

Stochastic Inflation as an Open Quantum System

Yue-Zhou Li^{✉*}

Department of Physics, Princeton University, Princeton, New Jersey 08540, USA



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We reinterpret Starobinsky's stochastic inflation as an open quantum system, where short-wavelength modes act as the environment for long-wavelength modes. Using the Schwinger-Keldysh formalism, we systematically trace out the environment and derive an effective theory for the reduced density matrix, including deviations from exact de Sitter space. The resulting master equation is a Lindblad equation, which reduces to a Fokker-Planck equation for the diagonal elements up to higher orders in the slow-roll expansion, while also yielding a more complete equation in phase space. Finally, we extend the formalism to global de Sitter space, for which the associated Fokker-Planck equation lacks equilibrium solutions until the late-time regime $aH \gg 1$.

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Introduction—Open quantum systems are quantum systems that interact with an environment. The relevant phenomena are ubiquitous, even in daily life, as they often encode the emergence of classical stochastic behavior from quantum theory [1–3]. In this philosophy, the “environment” refers to any degrees of freedom not explicitly tracked in our physical description (see, e.g., Refs. [4–6] for reviews of open quantum systems).

This raises a natural question: should we treat effective descriptions of gravitating systems, especially our universe, as closed or open, given that we cannot explicitly track all quantum degrees of freedom of gravity?

A first attempt to address this was made by Starobinsky in the context of inflationary cosmology [7]. Inflation is an epoch of exponential expansion before the hot big bang in our universe, during which the inflaton slowly rolls down a quite flat potential, and its quantum fluctuations are stretched to macroscopic scales, seeding the cosmic microwave background. However, when these fluctuations become comparable to the classical slow-roll motion, standard cosmological perturbation theories (see, e.g., Refs. [8–13]) become unreliable, and quantum effects can push the inflaton upward, leading to eternal inflation [14–17].

Starobinsky formulated this picture as a stochastic process, modeling the inflaton as undergoing a random walk governed by a Langevin equation and an associated Fokker-Planck (FP) equation [7,18]. This is known as stochastic inflation, and it has been widely used to understand eternal inflation [15,19–22] and the resummation of

secularly growing cosmological observables [18,23–32]. However, most formulations and generalizations rely on the Langevin equations and a postulated FP equation [33–45], with only limited understanding from underlying quantum field theory (QFT) in curved space [46–53] (see Ref. [54] for a review and references therein).

The key point is that the stochastic picture is essentially the semiclassical limit of an open quantum system [1,2]. A well-known example is the Brownian particle [1,55–58], which is precisely the phenomenon to which stochastic inflation draws an analogy. Therefore, a natural way to understand stochastic inflation is to treat it as an open quantum system, where short-wavelength modes act as the environment for the long-wavelength modes (see Refs. [59–67] for relevant discussions). Starobinsky's stochastic formulation then emerges as the semiclassical limit of the open quantum dynamics of the universe.

In this Letter, we develop this philosophy into a concrete and controlled framework. We treat the entire universe undergoing eternal slow-roll inflation as a quantum system, and explicitly trace out short-wavelength modes to obtain a mixed cosmological density matrix. Using the Schwinger-Keldysh (SK) path integral formalism [55,68], we derive the master equation of this density matrix in the Liouville space [69]. Restricting to its diagonal elements, we obtain the FP equation to higher orders in the slow-roll expansion (which recovers Starobinsky's result at leading order [7] and the higher-order results in Refs. [24,70]). Projecting to phase space (known as the Wigner function), we derive the corresponding master equation and find that, up to the order we compute, it takes the form of a Lindblad equation [4,69]. This provides, to our knowledge, the first derivation of a Lindblad equation from first principles in a gravitational QFT, obtained by explicitly integrating out ultra-violet modes as the environment.

This framework also generalizes to global de Sitter (dS) space, relevant to quantum cosmology [71–73], where we

*Contact author: liyuezhou@princeton.edu

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derive the corresponding FP equation and show that the early-time universe does not approach equilibrium states until its physical size far exceeds the Hubble radius.

In this philosophy, the FP equation and its phase space generalization should be interpreted as Lindblad extensions of the Wheeler-DeWitt (WdW) equation [74] in the minisuperspace of quantum gravity in the semiclassical limit.

