

How Zonal Fields Suppress Reversed Shear Alfvén Eigenmode in Tokamak Plasmas

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(Received 20 November 2025; accepted 26 June 2026; published 21 July 2026)

Understanding nonlinear saturation of reversed-shear Alfvén eigenmodes (RSAEs) in tokamaks is crucial for high-performance burning plasmas. Employing both nonlinear gyrokinetic simulations and theoretical analyses, we have discovered the novel result that, with energetic particle dynamics kept linear, the nonlinear suppression and eventual saturation of RSAE occur via the downward frequency chirping induced by the beat-driven zonal current. More specifically, as the mode frequency chirps downward, there is enhanced mode conversion to radially propagating electron Landau-damped kinetic Alfvén waves; resulting in enhanced convective (radiative) damping and, thereby, its suppression and saturation. Theoretical results are in good agreement with simulations both qualitatively and quantitatively. This Letter, thus, establishes a fundamental framework for zonal-current mediated saturation of Alfvén eigenmodes in burning plasmas.

DOI: [10.1103/hh65-rgwv](https://doi.org/10.1103/hh65-rgwv)

Introduction—In tokamak reactors, achieving high-performance plasma confinement is essential for efficient fusion energy production. Reversed-shear Alfvén eigenmodes (RSAEs) [1–3], ubiquitous in advanced scenarios with nonmonotonic safety factor profiles [4], however, can destabilize the plasma and lead to significant losses of energetic particles (EPs), which are crucial for sustaining fusion reactions. A comprehensive understanding of RSAE dynamics, particularly the mechanisms governing their nonlinear evolution and eventual saturation, is therefore crucial for optimizing plasma performance. A key candidate for mediating this saturation is zonal electromagnetic fields (ZFs), including zonal flows and zonal currents. ZFs are prevalent self-organized phenomena from planetary atmospheres to laboratory plasmas. In tokamaks, these toroidally symmetric, zero-frequency corrugations of plasma equilibria play a fundamental role in regulating microturbulence and mediating cross-scale couplings [5–8]. Understanding how ZFs govern RSAE nonlinear dynamics has been a crucial challenge in fusion plasma physics, and motivated

the present Letter, where we discover a novel nonlinear saturation mechanism based on zonal-current-induced downward frequency chirping. Our gyrokinetic simulations and theoretical analyses show that this chirping enhances mode conversion to electron Landau-damped kinetic Alfvén waves (KAWs) [9,10], resulting in substantially increased convective (radiative) damping. These findings reveal, for the first time, zonal-current-mediated energy channeling as one of the fundamental saturation mechanisms for Alfvén eigenmodes in burning plasmas.

Numerical simulations have established that self-interactions can lead to saturation of EP-driven RSAEs [11–13] via two main routes [14]: one involving nonlinear thermal plasma dynamics [12,15] yielding nonlinear beat-driven ZFs, and the other producing long-lived nonlinear modification of the EP phase-space structures, which are manifestation of EP nonlinear equilibria in the presence of a finite level of fluctuations and are called phase-space zonal structures (PSZS) [16–18]. The first route is typically responsible for nonlinear frequency shift and/or modification of the local current and safety factor profile, thereby enhancing continuum damping [12,15]; while the second route is responsible for the nonlinear modification of the EP driving rate [16–18]. Contrary to conventional expectations, recent simulations and theory [19] reveal that ZFs can enhance the EP drive of RSAEs, rather than suppressing it, suggesting that ZFs-induced suppression primarily takes place through the thermal plasma route. However, present

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understanding of this process relies solely on large-scale numerical simulations, lacking a first-principles theoretical framework capable of predicting the dominant nonlinear effect and its underlying physical mechanisms.

This Letter investigates the roles of ZFs due to the nonlinear dynamics of thermal plasmas only in the suppression and eventual saturation of a single- n RSAE. Nonlinear gyrokinetic simulations of the RSAE interacting with beat-driven ZFs exhibit clear downward frequency chirping, excitation of KAWs, and ultimate saturation. To understand the underlying physics, we formulate the RSAE eigenmode equations by self-consistently including both zonal flow and zonal current in the magnetohydrodynamic (MHD) framework. Analytical investigation reveals zonal current as the dominant mechanism responsible for downward frequency chirping, while zonal flow yields only a minor upward frequency shift. Theory further shows that Alfvén continuum resonant-damping is negligible. As the frequency approaches the shear Alfvén wave continuum, enhanced coupling to radially propagating KAWs introduces significant convective (radiative) damping. To illustrate these processes, we develop an integrated “RSAE-ZF-KAW” model incorporating finite ion Larmor radius (FLR) and electron-Landau damping dynamics, both indispensable for the proper description of KAW propagation and absorption. In order to quantitatively demonstrate this crucial point, we also solve a corresponding “RSAE-ZF-MHD” model, where the shear Alfvén continuous spectrum is considered in the ideal MHD limit; i.e., without KAW dynamics. Numerical solutions of the eigenvalue equations for the RSAE-ZF-MHD and RSAE-ZF-KAW models verify the analytically predicted downward chirping induced by zonal current and quantitatively demonstrate the essential role of KAW radiative damping in RSAE saturation.

Nonlinear gyrokinetic simulations of RSAE—We employ the gyrokinetic toroidal code (GTC) [20] to investigate how ZFs suppress RSAE through thermal plasma nonlinearity. Adopting DIII-D discharge No. 159243 parameters at 805 ms [21] featuring reversed magnetic shear (with minimum safety factor $q_m = 2.947$), we simulate an $n = 4$ RSAE with all poloidal harmonics included, retaining full nonlinearity for thermal plasmas to isolate this route while keeping EP dynamics linear. The simulation setup follows the well-established configuration of Refs. [19,22], utilizing a global field-aligned mesh that resolves kinetic scales, a low-noise δf scheme, and a time step capable of capturing both wave dynamics and electron motion. Electrons are treated with a fluid-kinetic hybrid electron model [23], while ions are fully gyrokinetic.

Our simulations demonstrate, in the present Letter, RSAE saturation mediated solely by thermal plasma nonlinearities. Even with linear EP drive, the mode saturates at finite amplitude [Fig. 1(a)], exhibiting three key signatures of the underlying mechanism: suppression of the nonlinear

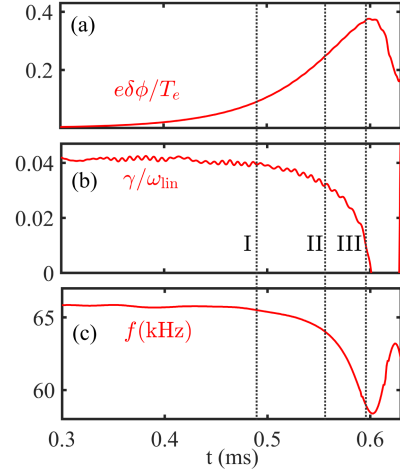


FIG. 1. Saturation of the $n = 4$ RSAE in GTC simulation with nonlinear thermal plasma and linear EP dynamics. Time evolution of (a) normalized electrostatic potential $e\delta\phi/T_e$, (b) normalized growth rate $\gamma/\omega_{\text{lin}}$, and (c) frequency f (kHz) at the initial q_m surface, all for the dominant poloidal harmonic $m = 12$. Vertical lines mark phases I–III; $\omega_{\text{lin}} = 2\pi \times 65.8$ kHz is the linear frequency.

growth rate [Fig. 1(b)], a pronounced downward frequency chirping [Fig. 1(c)], and excitation of KAWs. The KAW excitation is evidenced by a rapid, order-of-magnitude increase in the parallel electric field ratio $E_{\parallel}/E_{\parallel,\text{es}}$ [with $E_{\parallel} = -(\nabla_{\parallel}\delta\phi + \partial_t\delta A_{\parallel}/c)$ and $E_{\parallel,\text{es}} = -\nabla_{\parallel}\delta\phi$], as is quantitatively compared with theoretical predictions in Fig. 5, and by the resulting KAWs propagating radially away from q_m [Fig. 2(a)] with increased radial wave number k_r [Fig. 2(c)]. This mode conversion is triggered as the downward-chirping RSAE frequency approaches the accumulation point at q_m of the shear Alfvén wave continuum [Fig. 2(b)], opening an efficient channel for energy dissipation.

