a generic quantity $q(\mathbf{x}, t)$ with spatial Fourier transform $Q(\mathbf{k}, t)$. G_τ is a boxcar function with width τ . $\bar{q}$ contains wavenumbers $< \mathbf{k}$ and timescales $> \tau$. The corresponding density-weighted filter is $\hat{q} = \bar{\rho}/\bar{\rho}$ [54]. By applying this filter to Eq. (1), and calculating the global energy balance, we obtain

$$\begin{aligned} \partial_t \left\langle \frac{1}{2} \bar{\rho} \hat{u}^2 \right\rangle &= -W_P - \Pi_u + W_{\text{e.m.}} - D_u + I_u, \\ \partial_t \left\langle \frac{1}{2} \bar{B}^2 \right\rangle &= -\Pi_B - W_{\text{e.m.}} - D_B + I_B, \end{aligned} \quad (3)$$

where $\langle \cdot \rangle$ indicates the spatial average, and we assumed no transport across the system boundaries, causing all divergence terms to vanish when averaged, as in periodic systems (see Supplemental Material [55]). Energy transfer channels (ETCs) in Eq. (3) are

$$\begin{aligned} I_B &= \langle \bar{\mathbf{F}}_B \cdot \bar{\mathbf{B}} \rangle, & I_u &= \langle \bar{\mathbf{F}}_u \cdot \hat{\mathbf{u}} \rangle, \\ W_{\text{e.m.}} &= \langle -(\hat{\mathbf{u}} \times \bar{\mathbf{B}}) \cdot \bar{\mathbf{J}} \rangle, & \Pi_B &= \langle \boldsymbol{\tau}_E \cdot \bar{\mathbf{J}} \rangle, \\ \Pi_u &= \Pi_u^S + \Pi_u^L = \langle -\bar{\rho} \boldsymbol{\tau}_u : \nabla \hat{\mathbf{u}} \rangle + \langle -\boldsymbol{\tau}_B \cdot \hat{\mathbf{u}} \rangle, \\ W_P &= \langle -\bar{P} \nabla \cdot \hat{\mathbf{u}} \rangle, & D_B &= \langle \eta \bar{\mathbf{J}}^2 \rangle, & D_u &= \langle \bar{\Pi} : \nabla \hat{\mathbf{u}} \rangle, \end{aligned} \quad (4)$$

with I_B and I_u representing magnetic and kinetic energy injection rates, $W_{\text{e.m.}}$ and W_P are the electromagnetic (e.m.) and pressure works, D_B and D_u are magnetic and kinetic dissipation rates, all representing energy exchanges at scales $< \mathbf{k}$ and $> \tau$. Π_B and Π_u are the magnetic and kinetic cascade rates, quantifying energy transfers from scales $< \mathbf{k}$ and $> \tau$, to scales $> \mathbf{k}$ and $< \tau$. $\boldsymbol{\tau}_u = (\hat{\mathbf{u}} \hat{\mathbf{u}} - \hat{\mathbf{u}} \hat{\mathbf{u}})$, $\boldsymbol{\tau}_E = -(\hat{\mathbf{u}} \times \bar{\mathbf{B}} - \hat{\mathbf{u}} \times \bar{\mathbf{B}})$ and $\boldsymbol{\tau}_B = (\bar{\mathbf{J}} \times \bar{\mathbf{B}} - \bar{\mathbf{J}} \times \bar{\mathbf{B}})$ are the subscale stress tensor, electric field, and Lorentz force, characterizing small scales [56]. Hence, Π_B represents the interaction between small scale electric fields $\boldsymbol{\tau}_E$ and large scale currents $\bar{\mathbf{J}}$. Π_u includes Π_u^S , quantifying the interaction between small scale stresses $\boldsymbol{\tau}_u$ and the large scale strain tensor $\nabla \hat{\mathbf{u}}$, and Π_u^L , coupling the small scale Lorentz force $\boldsymbol{\tau}_B$ to large scale velocities $\hat{\mathbf{u}}$.