Density matrix evolution from Schwinger-Keldysh formalism—Let us briefly establish the key framework for exploring stochastic inflation as an open quantum system.

For any given theory with action $S_0[x, p]$, where x is the field variable and p its conjugate momentum, the density matrix of the “minimal energy” state can be prepared and evolved forward in time using the SK path integral, which connects Euclidean time to real time, as illustrated in Fig. 1.

The SK formalism doubles the degrees of freedom, introducing $(x^\pm, p^\pm)$, and treats $\pm$ distinctly as the ket and bra components. The Euclidean segment selects the vacuum initial state, which in our case is the Bunch-Davies (BD) state [76]. The density matrix can then be projected into any representation by specifying the boundary condition at the final time t , at which the density matrix is evaluated.

$$\langle\langle r[x^\pm, p^\pm]|\rho(t)\rangle\rangle = N^2 \int_{x^\pm(\pm i\infty=0)}^{r[x^\pm, p^\pm]} Dx^\pm Dp^\pm e^{iS_{L0}(x^\pm, p^\pm)}, \quad (1)$$

where $S_{L0} = S_0^+ - S_0^-$. In this formula, we adopt the Liouville space [69], in which the density matrix is promoted to a vector, leading to

$$S_{L0} = \int dt [p^c \dot{x}^q + p^q \dot{x}^c - H_L(x^c, x^q, p^c, p^q)], \quad (2)$$

where we introduce the Keldysh basis $x^c = [(x^+ + x^-)/2]$ the average field and $x^q = x^+ - x^-$ the response field. H_L is the Liouville superoperator, which plays the role of the Hamiltonian in Liouville space. In this Letter, we consider the representation $r[x^\pm, p^\pm]$ in two scenarios: (1) Diagonal density matrix $x^c(t) = x, x^q(t) = 0$

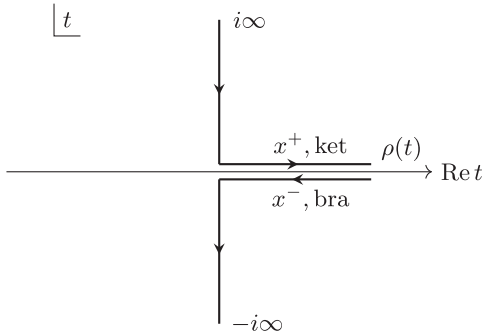


FIG. 1. The SK contour prepares and evolves the density matrix from Euclidean infinity to the real time at which it is measured. In the context of gravity, the doubled variables $x^\pm$ extend to include both the quantum fields and the geometry [71,75].

$$\langle\langle r|\rho(t)\rangle\rangle := P(x, t). \quad (3)$$

(2) The Wigner function $x^c(t) = x, p^c(t) = p$

$$\langle\langle r|\rho(t)\rangle\rangle = \int \frac{dx'}{\pi} \langle x + x'|\rho|x - x'\rangle e^{2ipx'} := W(x, p, t). \quad (4)$$

We now consider the case where x is coupled to an environment y . The variable y may represent an arbitrary auxiliary system; in particular, it can represent the short-wavelength modes of the same underlying system. In this context, the system-environment separation resembles the scale separation in Wilsonian effective field theories (EFTs). Tracing out the environment then corresponds to constructing an effective theory by integrating out y . This procedure introduces the Feynman-Vernon influence functional $e^{iS_{\text{IF}}}$ [77], encoding imprints of system-environment interactions

$$\langle\langle r[x^\pm, p^\pm]|\text{Tr}_y \rho(t)\rangle\rangle = N^2 \int_{x^\pm(\pm i\infty=0)}^{r[x^\pm, p^\pm]} Dx^\pm Dp^\pm e^{i(S_{L0} + S_{\text{IF}})}. \quad (5)$$

The effective theory is $S_L^{\text{eff}} = S_{L0} + S_{\text{IF}}$, with its Liouville Hamiltonian H_L^{eff} , subject to the appropriate boundary conditions. This gives the master equation for the reduced density matrix

$$\partial_t \rho_x(r[x^\pm, p^\pm], t) = N^2 \int_{x^\pm(\pm i\infty=0)}^{r[x^\pm, p^\pm]} Dx^\pm Dp^\pm (-iH_L^{\text{eff}}) e^{iS_L^{\text{eff}}}. \quad (6)$$