Below, in order to elucidate the underlying physics, we construct first-principles theoretical models to quantify the dominant damping mechanism and distinguish the respective roles of zonal current and zonal flow.

RSAE-ZF-MHD model—We first analyze RSAE dynamics with beat-driven ZFs in the MHD framework, considering a large aspect-ratio tokamak [$\epsilon \equiv r/R \sim \mathcal{O}(10^{-1}) \ll 1$] with circular magnetic surfaces. Following the theoretical approach of Ref. [7], the fluctuating electrostatic potential, $\delta\phi$, and parallel vector potential, $\delta A_{\parallel}$, are decomposed into RSAE components with toroidal mode number n_0 and zonal components

$$\delta\phi_0 = e^{-i\omega_0 t + in_0\xi} \sum_m \Phi_m e^{-im\theta} + \text{c.c.}, \quad \delta\phi_z = \Phi_z(r, t),$$

with analogous expressions for $\delta A_{\parallel}$. Here, the subscripts “0” and “z” denote the RSAE and zonal components; ξ , θ , and r are the toroidal-angle, poloidal-angle, and radial coordinates, respectively.

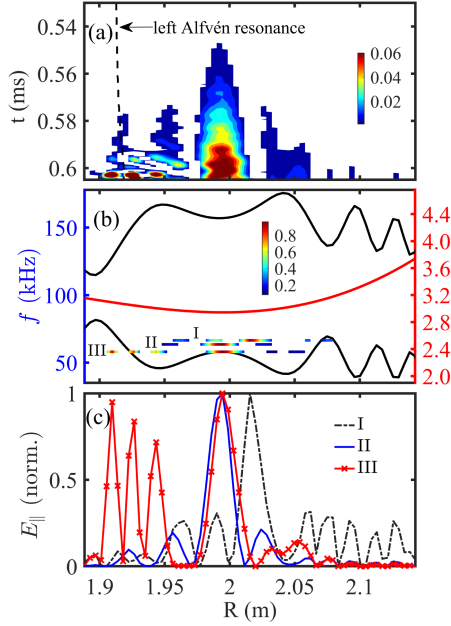


FIG. 2. KAW excitation and RSAE frequency evolution. (a) Time evolution of $E_{\parallel}$ radial profile, with Alfvén resonance inside q_m (dashed). (b) RSAE frequency relative to Alfvén continua (black) from MAS [24]; q profile (red, right axis). Horizontal lines mark phases I–III from GTC simulation (Fig. 1). (c) $E_{\parallel}$ radial profiles at phases I–III (normalized to their respective maxima), showing increasing k_r with downward chirping.

The zonal potentials beat driven by RSAE are derived in [19] and given below by

$$\begin{aligned}\delta\phi_z &= \frac{c}{B_0\omega_0^2} k_{\theta 0} \omega_{*pi} \frac{\partial}{\partial r} (|\delta\phi_0|^2), \\ \delta A_{\parallel z} &= \frac{c^2}{B_0\omega_0^2} k_{\theta 0} \frac{\partial}{\partial r} (k_{\parallel 0} |\delta\psi_0|^2).\end{aligned}\quad (1)$$

Here ω_0 is the RSAE frequency, $k_{\theta 0}$ and $k_{\parallel 0}$ are the poloidal and parallel wave numbers, respectively, $\omega_{*pj} = \omega_{*nj}(1 + \eta_j)$ is the diamagnetic drift frequency for species j , with $\omega_{*nj} = (cT/eB_0)_j (\mathbf{k} \times \mathbf{b}_0) \cdot \nabla \ln N_j$, $\mathbf{b}_0 = \mathbf{B}_0/B_0$, $\eta_j = |\nabla T_j|/|\nabla N_j|$, and $\delta\psi_0 = \omega_0 \delta A_{\parallel 0}/ck_{\parallel 0}$. These expressions are validated by GTC simulations [19].

The governing equations are the nonlinear gyrokinetic equation [25], quasineutrality condition [25], and gyrokinetic vorticity equation [26] (their explicit expressions are given in the End Matter Appendix A). The particle non-adiabatic perturbed distribution function for each particle species j is decomposed into linear $[\delta g_j^{(1)}]$ and nonlinear $[\delta g_j^{(2)}]$ components. For electrons, noting $|\omega_0|, |\omega_{de}|_0 \ll |k_{\parallel} v_{te}|_0$ and $|k_{\perp} \rho_e|_0^2 \ll 1$, we obtain the massless electron responses

$$\begin{aligned}\delta g_{e0}^{(1)} &\simeq -\frac{e}{T_e} F_{Me} \left(1 - \frac{\omega_{*ne}}{\omega}\right)_0 \delta\psi_0, \\ \delta g_{e0}^{(2)} &\simeq -i \frac{c}{B_0} \frac{k_{\theta 0} k_z}{\omega_0} (\delta\psi_0 \delta g_z - \delta\psi_z \delta g_0)_e,\end{aligned}\quad (2)$$

where $\delta\psi_z = \omega_0 \delta A_{\parallel z}/ck_{\parallel 0}$. For simplicity, we set $\eta_e = 0$ in the electron response. Meanwhile, for thermal ions with $|\omega_0| \gg |k_{\parallel} v_{ti}|_0$, we have

$$\begin{aligned}\delta g_{i0}^{(1)} &\simeq \frac{e}{T_i} F_{Mi} \left(1 - \frac{\omega_{*i}}{\omega}\right)_0 J_0 \delta\phi_0 \left(1 + \frac{\omega_{di}}{\omega}\right)_0, \\ \delta g_{i0}^{(2)} &\simeq i \frac{c}{B_0} \frac{k_{\theta 0} k_z}{\omega_0} (J_z \delta\phi_z \delta g_0 - J_0 \delta\phi_0 \delta g_z)_i.\end{aligned}\quad (3)$$

Here, $\omega_{*i} = \omega_{*ni}[1 + \eta_i(v^2/v_{ti}^2 - 3/2)]$ with v_{ti} being the ion thermal velocity, $\omega_{dj} = \mathbf{k}_{\perp} \cdot \mathbf{v}_{dj}$, with $\mathbf{v}_{dj} = \mathbf{b}_0 \times (\mu \nabla B_0 + v_{\parallel}^2 \boldsymbol{\kappa})_j / \Omega_j$, $\mu = v_{\perp}^2/2B_0$, $\boldsymbol{\kappa} = \mathbf{b}_0 \cdot \nabla \mathbf{b}_0$, and $\Omega_j = e_j B_0/m_j c$. The quantities δg_{ez} and δg_{iz} , respectively, denote the electron and ion responses to the ZFs as well as beat driven by RSAE derived in Ref. [19] (their explicit expressions are given in Appendix B). Substituting the δg_j 's expressions given by Eqs. (2) and (3) into the quasineutrality condition, Eq. (A2), yields the nonlinear Ohm's law [7]

$$\delta\phi_0 - \delta\psi_0 \simeq i \frac{c}{B_0} \frac{k_{\theta 0} k_z}{\omega_0} \delta\phi_0 \left(\delta\phi_z - \frac{\omega_0 \delta A_{\parallel z}}{ck_{\parallel 0}} \right). \quad (4)$$

Equation (4) describes the ZF-induced finite parallel electric field, and its derivations are given in the Appendix A. In this MHD description, the FILR corrections ($\propto k_{\perp}^2 \rho_i^2$) to Eq. (4) are neglected; they will be incorporated when we consider the mode conversion to KAWs [9,10].