We apply our CG method to a 3D MHD turbulence simulation, realized with Athena++ [57], solving Eq. (1). We consider a uniform periodic grid with 256×512^2 points, and size $L_z = 3L_y = 3L_x = 6\pi$ (in arbitrary units L_0). The plasma has zero initial velocity, homogeneous density ρ_0 , and guide field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. Pressure is isothermal, with beta $\beta = 2c_S^2/c_A^2 = 0.5$, where c_S and $c_A = B_0/\sqrt{\rho_0}$ are the sound and Alfvén speeds. Turbulence is driven by $\mathbf{F}_u$ and

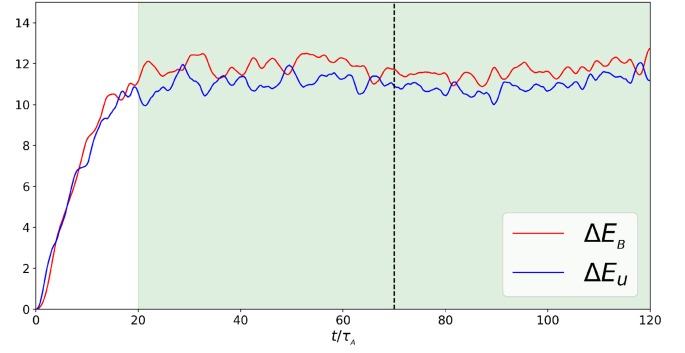


FIG. 1. Temporal evolution of magnetic and kinetic energy variations ΔE_B and ΔE_u .

$\mathbf{F}_B$, using the Langevin antenna (LA) scheme [58], with driving frequency $\omega_0 = 0.8\tau_A^{-1}$ and decorrelation rate $\gamma_0 = -0.7\tau_A^{-1}$ (where $\tau_A = L_0/c_A$). $\mathbf{F}_u$ and $\mathbf{F}_B$ drive incompressible magnetic and velocity fluctuations perpendicular to $\mathbf{B}_0$, with rms amplitudes $\delta B_{\text{rms}}/B_0 \simeq 0.18$ and $\delta u_{\text{rms}}/c_A \simeq 0.17$, cross helicity $\sigma_C \simeq 0$, and wavenumbers $1 \leq k_{\parallel} L_z/2\pi \leq 3$ and $1 \leq k_{\perp} L_x/2\pi \leq 4$. Our setup mimics near-Earth SW turbulence [59]. We set $\eta = \nu = 2.5 \times 10^{-4}$ (in L_0^2/τ_A units).

Figure 1 shows the temporal evolution of magnetic and kinetic energy variations, $\Delta E_B = E_B(t) - E_B(0)$ and $\Delta E_u = E_u(t) - E_u(0)$ (in $\rho_0 c_A^2 L_0^3$ units), growing and saturating after $20\tau_A$. We consider interval $T = [20\tau_A, 120\tau_A]$ for our analysis (green area), when turbulence is quasi-stationary. Figures 2(a) and 2(b) show $(k_{\parallel}, \omega)$ and $(k_{\perp}, \omega)$ projections of the magnetic field space-time Fourier spectrum P_B , calculated over interval T , with simulation data sampled every $\Delta t = 0.1\tau_A$. $(k_{\parallel}, \omega)$ and $(k_{\perp}, \omega)$ projections correspond to $P_B(k_{\parallel}, \omega) = \int P_B(k_{\perp}, k_{\parallel}, \omega) dk_{\perp}$ and $P_B(k_{\perp}, \omega) = \int P_B(k_{\perp}, k_{\parallel}, \omega) dk_{\parallel}$. Projections of the velocity spectrum P_u are shown in Figs. 2(c) and 2(d). We see that the energy distribution in $(k_{\parallel}, \omega)$ space quickly falls off toward large $k_{\parallel}$ and ω . $(k_{\perp}, \omega)$ projections also show that most energy lies around small ω , but with a wider distribution in $k_{\perp}$. Despite the incompressible driving, density fluctuations still develop, with amplitude $\delta \rho_{\text{rms}}/\rho_0 \simeq 0.08$. Figures 2(e) and 2(f) show $(k_{\parallel}, \omega)$ and $(k_{\perp}, \omega)$ projections of the density spectrum P_ρ . Density fluctuations are also concentrated around low ω , exhibiting a strong $k_{\perp} > k_{\parallel}$ anisotropy. Overall, (k, ω) spectra do not follow dispersion relations (dashed lines) of Alfvén waves (AW), slow modes (SM), and fast modes (FM), and most energy is stored in low ω , low $k_{\parallel}$ fluctuations, consistently with previous works [16–18].