This equation is not closed in general. Nevertheless, for a diagonal density matrix, it is clear that we should simply replace $p^q = -i\partial_x$; simultaneously, we evaluate other variables p^c using their classical solutions subject to the boundary conditions. For the Wigner function, the master equation becomes manifestly closed upon replacing $p^q = -i\partial_x$ and $x^q = i\partial_p$.

$$\begin{aligned} \partial_t P(x, t) &= -i[H_L^{\text{eff}}(x, x^q, p^c, -i\partial_x)]_{\text{saddle}} P(x, t), \\ \partial_t W(x, p, t) &= -iH_L^{\text{eff}}(x, i\partial_p, p, -i\partial_x) W(x, p, t). \end{aligned} \quad (7)$$

Effective theory from tracing out the short wavelength environment—We now apply this formalism to single-field inflation $S = S_{\text{grav}} + S_\phi$ where

$$\begin{aligned} S_{\text{grav}} &= \frac{1}{16\pi G} \left(\int d^4x \sqrt{-g} R - 2 \int d^3x \sqrt{h} K \right), \\ S_\phi &= - \int d^4x \sqrt{-g} \left(\frac{1}{2} (\nabla\phi)^2 + V(\phi) \right). \end{aligned} \quad (8)$$

We consider the slow-roll regime, defined by $16\pi G\epsilon = (V'/V)^2 \ll 1$. Gravity is treated semi-classically

after inflation starts at $\phi = \phi_0$ and $V(\phi_0) = 8\pi G H_0^2/3$, where $GH_0^2 \ll 1$. As a consequence, we work with a classical background metric $ds^2 = -dt^2 + a^2 dx^2$, extending it to SK geometry $(ds^\pm)^2$ and retaining only terms linear in the quantum component a^q in the SK action $S^+ - S^-$. These terms generate classical time evolution for the scalar field via the Liouville Hamiltonian constraint

$$(H_{L,\text{grav}} + H_{L,\phi})\rho = 0, \quad H_{L,\text{grav}} \simeq -i\dot{a}^c \partial_{a^c} = -i\partial_t. \quad (9)$$

We focus on the SK action of the scalar field in a fixed classical background $a^c = a_0^c(1 + \dots)$, $a_0^c = e^{H_0 t}$, with possible corrections to the exact dS (we slip off the superscript from now on). A detailed treatment is provided in Supplemental Material [78].

We nevertheless allow for eternal inflation with $H_0^2/\epsilon \sim M_{\text{pl}}^2$. In this regime, quantum fluctuations dominate and standard perturbation theory fails. However,

since the background remains classical and adiabatically evolving, we can perform a controlled expansion in $|V - V_0|/V_0 \sim \sqrt{\epsilon}\Delta\phi \ll 1$. The original picture is to think of quantum effects as noise inducing a random walk of the inflationary trajectory [7]. In our paradigm, this picture is made precise by treating the long modes as an open system that defines the trajectory, with short modes as its environment: $\phi = \phi_s + \phi_e$, where $\phi_e(k, t) = \theta(k - \epsilon a_0 H_0)\phi(k, t)$. The step function separation enforces the Markovian approximation [87]. We take the infrared parameter $\epsilon \rightarrow 0$ [7], so that ϕ_s captures zero modes. This realizes a scale separation analogous to Wilsonian EFT, though it goes beyond the standard Wilsonian treatment due to the time dependence of the cutoff $\epsilon a_0 H_0$.

The reduced density matrix of ϕ_s is then computed by tracing out the short modes as the environment, replacing $(x, p) \rightarrow (\phi_s, \Pi_s)$ and $y \rightarrow \phi_e$ in Eq. (5). We have

$$\begin{aligned} S_{L0} &= \int dt d^3x (a^3 \dot{\phi}_s^q \dot{\phi}_s^c - a^3 v'(\phi_s^c) \phi_s^q - a \partial_i \phi_s^q \partial^i \phi_s^c + \mathcal{O}((\phi_s^q)^3)), & e^{iS_{\text{IF}}} &= \int_{\text{BD}} D\phi_e^\pm e^{i(S_{Le0} + S_{\text{int}})} \\ S_{Le0} &= \int dt d^3x (a^3 \dot{\phi}_e^q \dot{\phi}_e^c - a \partial_i \phi_e^q \partial^i \phi_e^c), & S_{\text{int}} &= \int dt d^3x (a^3 \dot{\phi}_s^q \dot{\phi}_e^c - a \partial_i \phi_s^q \partial^i \phi_e^c + a^3 \delta v_{\text{int}}), \end{aligned} \quad (10)$$