The other field equation, the gyrokinetic vorticity equation [Eq. (A3)] [26], which combines the $\nabla \cdot \delta \mathbf{J} = 0$ and parallel Ampère's law $\nabla_{\perp}^2 \delta A_{\parallel} = -4\pi \delta J_{\parallel}/c$, can be cast as

$$\nabla \cdot \delta \mathbf{J}_k^{(1)} + \nabla \cdot \delta \mathbf{J}_k^{(2)} = 0, \quad (5)$$

where $\nabla \cdot \delta \mathbf{J}_0^{(1)}$ is the well-known linear response for the RSAE [26]:

$$\begin{aligned}\nabla \cdot \delta \mathbf{J}_0^{(1)} &= \frac{\delta \mathbf{B}_{\perp 0}}{B_0} \cdot \nabla J_{\parallel 0} - ik_{\parallel 0} \left(\frac{c}{4\pi} \right) \nabla_{\perp}^2 \delta A_{\parallel 0} + i \frac{c^2}{4\pi} \nabla_{\perp} \cdot \frac{(\omega_0 - \omega_{*pi})}{v_A^2} \nabla_{\perp} \delta\phi_0 - i \frac{c^2}{4\pi} \nabla_{\perp} \cdot \frac{(1 - \omega_{*pi}/\omega_0) \omega_{\text{gam}}^2}{v_A^2} \nabla_{\perp} \delta\phi_0 + \lambda_a,\end{aligned}\quad (6)$$

where the terms represent, respectively, the kink drive, field-line bending, ion inertia including diamagnetic

drift, and the last two curvature-pressure couplings terms, with $\lambda_a = -(c/B_0)\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla [\sum_j \langle (em/T)F_M \rangle_j (\omega_{*j}/\omega_0) [(v_\perp^2/2) + v_{||}^2]_v]_s \delta\phi_0$ representing the favorable surface-averaged curvature [27]. Here, $\langle \dots \rangle_v$ and $[\dots]_s$ denote velocity-space integration and the flux-surface averaging, respectively, and $\omega_{\text{gam}}^2 = (v_{ti}/R)^2(\tau + 7/4)$ is the geodesic-acoustic frequency [28–30] with $\tau = T_e/T_i$. For clarity, Eq. (6) retains only the reactive components, neglecting wave-particle resonances in the present analysis.

$\nabla \cdot \delta \mathbf{J}_k^{(2)}$ in Eq. (5) corresponds to nonlinear contributions due to the Maxwell stress and gyrokinetic Reynolds stress [26]; that is, for RSAE,

$$\nabla \cdot \delta \mathbf{J}_0^{(2)} = \frac{c^3 k_{\theta 0} k_z}{4\pi B_0 v_A^2} (k_z^2 - k_{\perp 0}^2) \left[\left(\frac{k_{||0} v_A}{\omega_0} \right) \left(\frac{v_A}{c} \right) \delta A_{||z} - \left(1 - \frac{\omega_{*pi}}{\omega_0} \right) \delta \phi_z \right] \delta \phi_0. \quad (7)$$

The derivations of Eqs. (6) and (7) are detailed in Appendix C.

Combining Eqs. (4)–(7) yields the “RSAE-ZF-MHD” eigenmode equation (see End Matter IV for details):

$$\nabla_\perp \cdot (\epsilon_A \nabla_\perp \delta \phi_0) + \Lambda \delta \phi_0 + \nabla \cdot [(\nabla \alpha_{\phi 2}) \delta \phi_0] = 0, \quad (8)$$

where $\epsilon_A = \epsilon_{A0} + \alpha_{\phi 1} + \alpha_A$, and

$$\begin{aligned} \epsilon_{A0} &= [(1 - \omega_{*pi}/\omega_0)(\omega_0^2 - \omega_{\text{gam}}^2)] - k_{||0}^2 v_A^2, \\ \alpha_{\phi 1} &= \omega_0 \frac{c}{B_0} k_{\theta 0} [(k_{||0} v_A/\omega_0)^2 + 1 - \omega_{*pi}/\omega_0] (\partial \delta \phi_z / \partial r), \\ \alpha_A &= -2\omega_0 \frac{c}{B_0} k_{\theta 0} (k_{||0} v_A/\omega_0) (v_A/c) (\partial \delta A_{||z} / \partial r), \\ \alpha_{\phi 2} &= \omega_0 \frac{c}{B_0} k_{\theta 0} [(k_{||0} v_A/\omega_0)^2 - 1 + \omega_{*pi}/\omega_0] (\partial \delta \phi_z / \partial r). \end{aligned} \quad (9)$$

Here, Λ incorporates effects from favorable average curvature and toroidal couplings [27,31].

Variational analysis—We perform a variational analysis of the “RSAE-ZF-MHD” eigenmode equation, Eq. (8), for a RSAE localized near $r = r_m$, using the normalized radial coordinate $\zeta = k_{\theta 0}(r - r_m)$ with $k_{\theta 0} = m_0/r_m$. Introducing $\psi = \epsilon_A^{1/2} \delta \phi_0$, Eq. (8) leads to the variational form

$$\mathcal{L}[\psi] = \frac{1}{2} \int_{-\infty}^{\infty} d\zeta \left\{ - \left| \frac{d\psi}{d\zeta} \right|^2 + |\psi|^2 \left[-1 + \frac{1}{4} \left(\frac{\hat{\epsilon}'_A}{\hat{\epsilon}_A} \right)^2 - \frac{1}{2} \left(\frac{\hat{\epsilon}''_A}{\hat{\epsilon}_A} \right) + \frac{\hat{\Lambda}}{\hat{\epsilon}_A} + \frac{1}{2} \frac{\hat{\alpha}''_{\phi 2}}{\hat{\epsilon}_A} \right] \right\}, \quad (10)$$

where $\hat{\epsilon}_A = \hat{\epsilon}_{A0} + \hat{\alpha}_{\phi 1} + \hat{\alpha}_A$, $\hat{\epsilon}_{A0} = (1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{\text{gam}}^2) - K_{||0}^2$, $\Omega_0 = \omega_0/\omega_A$ with $\omega_A = v_A/(q_m R)$ and same normalization adopted for other frequency variables,

$K_{||0} = (n_0 q - m_0)$, $\hat{\Lambda} = \Lambda/(k_{\theta 0} \omega_A)^2$, $g' \equiv dg/d\zeta$, $g'' \equiv d^2g/d\zeta^2$, and all other hatted quantities are normalized by ω_A^2 .