We investigate the origin of such spectral features using our spatiotemporal CG method, tracking energy transfers in $(k_{\perp}, k_{\parallel}, \tau)$ space (with $\tau \simeq 2\pi/\omega$). The goal is understanding how low ω (large τ) fluctuations develop, and their dynamical role. We analyze two projections of ETCs in Eq. (4): $(k_{\parallel}, \tau)$ projections, obtained by filtering in $k_{\parallel}$ and τ ,

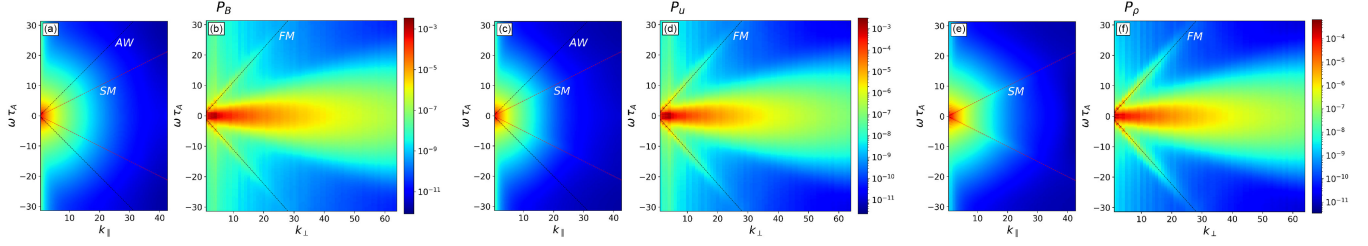


FIG. 2. $(k_{\parallel}, \omega)$ and $(k_{\perp}, \omega)$ projections of magnetic field, velocity, and density spectra P_B (a),(b), P_u (c),(d), and P_{ρ} (e),(f). Dashed lines indicate dispersion relations of Alfvén wave and slow modes for $k_{\perp} = 0$, and fast modes for $k_{\parallel} = 0$.

retaining all $k_{\perp}$; $(k_{\perp}, \tau)$ projections, calculated by filtering in $k_{\perp}$ and τ , keeping all $k_{\parallel}$. For the temporal filtering, we center the boxcar G_{τ} at $t = 70\tau_A$ (vertical dashed line in Fig. 1), varying its width τ from $0.2\tau_A$ to $100\tau_A$, covering the whole interval T . ETCs are shown in Fig. 3, with black lines indicating their isocontours. As I_B and I_u exhibit analogous $(k_{\perp}, k_{\parallel}, \tau)$ dependence and comparable amplitude, we analyze $I_{\text{tot}} = I_B + I_u$. For the same reason, we consider $D_{\text{tot}} = D_B + D_u$ (see Supplemental Material [55]). For each ETC, $(k_{\parallel}, \tau)$ and $(k_{\perp}, \tau)$ projections exhibit similar features, being anisotropic in $(k_{\parallel}, k_{\perp})$ space. The ETCs anisotropy causes the $k_{\perp} > k_{\parallel}$ anisotropy observed in spectra [60,61]. As references, we show parallel and perpendicular integral scales $k_{\parallel}^{\text{int}}$ and $k_{\perp}^{\text{int}}$ (vertical green

dashed lines) [62]

$$(k_{\perp}^{\text{int}}, k_{\parallel}^{\text{int}}) = \frac{\int \int (k_{\perp}, k_{\parallel}) P_u dk_{\perp} dk_{\parallel} d\omega}{\int \int P_u dk_{\perp} dk_{\parallel} d\omega}, \quad (5)$$

representing the scale of energy containing fluctuations, and the corresponding nonlinear time (horizontal green dashed lines) $\tau_{nl} = 2\pi/(k_{\perp}^{\text{int}} \delta u_{\text{rms}})$. Starting from I_{tot} , Figs. 3(a) and 3(b), we see it is positive and increases with wavenumber, up to $k_{\parallel} \simeq 1.3$ and $k_{\perp} \simeq 6$, after which it saturates, indicating no contribution from larger wavenumbers. Regarding its τ dependence, I_{tot} is not peaked at the driving time $2\pi/\omega_0 \simeq 7.8\tau_A$, but exhibits a broad frequency response, growing from $\tau \simeq 15$ to $\tau \simeq 1.5$, while