where $\delta v_{\text{int}} = v(\phi_s^+) - v(\phi_s^-) - v(\phi^+) + v(\phi^-)$ with $v = V - V_0$. In this Letter, we only retain ϕ_s^q terms up to quadratic order, capturing the leading effects of diffusion and decoherence.

Tracing out the environment leads to an effective theory to order $\mathcal{O}(v^2, (\phi_s^q)^2, \partial_t^3, k^2)$, with $S_L^{\text{eff}} = S_{Lr} + S_{\text{IF}}$ where [89]

$$\begin{aligned} S_{Lr} &= \int dt (a^3 \dot{\phi}_s^q \dot{\phi}_s^c - a^3 v'_r(\phi_s^c) \phi_s^q + \mathcal{O}((\phi_s^q)^3, k^2)), \\ S_{\text{IF}} &= i \int dt a^6 (C_1(\phi_s^c)(\dot{\phi}_s^q)^2 + C_2(\phi_s^c)\dot{\phi}_s^q \phi_s^q + C_3(\phi_s^c)(\phi_s^q)^2 + \mathcal{O}((\phi_s^q)^3, k^2, \partial_t^3, \delta C_{ir})). \end{aligned} \quad (11)$$

The diffusion coefficients C_i are determined by Feynman diagrams involving virtual short modes [90]. $v_r, \delta C_{ir}$ refer to the renormalization effects. The new operators $(\dot{\phi}_s^q)^2, \dot{\phi}_s^q \phi_s^q, (\phi_s^q)^2$ encode the diffusion in the phase space. More importantly, $(\phi_s^q)^2$ term is responsible for the decoherence in the pointer basis $\phi^\pm$. The system undergoes the decoherence in $\phi^\pm$ basis as far as $C_3 > 0$, allowing for possible quantum-to-classical transition [46,59,61,93–96]. However, the actual quantum-to-classical transition should be captured by the phase space decoherence [61,65–67,97,98], where it is relevant to $\mathcal{C} = \det \begin{pmatrix} 2C_1 & C_2 \\ C_2 & 2C_3 \end{pmatrix}$.

Assumptions and Feynman rules: We now detail the approximations entering the EFT (11). We consider two standard assumptions commonly used in open quantum systems [69]: (1) the Born approximation, and (2) the secular approximation. The Born approximation assumes weak system-environment coupling, allowing a perturbative expansion in terms of Feynman diagrams. The secular approximation suppresses fast-oscillating terms relative to

the system's time scale and corresponds to large time-scale separation in the Wilsonian EFT sense. This separation leads to a multipole expansion in time, e.g., $\phi_s(t_2) = \phi_s(t_1) + \partial_{t_1} \phi_s(t_1)(t_2 - t_1) + \dots$, such that the system field acts as an effectively static source of environment and does not enter the t_2 integral, resulting in Markovian Langevin dynamics [54,99]. The Markovian approximation of the density matrix evolution [100] is perturbatively ensured by the step-function separation between the system and the environment, which makes the environment's response and noise local-in-time $\delta(t - t')$ [1,4]. Particularly, as the environment's response is projected to equal times, it identically vanishes in the BD vacuum, $G_R(t, t) = 0$. A similar scale separation exists in spatial momentum k , enabling a multipole expansion in space as well.