Considering first the case without ZFs. For $n_0 q_m \gg 1$, the RSAE is strongly localized around $\zeta = 0$, allowing the approximation $K_{||0} \simeq K_m + \frac{1}{2}(S^2/n_0 q_m)\zeta^2$, with $K_m = (n_0 q_m - m_0) < 0$ and $S^2 \equiv r_m^2(d^2q/dr^2)/q_m$. This yields $\epsilon_{A0} \simeq [|K_m|S^2/n_0 q_m](\zeta^2 + \zeta_0^2)$ with $\zeta_0^2 = [(1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{\text{gam}}^2) - K_m^2](n_0 q_m)/(S^2|K_m|)$. Using the trial function $\psi_t = \exp(-|\zeta|)$, Eq. (10) then readily yields the linear RSAE variational dispersion relation without ZFs:

$$\zeta_0 = (\pi/2)[\hat{\Lambda} n_0 q_m / (|K_m|S^2) - 1/2], \quad (11)$$

in agreement with previous analytic results [27,32].

Continuum damping can be incorporated by evaluating the Alfvén resonant absorption near $\zeta = \pm \zeta_2$, where $\epsilon_{A0} = 0$. Near the resonance, $\epsilon_{A0} \simeq -[(1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{\text{gam}}^2)]^{1/2}(S^2/n_0 q_m)(\zeta^2 - \zeta_2^2)$ with $\zeta_2^2 = (2n_0 q_m/S^2)\{[(1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{\text{gam}}^2)]^{1/2} + |K_m|\}$. The modified dispersion relation including continuum resonant-absorption damping becomes

$$\zeta_0(1 + i\delta_c) = (\pi/2)[\hat{\Lambda} n_0 q_m / (|K_m|S^2) - 1/2], \quad (12)$$

where $\delta_c = (\pi/2) \exp(-2\zeta_2)$ is the continuum damping rate, and $\text{Re}(\zeta_0), \text{Re}(\zeta_2) > 0$. This result, which recovers established analytical theory [27], shows that δ_c is typically negligible due to the exponential factor [33].

We now extend the same approach to include ZFs. Introducing $\alpha_z = \hat{\alpha}_{\phi 1} + \hat{\alpha}_A$ and expanding around $\zeta = 0$, we obtain $\hat{\epsilon}_A = \Delta_z(\zeta^2 + \zeta_{0z}^2)$ with

$$\begin{aligned} \Delta_z &= \frac{1}{(n_0 q_m)^2} \left[|K_m| n_0 \left(r^2 \frac{d^2 q}{dr^2} \right)_m + \frac{1}{2} \left(r^2 \frac{d^2 \alpha_z}{dr^2} \right)_m \right], \\ \zeta_{0z}^2 &= [(1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{\text{gam}}^2) - K_m^2 + \alpha_{zm}]/\Delta_z, \end{aligned} \quad (13)$$

where the subscript “m” denotes evaluation at $r = r_m$. Using the same trial function $\psi_t = \exp(-|\zeta|)$ yields the RSAE-ZFs variational dispersion relation:

$$\zeta_{0z}(1 + i\delta_c) = (\pi/2) \left(\hat{\Lambda}_z / \Delta_z - 1/2 \right), \quad (14)$$

with $\hat{\Lambda}_z = \hat{\Lambda} + (d^2 \hat{\alpha}_{\phi 2} / d\zeta^2)_m / 2$.

The nonlinear terms in Eqs. (13) and (14) can be evaluated explicitly using the beat-driven ZFs from Eq. (1). For α_{zm} , we have

$$\begin{aligned} \alpha_{zm} &= -2 \frac{c^2}{B_0^2 \omega_A^2} \left(\frac{n_0 q}{r} \right)_m^4 \left\{ \left(\frac{K_{||0}}{\Omega_0} \right)^2 - \frac{1}{2} \frac{\Omega_{*pi}}{\Omega_0} \left[\left(\frac{K_{||0}}{\Omega_0} \right)^2 + 1 - \frac{\Omega_{*pi}}{\Omega_0} \right] \right\}_m \frac{\partial^2 |\delta \phi_0|^2}{\partial \zeta^2} \Big|_m, \end{aligned} \quad (15)$$

while $d^2\hat{\alpha}_{\phi 2}/d\zeta^2|_m$ is obtained analogously:

$$\begin{aligned} \frac{d^2\hat{\alpha}_{\phi 2}}{d\zeta^2}\Big|_m &= \frac{c^2}{B_0^2\omega_A^2} \left(\frac{n_0 q}{r}\right)_m \frac{\Omega_{*pi}}{\Omega_0} \left[\left(\frac{K_{||0}}{\Omega_0}\right)^2 - 1 \right. \\ &\quad \left. + \frac{\Omega_{*pi}}{\Omega_0} \right] \frac{\partial^4 |\delta\phi_0|^4}{\partial \zeta^4}\Big|_m. \end{aligned} \quad (16)$$

For a mode peaking at $\zeta = 0$, we have $\partial^2 |\delta\phi_0|^2 / \partial \zeta^2 < 0$. Under the typical ordering $|\Omega_0| \approx |K_m| > |\Omega_{*pi}|$, Eq. (15) gives $\alpha_{zm} > 0$. This positive α_{zm} lowers the SAW continuum accumulation frequency at r_m via $(1 - \Omega_{*pi}/\Omega_0)(\Omega_0^2 - \Omega_{gam}^2) = K_m^2 - \alpha_{zm}$, an effect dominated by zonal current ($\delta A_{||z} \propto K_{||0}$); zonal flow ($\delta\phi_z \propto \Omega_{*pi}$) alone would give an opposite smaller upward frequency shift. Similarly, $|d^2\hat{\alpha}_{\phi 2}/d\zeta^2| \propto |\Omega_{*pi}|$ gives a subdominant small correction.

This analysis predicts that ZF-induced downward frequency chirping shifts the RSAE frequency toward the accumulation point of the continuum spectrum, increasing the effective radial wave number k_r and, thereby, enhancing mode conversion to KAWs. To self-consistently capture this physics, we extend our theoretical framework to include the KAW dynamics and electron Landau damping.

RSAE-ZF-KAW model—For the coupled RSAE-KAW system where electron Landau resonance becomes relevant ($|\omega_0| \sim |k_{||0}v_{te}|$), the nonadiabatic electron response is given by

$$\delta g_e \simeq -\frac{e}{T_e} F_{Me} \left(1 - \frac{\omega_{*ne}}{\omega_0}\right) \left(\delta\psi_0 - \frac{\omega_0 \delta\phi_{||}}{k_{||0}v_{te} - \omega_0}\right), \quad (17)$$

where $\delta\phi_{||} \equiv \delta\phi_0 - \delta\psi_0$. The ion response remains unchanged from the MHD description and is given by $\delta g_{i0}^{(1)}$ in Eq. (3). Substituting these responses into the quasineutrality condition, Eq. (A2), yields $\delta\psi_0 = \hat{\sigma}_k \delta\phi_0$, where $\hat{\sigma}_k = 1 + \tau(1 - \Gamma_0)(1 - \omega_{*pi}/\omega_0)/[(1 - \omega_{*ne}/\omega_0)(1 + \zeta_e Z(\zeta_e))]$ incorporates FILR effects via $\Gamma_0 = I_0(b_i) \exp(-b_i)$ and electron kinetics via the plasma dispersion function $Z(\zeta_e)$, with $\zeta_e = \omega_0/(|k_{||0}v_{te}|)$. Here, I_0 is the zeroth-order modified Bessel function, and $b_i = k_{\perp}^2 \rho_i^2$.