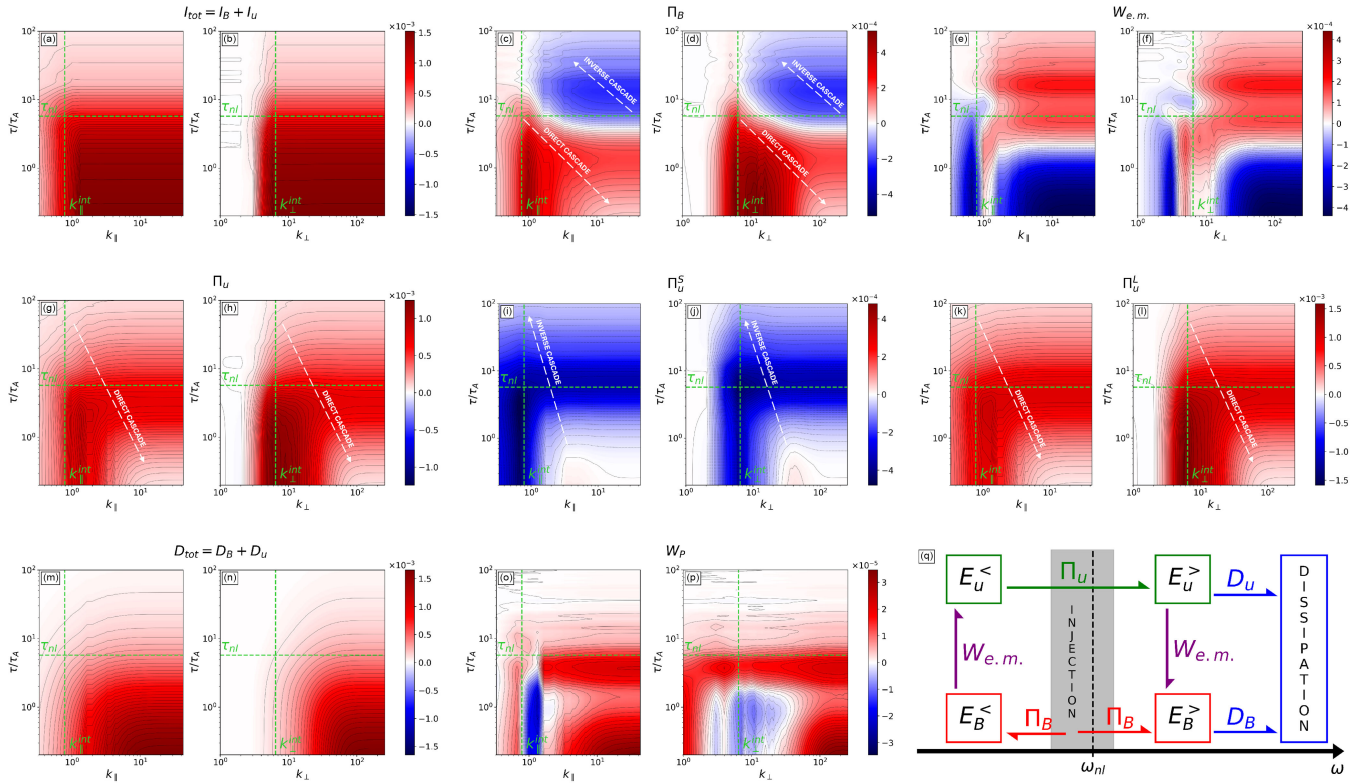


FIG. 3. $(k_{\parallel}, \tau)$ and $(k_{\perp}, \tau)$ projections of $I_B + I_u$ (a),(b), Π_B (c),(d), $W_{\text{e.m.}}$ (e),(f), Π_u (g),(h), Π_u^S (i),(j), Π_u^L (k),(l), $D_B + D_u$ (m),(n), and W_P (o),(p), with vertical green dashed lines indicating $k_{\parallel}^{\text{int}}$ and $k_{\perp}^{\text{int}}$, while horizontal green dashed lines represent τ_{nl} . Schematic of energy transfers in frequency space (q).

being almost constant elsewhere. This broadening is determined by the decorrelation rate γ_0 of the LA driver, typically chosen to be $|\gamma_0| \leq \omega_0$. Different drivers imply different I_{tot} features, but we choose the LA approach since it reproduces key properties of SW turbulence [58,63].