We summarize here the relevant Feynman rules used to trace out ϕ_e (with conformal time $\eta H_0 = -e^{-H_0 t}$). We use vertices

$$-\frac{qc}{t} = a^3 \dot{\phi}_s^q \partial_t - a k^2 \phi_s^q, \quad \frac{c}{t} \overset{c}{\underset{q}{\otimes}} = v''(\phi_s^c(t)). \quad (12)$$

The propagator is

$$\begin{aligned} \frac{c}{t_1} \frac{k}{t_2} \frac{c}{t_2} &= \frac{H_0^2 ((\eta_1 \eta_2 k^2 + 1) \cos((\eta_1 - \eta_2)k) + (\eta_1 - \eta_2)k \sin((\eta_1 - \eta_2)k))}{2k^3} \theta\left(k + \frac{\varepsilon}{\eta_1}\right) \theta\left(k + \frac{\varepsilon}{\eta_2}\right), \\ \frac{c}{t_1} \frac{k}{t_2} \frac{q}{t_2} &= -i \frac{H_0^2 ((\eta_1 \eta_2 k^2 + 1) \sin((\eta_1 - \eta_2)k) + (\eta_2 - \eta_1)k \cos((\eta_1 - \eta_2)k))}{k^3} \theta(t_1 - t_2) \theta\left(k + \frac{\varepsilon}{\eta_1}\right) \theta\left(k + \frac{\varepsilon}{\eta_2}\right). \end{aligned} \quad (13)$$

We use a solid line for ϕ_e and dashed line for ϕ_s . Similar Feynman rules are also used in Refs. [50,51].

Feynman diagrams and diffusion coefficients: For example, if we consider an exact dS $a_0 = e^{H_0 t}$, the low-lying relevant diagrams that generate the diffusion up to v^2 are

$$\begin{aligned} &\text{Diagram 1: } \frac{q}{t_1} \frac{c}{t_2} \frac{q}{t_2} + 2 \times \text{Diagram 2: } \frac{q}{t_1} \frac{c}{t_3} \frac{q}{t_2} \frac{c}{t_2} + 2 \times \text{Diagram 3: } \frac{q}{t_1} \frac{c}{t_3} \frac{c}{t_4} \frac{q}{t_2} \frac{c}{t_2} + \text{Diagram 4: } \frac{q}{t_1} \frac{c}{t_3} \frac{c}{t_4} \frac{c}{t_2} \frac{q}{t_2} \end{aligned} \quad (14)$$

The intermediate times (t_3, t_4) are integrated using a multipole expansion, e.g., $v[\phi^c(t_3)] \sim v[\phi^c(t_1)] + \dots$. At leading order, the scale separation in the wavelength sets all momenta equal, $k_1 = k_2 = \dots$, so the “potential” insertions act as static sources. We retain only the leading term in both temporal and spatial multipole expansions, as higher-order terms correspond to higher-dimensional operators in EFT that are not considered in this Letter. For example, the first diagram gives the standard noise contribution $H_0^3/(4\pi^2)$ [7]. We leave the details to Supplemental Material [78].

In addition, many other diagrams exist, which are systematically captured by the standard Wilsonian coarse-graining. These renormalize the potential $v(\phi^c) \rightarrow v_r(\phi^c)$ and the diffusion terms, and generate a tower of higher-dimensional operators in EFT. We neglect these terms (but keep the superscript $v_r(\phi_s^c)$), which corresponds to determining the leading Wilson coefficients by matching relevant diagrams in the full theory [62,63,70].

The diffusion coefficients are

$$\begin{aligned} C_1(\phi_s^c) &= \frac{H_0^3}{8\pi^2} \left[1 + \frac{2}{3H_0^2} v''(\phi_s^c) \log + \frac{2}{9H_0^4} v''(\phi_s^c)^2 \left(\log^2 + \frac{\log}{3} + \frac{\pi^2 - 8}{4} \right) \right], \\ C_2(\phi_s^c) &= \frac{H_0^2}{12\pi^2} v''(\phi_s^c) + \frac{1}{18\pi^2} v''(\phi_s^c)^2 \left(\log + \frac{1}{6} \right), \quad C_3(\phi_s^c) = \frac{H_0}{72\pi^2} v''(\phi_s^c)^2, \end{aligned} \quad (15)$$

where $\log = \log \frac{\varepsilon}{2} - \psi(\frac{3}{2})$. C_i s can also be obtained by treating the short modes as effectively massive, with mass $\sim v''(\phi_s^c)$, as shown in Supplemental Material [78]. Nevertheless, we see $\mathcal{C} = 0$, indicating there is no decoherence in phase space at this order.