Assuming $|k_{\perp}\rho_i|^2 < 1$, the nonlinear gyrokinetic vorticity equation, Eq. (A3), then yields, after some straightforward manipulations, the fourth-order eigenmode equation for the RSAE-ZF-KAW model:

$$\begin{aligned} \rho_i^2 \nabla_{\perp}^2 (1 - \omega_{*pi}/\omega_0) \left\{ \frac{3}{4} (\omega_0^2 - \omega_{gam}^2) \right. \\ \left. + \tau k_{||}^2 v_A^2 / [(1 - \omega_{*ne}/\omega_0)(1 + \zeta_e Z(\zeta_e))] \right\} \nabla_{\perp}^2 \delta\phi_0 \\ + \nabla_{\perp} \cdot (\epsilon_A \nabla_{\perp} \delta\phi_0) + \Lambda \delta\phi_0 + \nabla \cdot [(\nabla \alpha_{\phi 2}) \delta\phi_0] = 0. \end{aligned} \quad (18)$$

This extended model self-consistently describes the coupling between RSAEs, ZFs, and KAWs, incorporating both FILR effects and electron Landau damping. The FILR terms not only regularize the singularity at $\epsilon_A = 0$ present in the MHD description, Eq. (8), but also capture the essential physics of mode conversion from RSAEs to KAWs. Thereby, the model provides a foundation for analyzing the enhanced radiative damping responsible for the nonlinear suppression and eventual saturation of the RSAE.

Numerical validations—We present numerical solutions of the RSAE-ZF-MHD and RSAE-ZF-KAW models to quantitatively validate theoretical predictions and distinguish the roles of zonal flow and zonal current, with direct comparison to those of gyrokinetic simulations.

Eigenmode solutions confirm the theoretical prediction of zonal current dominance. When the both zonal components are included, the RSAE-ZF-MHD model exhibits clear downward chirping [Fig. 3(a), blue]. Artificially removing zonal current reverses this trend, yielding a smaller upward frequency chirping [Fig. 3(b), blue]; thus, demonstrating that zonal current dominates the nonlinear frequency shift. In both cases, continuum damping remains negligible, of the order of 10^{-4} to 10^{-3} [Figs. 3(a) and 3(b), red], in excellent agreement with theoretical expectations.

Radiative damping via KAWs is identified as the primary mechanism by which ZFs yield RSAE saturation. This conclusion is supported by three key findings from the RSAE-ZF-KAW model: (1) it reproduces the downward frequency chirping and yields a radiative damping rate nearly two orders of magnitude stronger than MHD continuum damping [Fig. 4(a)]; (2) it predicts the development of fine-scale radial structures in the perturbed electrostatic potential [Fig. 4(b)], confirming the increase in k_r predicted by the eigenmode theory and in agreement with GTC observations [Figs. 2(a) and 2(c)]; (3) most

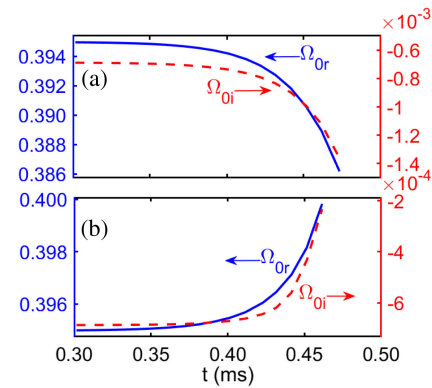


FIG. 3. (a) Time evolution of normalized frequency $\Omega_{0r} = \text{Re}(\omega_0)/\omega_A$ and damping rate $\Omega_{0i} = \text{Im}(\omega_0)/\omega_A$ from the RSAE-ZF-MHD model with zonal flow and zonal current. (b) Same with zonal flow only. Here Ω_{0i} corresponds to the MHD continuum resonant absorption; electron Landau damping and EP destabilizing and damping effects are not included.

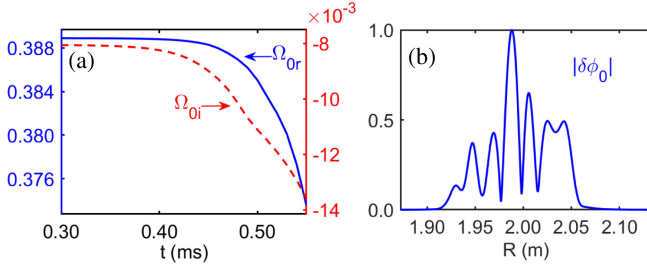


FIG. 4. (a) Time evolution of Ω_{0r} and Ω_{0i} from the RSAE-ZF-KAW model with zonal flow and zonal current. Here Ω_{0i} includes radiative (convective) damping from KAW mode conversion. (b) Radial profile of normalized $|\delta\phi_0|$ at $t = 0.55$ ms.

compellingly, it captures a rapid, order-of-magnitude increase in $E_{\parallel}/E_{\parallel,es}$ (Fig. 5, red squares), in excellent agreement, again, with GTC simulations both qualitatively and quantitatively (Fig. 5, black). This enhancement is a clear signature of KAW excitation, which the RSAE-ZF-MHD model (Fig. 5, blue circles) fails to describe as the RSAE merges into the Alfvén continuous spectrum. In this case, the ZF contribution to $E_{\parallel}$ remains over an order of magnitude smaller throughout, confirming KAW generation as the primary mechanism responsible for the strong enhancement of $E_{\parallel}$.

Conclusions and discussions—This Letter establishes a comprehensive physical picture of RSAE suppression and saturation via the thermal plasma nonlinearity route. Combining gyrokinetic simulations and analytical theory, we find that, in contrast to previous theoretical expectations emphasizing zonal flow effects [5–7] or continuum damping [12,15], the nonlinear frequency chirping is dominated by zonal current rather than zonal flow, and energy dissipation is governed by convective (radiative) damping via mode-converted KAWs rather than continuum damping. The developed RSAE-ZF-KAW model qualitatively and quantitatively captures this physics and provides a crucial extension of the current GTC simulation framework by incorporating the full electron kinetics necessary for describing long-time global KAW dynamics. We emphasize that this work isolates

the thermal-plasma nonlinearity route by keeping EP dynamics linear; the complementary role of EP nonlinearities (which can generate beat-driven zonal fields and induce frequency chirping [21,34]), zonal fields from modulational instability [7], and the parametric regime in which each nonlinearity dominates will be addressed elsewhere.

Our Letter suggests that one may expect the processes described to be generally applicable to Alfvén eigenmode dynamics. They not only illuminate the key ambiguity regarding the role of zonal electromagnetic fields in the suppression and saturation of Alfvénic fluctuations, but also unveil a novel energy transfer channel—from unstable EP-driven-eigenmodes to damped KAWs, mediated by zonal current. This paradigm, centered on zonal-current-induced frequency chirping and subsequent enhanced radiative damping, provides a fundamental framework for understanding Alfvén eigenmode nonlinear evolution in toroidal plasmas. Furthermore, the identified energy channeling process opens up new avenues for future research into alpha-particle transport in burning plasmas.