Energy injected by I_{tot} is redistributed by Π_B and Π_u . Starting from Π_B , Figs. 3(c) and 3(d), we find it is negative for $\tau \gtrsim \tau_{nl}$ and wavenumbers larger than $k_{\parallel}^{\text{int}}$ and $k_{\perp}^{\text{int}}$, while being positive for $\tau < \tau_{nl}$, where it peaks around integral scales. This indicates a frequency space bifurcation in the magnetic energy cascade, with large τ (low ω) fluctuations exhibiting an inverse cascade (by “inverse cascade” we mean energy transfer from high to low frequencies) toward even lower frequencies and smaller wavenumbers in (k, τ) space, while small τ (high ω) magnetic modes undergo a direct cascade, with a stronger cascade rate around $k_{\parallel}^{\text{int}}$ and $k_{\perp}^{\text{int}}$. Hence, part of the injected magnetic energy flows to low (k, ω) fluctuations, while another fraction goes to high (k, ω) . We first follow low ω magnetic fluctuations. Magnetic energy transferred to large τ by the inverse cascade can be converted into kinetic energy by $W_{\text{e.m.}}$. Figures 3(e) and 3(f) show that $W_{\text{e.m.}}$ is positive where Π_B is negative, for $\tau \gtrsim \tau_{nl}$ and wavenumbers larger than $k_{\parallel}^{\text{int}}$ and $k_{\perp}^{\text{int}}$, with similar magnitude, indicating that low ω magnetic energy is converted into low ω kinetic energy. Such balance between the inverse magnetic energy cascade and a low ω energy sink, $W_{\text{e.m.}}$ in this case, is expected since turbulence is quasistationary. The combination of the inverse magnetic energy cascade and the magnetic-to-kinetic energy conversion at low ω , implies an accumulation of magnetic and kinetic energy at small frequencies, explaining the origin of low ω fluctuations in energy spectra. Unlike Π_B , the kinetic energy cascade rate Π_u , Figs. 3(g) and 3(h), is always positive, peaking around integral scales and at $\tau \sim \tau_{nl}$, indicating a direct kinetic energy cascade at all frequencies. Combining Π_B , $W_{\text{e.m.}}$, and Π_u , we find that low ω magnetic fluctuations contribute to driving turbulence, providing magnetic energy that is first converted into low ω kinetic energy by $W_{\text{e.m.}}$, and finally transferred to higher (k, ω) fluctuations by Π_u . Since $W_{\text{e.m.}}$ is negative for large k and small τ , Figs. 3(e) and 3(f), part of the small scale kinetic energy resulting from the direct cascade is converted to small scale magnetic energy.

Additional insights are obtained decomposing Π_u into Π_u^S and Π_u^L , in Figs. 3(i)–3(l). We find that Π_u^S is essentially always negative (except at small scales, where it is weakly positive), indicating an inverse cascade. This is strikingly different from hydrodynamic turbulence, where Π_u^S is typically positive, being the only cascade term [64]. Conversely, Π_u^L is positive at all scales, and larger than Π_u^S , producing a net direct cascade when combined with Π_u^S . Hence, the direct kinetic energy cascade in MHD turbulence is mainly driven by e.m. interactions, quantified

by Π_u^L , while the hydrodynamic term Π_u^S opposes the direct Π_u^L cascade.