We can also include the classical effect of deviation from exact dS, with the scale factor $a = e^{H_0 t} [1 + (8\pi G/6H_0^2) \int^t dt' \delta(\phi_s^c(t'))]$ such that $H = H_0 + (8\pi G/6H_0) \delta(\phi_s^c)$. This deviation is suppressed in $G \rightarrow 0$ and the function $\delta(\phi_s^c)$ depends on the inflationary trajectory. Under our approximations, the deviation generates new Feynman diagrams at leading order in δ and leads to a shift in the diffusion coefficient (setting $8\pi G = 1$) $v''(\phi_s^c) \rightarrow v''(\phi_s^c) - \frac{3}{4} \delta(\phi_s^c)$ up to $\mathcal{O}(v\delta)$. See Supplemental Material [78].

Master equations and the Lindbladian—We can now apply Eq. (7) to derive the master equations for stochastic inflation (11). We find

$$\begin{aligned} H_L^{\text{eff}} &= \frac{\Pi^c \Pi^q}{a^3} + a^3 \phi^q v_r'(\phi^c) \\ &\quad - i[C_1(\phi^c)(\Pi^q)^2 + a^3 C_2(\phi^c) \Pi^q \phi^q + a^6 C_3(\phi^c)(\phi^q)^2]. \end{aligned} \quad (16)$$

(1) Fokker-Planck equation: For the diagonal density matrix, we evaluate the Hamiltonian at the saddle point on the time surface where we evaluate the density matrix, including the deviation from exact dS. In the slow-roll limit we find $\phi_{\text{sad}}^q = \Pi_{\text{sad}}^q = 0$, $\delta = v_r + \mathcal{O}(v_r^2)$ and

$$\Pi_{\text{sad}}^c = -\frac{a^3}{3H(\phi^c)} \left(v_r'(\phi^c) + \frac{v_r'(\phi^c) v_r''(\phi^c)}{[3H(\phi^c)]^2} + \mathcal{O}(v^3) \right), \quad (17)$$

where $H(\phi^c) = H_0 + [\delta(\phi^c)/6H_0] + \mathcal{O}(v^2)$. Applying Eq. (7) leads to a Fokker-Planck equation [101]

$$\partial_t P(\phi, t) = -\partial_\phi [a^{-3} \Pi_{\text{sad}}^c P(\phi, t)] + \partial_\phi^2 (\tilde{C}_1(\phi) P(\phi, t)), \quad (18)$$

where $\tilde{C}_1$ is the modified diffusion coefficient by $\delta(\phi)$. The linear v in Eq. (18) reproduces the original Starobinsky FP equation [7]. Setting $v = \lambda/4\phi^4$ in exact dS $\delta = 0$ in Eq. (18) reproduces the FP equation derived in Refs. [24,70] to the order $\mathcal{O}(\lambda^2, (\phi')^2)$ [104]. We present the equilibrium states and the one-point function $\langle \phi^2 \rangle$ for different potentials in Supplemental Material [78].

(2) Wigner function and the Lindblad equation: We now consider the Wigner function in the stochastic inflation [105], which encodes the decoherence.

We first consider exact dS. Using Eqs. (7) and (16) yields

$$\begin{aligned} \partial_t W(\phi, \Pi, t) = & -a^{-3} \Pi \partial_\phi W(\phi, \Pi, t) + a^3 v'_r(\phi) \partial_\Pi W(\phi, \Pi, t) \\ & + \partial_\phi^2 [C_1(\phi) W(\phi, \Pi, t)] \\ & - a^3 \partial_\phi \partial_\Pi [C_2(\phi) W(\phi, \Pi, t)] \\ & + a^6 \partial_\Pi^2 [C_3(\phi) W(\phi, \Pi, t)]. \end{aligned} \quad (19)$$

We show in Supplemental Material [78] that Eq. (19) implies the FP equation (18).

Equation (19) is essentially a Lindblad equation for the long-wavelength inflaton. We can show Eq. (19) is equivalent to projecting the Lindblad equation $\partial_t \rho = -i[H_r, \rho] + \gamma(L\rho L - \frac{1}{2}\{L^2, \rho\}) = -iH_{Lr}\rho - (\gamma/2)(L^q)^2\rho$ into Wigner representation with the jump operator

$$\begin{aligned} L &= (1 + \alpha)\Pi + \frac{a^3 v''(\phi^c)(1 + \beta)}{3H_0} \phi, \quad \gamma = \frac{H_0^3}{4\pi^2}, \\ \alpha &= \frac{v''(\phi^c)}{3H_0^2} \log + \frac{v''(\phi^c)^2}{108H_0^4} (6\log^2 + 4\log + 3\pi^2 - 24), \\ \beta &= \frac{v''(\phi^c)}{3H_0^2} (1 + 3\log). \end{aligned} \quad (20)$$

Our method can nevertheless systematically go beyond the Lindbladian structure. Including the deviation from dS shifts $v'' \rightarrow v'' - 3/4\delta$ up to $\mathcal{O}(v\sigma)$ with $\delta = v_r$, as shown in the Supplemental Material [78].