Acknowledgments—This work was supported by the National key R&D Program of China (No. 2022YFE03040002 and No. 2024YFE03170000), the CNNC Basic Research Program (No. CNNC-JCKY-202640), the Italian Ministry of Foreign Affairs (No. CN23GR02), the Natural Science Foundation of Sichuan (No. 2025ZNSFSC0064), the Strategic Priority Research Program of the Chinese Academy of Sciences (CAS) (No. XDB0500302), and the startup funding of the Institute of Physics, CAS (No. E3CB031R21). Part of this work was carried out within the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No. 101052200-EUROfusion).

Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

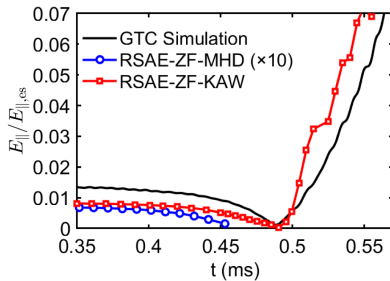


FIG. 5. Quantitative comparison of the evolution of $E_{\parallel}/E_{\parallel,es}$: GTC Simulation (black), RSAE-ZF-MHD model [Eq. (8), blue circles, scaled $\times 10$], and RSAE-ZF-KAW model [Eq. (18), red squares]. The GTC result is evaluated at the initial q_m surface for $m = 12$.

- [1] H. Kimura, Y. Kusama, M. Saigusa, G. J. Kramer, K. Tobita, M. Nemoto, T. Kondoh, T. Nishitani, O. D. Costa, T. Ozeki *et al.*, *Nucl. Fusion* **38**, 1303 (1998).
- [2] S. E. Sharapov, D. Testa, B. Alper, D. N. Borba, A. Fasoli, N. C. Hawkes, R. F. Heeter, M. Mantsinen, and M. G. V. Hellermann (The EFDA-JET Work Programme Collaboration), *Phys. Lett. A* **289**, 127 (2001).
- [3] R. Nazikian, G. J. Kramer, C. Z. Cheng, N. N. Gorelenkov, H. L. Berk, and S. E. Sharapov, *Phys. Rev. Lett.* **91**, 125003 (2003).
- [4] P. Liu, X. Wei, Z. Lin, G. Brochard, G. J. Choi, W. W. Heidbrink, J. H. Nicolau, and G. R. McKee, *Phys. Rev. Lett.* **128**, 185001 (2022).
- [5] L. Chen, Z. Lin, R. White, and F. Zonca, *Nucl. Fusion* **41**, 747 (2001).

- [6] P. H. Diamond, S. I. Itoh, K. Itoh, and T. S. Hahm, *Plasma Phys. Controlled Fusion* **47**, R35 (2005).
- [7] L. Chen and F. Zonca, *Phys. Rev. Lett.* **109**, 145002 (2012).
- [8] J. Li and Y. Kishimoto, *Phys. Rev. Lett.* **89**, 115002 (2002).
- [9] A. Hasegawa and L. Chen, *Phys. Rev. Lett.* **35**, 370 (1975).
- [10] A. Hasegawa and L. Chen, *Phys. Fluids* **19**, 1924 (1976).
- [11] Y. Chen, G. Fu, C. Collins, S. Taimourzadeh, and S. Parker, *Phys. Plasmas* **25**, 032304 (2018).
- [12] T. Wang, S. Wei, S. Briguglio, G. Vlad, F. Zonca, and Z. Qiu, *Plasma Sci. Technol.* **26**, 053001 (2024).
- [13] P. Liu, X. Wei, Z. Lin, G. Brochard, G. Choi, and J. Nicolau, *Rev. Mod. Plasma Phys.* **7**, 15 (2023).
- [14] L. Chen and F. Zonca, *Phys. Plasmas* **20**, 055402 (2013).
- [15] S. Wei, T. Wang, N. Chen, and Z. Qiu, *J. Plasma Phys.* **87**, 905870505 (2021).
- [16] F. Zonca, L. Chen, S. Briguglio, G. Fogaccia, G. Vlad, and X. Wang, *New J. Phys.* **17**, 013052 (2015).
- [17] F. Zonca, L. Chen, M. V. Falessi, and Z. Qiu, *J. Phys. Conf. Ser.* **1785**, 012005 (2021).
- [18] M. V. Falessi, L. Chen, Z. Qiu, and F. Zonca, *New J. Phys.* **25**, 123035 (2023).
- [19] L. Chen, P. Liu, R. Ma, Z. Lin, Z. Qiu, W. Wang, and F. Zonca, *Nucl. Fusion* **65**, 016018 (2025).
- [20] Z. Lin, T. Hahm, W. Lee, W. Tang, and R. White, *Science* **281**, 3 (1998).
- [21] C. S. Collins, W. W. Heidbrink, M. E. Austin, G. J. Kramer, D. C. Pace, C. C. Petty, L. Stagner, M. A. Van Zeeland, R. B. White, and Y. B. Zhu (DIII-D Team Collaboration), *Phys. Rev. Lett.* **116**, 095001 (2016).
- [22] P. Liu, X. Wei, Z. Lin, W. Heidbrink, G. Brochard, G. Choi, J. Nicolau, and W. Zhang, *Nucl. Fusion* **64**, 076007 (2024).
- [23] Z. Lin and L. Chen, *Phys. Plasmas* **8**, 1447 (2001).
- [24] J. Bao, W. Zhang, D. Li, Z. Lin, G. G. Dong, C. Liu, H. Xie, G. Meng, J. Cheng, C. Dong *et al.*, *Nucl. Fusion* **63**, 076021 (2023).
- [25] E. A. Frieman and L. Chen, *Phys. Fluids B* **25**, 502 (1982).
- [26] L. Chen and F. Zonca, *Rev. Mod. Phys.* **88**, 015008 (2016).
- [27] F. Zonca, S. Briguglio, L. Chen, S. Dettrick, G. Fogaccia, D. Testa, and G. Vlad, *Phys. Plasmas* **9**, 4939 (2002).
- [28] N. Winsor, J. Johnson, and J. Dawson, *Phys. Fluids* **11**, 2448 (1968).
- [29] A. D. Turnbull, E. J. Strait, W. W. Heidbrink, M. S. Chu, H. H. Duong, J. W. Greene, L. L. Lao, T. S. Taylor, and S. J. Thompson, *Phys. Fluids B* **5**, 2546 (1993).
- [30] F. Zonca, L. Chen, and R. A. Santoro, *Plasma Phys. Controlled Fusion* **38**, 2011 (1996).
- [31] L. Yu, G.-Y. Fu, and Z.-M. Sheng, *Phys. Plasmas* **16**, 072505 (2009).
- [32] H. L. Berk, R. Mett, and D. Lindberg, *Phys. Fluids B* **5**, 3969 (1993).
- [33] W. Deng, Z. Lin, I. Holod, Z. Wang, Y. Xiao, and H. Zhang, *Nucl. Fusion* **52**, 043006 (2012).
- [34] W. W. Heidbrink, C. S. Collins, M. Podestà, G. J. Kramer, D. C. Pace, C. C. Petty, L. Stagner, M. A. Van Zeeland, R. B. White, and Y. B. Zhu, *Phys. Plasmas* **24**, 056109 (2017).