Magnetic and kinetic energy cascading to small scales is eventually dissipated. Figures 3(m) and 3(n) show D_{tot} to be weak at large scales, growing to large positive values toward high k and small τ , indicating that dissipation mainly affects high frequencies and wavenumbers. Consequently, low ω fluctuations resulting from the inverse cascade are not dissipated. Turbulent SW dissipation is mainly caused by kinetic processes [65], rather than by resistive and viscous effects as in our simulation. However, kinetic scale dissipation is typically mediated by wave-particle interactions [65,66] and intermittent events, like magnetic reconnection [67,68], with timescales comparable to particle gyroperiods. Hence, we expect kinetic effects not to affect low ω fluctuations, whose timescales are larger than τ_{nl} , and thus much larger than particle gyroperiods [17,18].

Another channel for dissipation is W_P , quantifying compressibility. Figures 3(o) and 3(p) show W_P to be negligible for $\tau > \tau_{nl}$, meaning that low ω fluctuations are nearly incompressible, while for $\tau \leq \tau_{nl}$ it is negative (dilatation) around integral scales, and positive (compression) elsewhere. However, W_P is about 1 order of magnitude smaller than other ETCs, and thus energetically irrelevant.

The interplay among all ETCs in frequency space is summarized in Fig. 3(q), highlighting the origin and role of low ω fluctuations. Energy injection is represented by the gray area around $\omega_{nl} \simeq 2\pi/\tau_{nl}$. Part of the injected magnetic energy exhibits an inverse cascade, driven by Π_B , producing low ω magnetic energy $E_B^<$. The latter is transferred to low ω kinetic energy $E_u^<$ by $W_{\text{e.m.}}$. $E_u^<$ undergoes a direct cascade, driven by Π_u , toward high ω kinetic energy $E_u^>$. Energy flows into high ω magnetic energy $E_B^>$ through two paths: a direct cascade from injection to $E_B^>$, driven by Π_B , and the kinetic-to-magnetic energy transfer driven by $W_{\text{e.m.}}$ at high ω , converting $E_u^>$ into $E_B^>$. Finally, $E_B^>$ and $E_u^>$ are dissipated by D_B and D_u . $E_B^<$ and $E_u^<$ are not dissipated, making low ω fluctuations a stable energy reservoir that contributes to driving turbulence.

The main new mechanism unveiled by our spatiotemporal CG analysis is the magnetic energy cascade bifurcation in frequency space, with magnetic fluctuations exhibiting an inverse cascade for $\omega < \omega_{nl}$, and a direct cascade for $\omega > \omega_{nl}$. Inverse cascades in frequency space occur also in hydrodynamic turbulence [69–74]. To test the generality of our findings, we applied our CG analysis to a ($\beta = 18, \sigma_C \simeq 0$) run, representing outer heliosphere turbulence [75], and to a ($\beta = 0.5, \sigma_C \simeq 0.73$) run, reproducing imbalanced near-Sun turbulence [76]. Both simulations exhibit the same spatiotemporal behavior of our main ($\beta = 0.5, \sigma_C \simeq 0$) run, with an inverse magnetic energy cascade occurring at $\tau \gtrsim \tau_{nl}$ (see Supplemental Material [55]). Besides β and σ_C , another key quantity is the magnetic

helicity σ_B , undergoing an inverse wavenumber space cascade [77,78]. The random phase nature of $\mathbf{F}_B$ implies no net σ_B injection, suggesting that the inverse frequency space cascade we observe is not related to the inverse σ_B cascade. Our results also differ from the turbulent dynamo, where large scale magnetic fields are produced from small scale fluctuations, implying an inverse wavenumber space cascade [47]. This scenario is different from our simulations, where magnetic perturbations are injected at large scales, as in the SW, and we find that magnetic fluctuations with different ω contribute differently to turbulence, with some of them exhibiting an inverse cascade, while others undergo a direct cascade.

Our results suggest that low frequency SW fluctuations may originate not only in the solar corona, but also locally, produced by turbulence. By “frequency” we mean time-scales ω , unambiguously separated from spatial scales k . Such space-time decomposition can be achieved via multi-spacecraft methods, producing (k, ω) spectra consistent with Fig. 2 [17,18]. On the other hand, single spacecraft spectra are functions of the sampling frequency f , not necessarily related to timescales ω , and often converted to spatial scales k via the Taylor hypothesis [65]. Therefore, by “low frequency fluctuations” we mean small ω in (k, ω) spectra, not the low frequency $1/f$ range of single spacecraft SW spectra.

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