Generalization to global slicing—Our previous discussion has been restricted to the flat slicing, as is standard in stochastic inflation [7]. However, the open-system paradigm and our framework can be systematically generalized to other coordinates, such as the global slicing relevant in quantum cosmology [106]. This allows curvature fluctuations large enough to create a sphere Ω_3 , i.e., $ds^2 = -d\tau^2 + (a^2/H_0^2)d\Omega_3^2$, with $a = \cos(H_0\tau)$ for exact dS. This setup captures the physics of earlier times ϕ_0 , when the number of e-folds is not yet large enough to flatten the spatial geometry.

In the global slicing, the field ϕ is decomposed into spherical harmonics as $\phi = \sum_{Jm_1m_2} \phi_\ell Y_{Jm_1m_2}(\Omega)$. The canonical modes and their SK correlators are presented in Supplemental Material [78]. In this setting, short modes

are defined by $J > \epsilon a_0 H_0 \gg 1$, and we trace them out in the limit $\epsilon \ll 1$. Our analysis shows that the resulting diffusion coefficients are consistent with the expectation that short modes remain locally correlated by flat-slice approximation, although the spatial geometry remains as a sphere (see Supplemental Material [78]). For example, at leading order we have $C_1(\phi_s^c, \tau) = (H_0^3/8\pi^2) \tanh(H_0\tau)$.

We now present the leading FP equation in global slicing of exact dS. This is an equation for the global diagonal Hartle-Hawking density matrix [71,75] in minisuperspace $P(\phi, a)$

$$\begin{aligned} & \sqrt{H_0^2 a^2 - 1} \partial_a P(\phi, a) \\ &= \partial_\phi \left(\frac{a v'_r(\phi) P(\phi, a)}{3\sqrt{H_0^2 a^2 - 1}} \right) + \partial_\phi^2 \left(\frac{H_0^2 \sqrt{H_0^2 a^2 - 1}}{8\pi^2 a} P(\phi, a) \right). \end{aligned} \quad (21)$$

In the late-time limit $aH_0 \gg 1$, Eq. (21) reduces to the flat-slice FP equation. Nevertheless, it also shows that the early universe is not in an equilibrium state $\partial_a P = 0$ prior to late times, because of the nontrivial a dependence in diffusion. Conceptually, Eq. (21) should be interpreted as the semiclassical limit of a Lindblad-type extension of the WdW equation [74] in the minisuperspace.

Conclusions—In this Letter, we clarify that stochastic inflation is naturally understood as an open quantum system, with short-wavelength modes acting as the environment. We develop a systematic framework to study its reduced density matrix using the SK formalism. The approximations involved in tracing out the environment are made explicit and can be viewed as a generalization of Wilsonian coarse-graining with a time-dependent scale. We then present a diagrammatic rule for computing the effective theory governing the reduced density matrix, focusing on diffusion terms and allowing for deviations from exact dS. We obtain the corresponding master equation, yielding the FP equation to higher orders in the slow-roll expansion for the diagonal elements, and its phase-space generalization for the Wigner function, which we identify as a Lindblad equation. Finally, we generalize this framework to global dS and derive the corresponding FP equation, which intrinsically does not have an equilibrium state until very late times.

Our analysis confirms that gravitational effective theories can be nonunitary, describing open quantum systems, even if their short-distance completions are closed. On the other hand, our Letter also illustrates that EFT provides a powerful framework for open quantum systems, which could in principle include RG [108,109] and systematic corrections beyond the Lindbladian.

We emphasize that our analysis, particularly the generalization to global dS, initiates a program to find Lindblad equations in quantum gravity, generalizing the WdW equation. This may offer new insights into the measure problem in eternal inflation [16,110].

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