End Matter

Appendix A: Nonlinear gyrokinetic formalism—The nonlinear gyrokinetic equation for the perturbed distribution function δg_j of species j ($j = i, e$) is [25]

$$\begin{aligned} & (\partial_t + v_{\parallel} \mathbf{b}_0 \cdot \nabla + \mathbf{v}_d \cdot \nabla) \delta g_{jk} \\ &= i \left(\frac{e}{m} \right)_j Q F_{Mj} \langle \delta L_k \rangle \\ & - \left(\frac{c}{B_0} \right) \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \mathbf{b}_0 \cdot (\mathbf{k}'' \times \mathbf{k}') \langle \delta L_{k'} \rangle \delta g_{jk''}, \quad (\text{A1}) \end{aligned}$$

with $Q F_{Mj} = (i \partial_t \partial / \partial \varepsilon + \hat{\omega}_{*k}) F_{Mj}$, $\varepsilon = v^2/2$, $\hat{\omega}_{*k} F_{Mj} = -i(\mathbf{b}_0 / \Omega_j \times \nabla F_{Mj}) \cdot \nabla$, $\delta L_k = (\delta \phi - v_{\parallel} \delta A_{\parallel} / c)_k$, and

$\langle \delta L_k \rangle = J_k \delta L_k$, $J_k = J_0(k_{\perp} \rho_j)$, J_0 the Bessel function, $\rho_j = v_{\perp} / \Omega_j$.

The quasineutrality condition is [25]

$$\frac{N_0 e^2}{T_e} \left(1 + \frac{T_e}{T_i} \right) \delta \phi_k = \sum_j \langle e_j J_k \delta g_{jk} \rangle_v, \quad (\text{A2})$$

with $\langle \dots \rangle_v$ denoting integration in velocity space. The nonlinear vorticity equation is [26]

$$\begin{aligned} & B_0 \left(\nabla_{\parallel} + \frac{\delta \mathbf{B}_{\perp}}{B_0} \cdot \nabla \right) \left(\frac{\delta J_{\parallel}}{B_0} \right) - \nabla \cdot \sum_j \left\langle \left(\frac{e^2}{m} \frac{2\mu}{\Omega^2} B_0 \frac{\partial F_{Mj}}{\partial \varepsilon} \right) \left(\frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \nabla_{\perp} \partial_t \delta \phi - \sum_j e c \mathbf{b}_0 \times \nabla \left\langle \frac{2\mu}{\Omega^2} F_{Mj} \left(\frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \\ & \cdot \nabla \nabla_{\perp}^2 \delta \phi + \sum e \nabla_{\perp} \cdot \langle \mathbf{v}_d J_0 \delta g_j \rangle_v + \delta \mathbf{B}_{\perp} \cdot \nabla \left(\frac{J_{\parallel 0}}{B_0} \right) + \sum_j e \left\langle J_0 \left[\frac{c}{B_0} \mathbf{b}_0 \times \nabla (J_0 \delta \phi) \cdot \nabla \delta g_j \right] \right. \\ & \left. - \frac{c}{B_0} \mathbf{b}_0 \times \nabla \delta \phi \cdot \nabla (J_0 \delta g_j) \right\rangle_v = 0, \quad (\text{A3}) \end{aligned}$$

with $\lambda^2 = 2\mu B_0 k_\perp^2 / \Omega$. These three equations form the foundation of our analysis.

Appendix B: Responses to ZFs and derivation of nonlinear Ohm's law—The explicit expressions of δg_{ez} and δg_{iz} , which appear in Eqs. (2) and (3) are derived in Ref. [19] and given below.

For electrons, with trapped and circulating populations:

$$\begin{aligned}\delta g_{ez,t}^{(1)} &= -\frac{e}{T_e} F_{Me} \delta \phi_z, \\ \delta g_{ez,t}^{(2)} &\simeq \frac{c}{B_0} \frac{e}{T_e} F_{Me} \frac{1}{\omega_0^2} \frac{\partial}{\partial r} [(k_\theta \omega_{*e})_0 |\delta \phi_0|^2], \\ \delta g_{ez,c}^{(1)} &= -\frac{e}{T_e} F_{Me} (\delta \phi - \bar{v}_\parallel \delta A_\parallel / c)_z, \\ \delta g_{ez,c}^{(2)} &\simeq \frac{c}{B_0} \frac{e}{T_e} F_{Me} \frac{1}{\omega_0^2} \frac{\partial}{\partial r} [(k_\theta \omega_{*e} - \bar{v}_\parallel k_\theta k_\parallel)_0 |\delta \phi_0|^2],\end{aligned}\quad (\text{B1})$$

where $\bar{v}_\parallel$ is the transit-averaged parallel velocity.

For ions, noting $|\omega_0| \gg |k_{\parallel 0} v_{ti}|$ for RSAE, we ignore the $(v_\parallel \delta A_\parallel / c)_k$ contributions from RSAE in the nonlinear response [19]:

$$\begin{aligned}\delta g_{iz,t}^{(1)} &\simeq \left(\frac{e F_{Mi}}{T_i} \right) J_z \mathcal{J}_{z0} (\mathcal{J}_{z0} \delta \phi - \mathcal{J}_{z1} \delta A_\parallel)_z, \\ \delta g_{iz,c}^{(1)} &\simeq \left(\frac{e F_{Mi}}{T_i} \right) \mathcal{J}_{z0}^2 J_z (\delta \phi - \bar{v}_\parallel \delta A_\parallel / c)_z, \\ \delta g_{iz}^{(2)} &\simeq -\frac{c}{B_0} \frac{e}{T_i} \frac{F_{Mi}}{\omega_0^2} \mathcal{J}_{z0}^2 J_0^2 \frac{\partial}{\partial r} [k_{\theta 0} \omega_{*pi} |\delta \phi_0|^2],\end{aligned}\quad (\text{B2})$$

where $\mathcal{J}_{z0} = \overline{\exp(i\lambda_{di})}$ and $\mathcal{J}_{z1} = \overline{(v_\parallel / c) \exp(i\lambda_{di})}$ account for finite drift-orbit-width and banana-orbit-width effects, with $\lambda_{di} = k_z \rho_{di}$ and $\overline{(\dots)}$ denoting bounce averaging.

With $\delta g_{j0} = \delta g_{j0}^{(1)} + \delta g_{j0}^{(2)}$ given by Eqs. (2) and (3), the quasineutrality condition [Eq. (A2)] for RSAE becomes, assuming $|k_{\perp 0} \rho_i|^2 < 1$,

$$\frac{N_0 e^2}{T_e} \left(1 - \frac{\omega_{*ne}}{\omega_0} \right) (\delta \phi_0 - \delta \psi_0) = \underbrace{\langle e J_0 \delta g_{i0}^{(2)} \rangle_v - \langle \delta g_{e0}^{(2)} \rangle_v}_{\text{R.H.S.}} \quad (\text{B3})$$

In R.H.S., noting $\delta \phi_0 \simeq \delta \psi_0$ and the quasi-neutrality condition for the zonal component $\delta \phi_z$, we find

$$\delta \psi_0 [\langle e \delta g_{ez} \rangle_v - \langle e \delta g_{iz} \rangle_v] \simeq -\frac{N_0 e^2}{T_e} \left(1 + \frac{T_e}{T_i} \right) \delta \phi_0 \delta \phi_z. \quad (\text{B4})$$

Meanwhile, in $\delta g_{j0}^{(2)}$, $\delta g_{j0} \simeq \delta g_{j0}^{(1)}$. We then derive, after some straightforward algebra,

$$\text{R.H.S} = i \frac{c}{B_0} \frac{k_{\theta 0} k_z}{\omega_0} \frac{N_0 e^2}{T_e} (\delta \psi_z - \delta \phi_z) \left(1 - \frac{\omega_{*ne}}{\omega_0} \right) \delta \phi_0. \quad (\text{B5})$$

Combining Eqs. (B3) and (B5) yields Eq. (4) of the main text.

Appendix C: Linear and nonlinear vorticity responses—The linear RSAE response $\nabla \cdot \delta \mathbf{J}_0^{(1)}$ in Eq. (6) follows from Eq. (A3) [26] for single- n RSAE and neglecting wave-particle resonances as well as FILR KAW physics:

$$\nabla \cdot \delta \mathbf{J}_0^{(1)} = \frac{\delta \mathbf{B}_{\perp 0}}{B_0} \cdot \nabla J_{\parallel 0} + B_0 \nabla_\parallel \left(\frac{\delta J_\parallel}{B_0} \right) + \nabla \cdot \delta \mathbf{J}_{\perp 0}^{(1)}, \quad (\text{C1})$$

with

$$\nabla \cdot \delta \mathbf{J}_{\perp 0}^{(1)} = -\frac{c^2}{4\pi} \nabla \cdot \left(\frac{1}{v_A^2} \right) \nabla_\perp \partial_t \delta \phi_0 + \frac{c^2}{4\pi} \mathbf{b}_0 \times \nabla \left[\frac{4\pi}{B_0^2} \left(\frac{P_{\perp 0}}{\Omega} \right)_i \right] \cdot \nabla \nabla_\perp^2 \delta \phi_0 + \underbrace{\frac{c}{B_0} \mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \sum_j \left\langle m_j \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right)_j J_0 \delta g_j \right\rangle_v}_{\mathcal{I}}, \quad (\text{C2})$$

where the first two terms combine to give $(ic^2/4\pi) \nabla_\perp \cdot [(\omega_0 - \omega_{*pi})/v_A^2] \nabla_\perp \delta \phi_0$ in Eq. (6), representing the inertial and ion diamagnetic response, while the last term $\mathcal{I}$ corresponds to the ballooning-interchange contribution discussed in the main text following Eq. (6).

Substituting the linear responses $\delta g_j^{(1)}$ from Eqs. (2) and (3) into term $\mathcal{I}$ and assuming $|k_{\perp 0} \rho_i|^2 < 1$, we have

$$\mathcal{I} = -\frac{c}{B_0} \mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \sum_j \left\langle \left(\frac{em}{T} F_M \right)_j \frac{\omega_{*j}}{\omega_0} \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right)_j \right\rangle_v \delta \phi_0 + \frac{c}{B_0} \mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \left\langle \frac{em_i}{T_i} F_{Mi} \left(1 - \frac{\omega_{*i}}{\omega_0} \right) \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right)_i \frac{\omega_{di}}{\omega_0} \right\rangle_v \delta \phi_0. \quad (\text{C3})$$

The first term gives rise to λ_a in Eq. (6), while the second term gives the geodesic-acoustic frequency (ω_{gam}^2) arising from thermal ion compressibility [30] [via $v_{ti}^2(\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla)^2 \propto k_{\perp}^2 v_{ti}^2 / R^2$]. Combining these expressions recovers Eq. (6) of the main text.

The nonlinear response $\nabla \cdot \delta \mathbf{J}_0^{(2)}$ in Eq. (7) arises from Maxwell stress and gyrokinetic Reynolds stress in Eq. (A3) [26].

The Maxwell stress term, originating from $(\delta \mathbf{B}_{\perp} / B_0) \cdot \nabla (\delta J_{\parallel} / B_0)$, can be expressed in wave-number space using $\delta \mathbf{B}_{\perp} = \nabla_{\perp} \times (\delta A_{\parallel} \mathbf{b}_0)$ and parallel Ampère's law $\nabla_{\perp}^2 \delta A_{\parallel} = -4\pi \delta J_{\parallel} / c$:

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}|_{\text{MX}} = -\frac{c}{4\pi B_0^2} \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \left[\mathbf{k}'_{\perp} \times (\mathbf{b}_0 \delta A_{\parallel \mathbf{k}'}) \right] \cdot \mathbf{k}'' k_{\perp}^{\prime 2} \delta A_{\parallel \mathbf{k}''}. \quad (\text{C4})$$

For beat-driven zonal fields generated by RSAE self-coupling ($\mathbf{k} = \mathbf{k}_z + \mathbf{k}_0$ with $\mathbf{k}_z = k_z \mathbf{r}$), the two combinations $(\mathbf{k}', \mathbf{k}'') = (\mathbf{k}_z, \mathbf{k}_0)$ and $(\mathbf{k}_0, \mathbf{k}_z)$ contribute. Using $(\mathbf{b}_0 \times \mathbf{k}_z) \cdot \mathbf{k}_0 = -k_{\theta 0} k_z$ and $\delta A_{\parallel 0} = (c k_{\parallel 0} / \omega_0) \delta \psi_0$ with the ideal MHD limit $\delta \psi_0 = \delta \phi_0$, we obtain

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}|_{\text{MX}} = \frac{c^3}{4\pi B_0 v_A^2} k_{\theta 0} k_z (k_z^2 - k_{\perp 0}^2) \frac{k_{\parallel 0} v_A}{\omega_0} \frac{v_A \delta A_{\parallel z}}{c} \delta \phi_0. \quad (\text{C5})$$

Meanwhile, the Reynolds stress contribution comes from the last two terms of Eq. (A3). Taking $J_0 \simeq 1 - k_{\perp}^2 \rho_i^2 / 4$, we then find

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}|_{\text{RS}} \simeq \frac{c}{4B_0} [(k_z^2 - k_{\perp 0}^2)(\mathbf{b}_0 \times \mathbf{k}_z) \cdot \mathbf{k}_{\perp 0} \langle e \rho_i^2 \delta g_{i0} \rangle_v \delta \phi_z - (k_z^2 - k_{\perp 0}^2)(\mathbf{b}_0 \times \mathbf{k}_{\perp 0}) \cdot \mathbf{k}_z \langle e \rho_i^2 \delta g_{iz} \rangle_v \delta \phi_0]. \quad (\text{C6})$$

Noting that $\delta g_{iz} = \delta g_{iz}^{(1)} + \delta g_{iz}^{(2)}$ given by Eq. (B2) and $\delta \phi_z$ given by Eq. (1), we find $|\langle \rho_i^2 \delta g_{iz} \rangle_v \delta \phi_0| / |\langle \rho_i^2 \delta g_{i0} \rangle_v \delta \phi_z| \approx \mathcal{O}(|k_z \rho_{di}|^2) \ll 1$. Taking $\delta g_{i0} \simeq \delta g_{i0}^{(1)}$ in Eq. (C6), we then readily derive

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}|_{\text{RS}} \simeq -\frac{c^3 k_{\theta 0} k_z}{4\pi B_0 v_A^2} \left(1 - \frac{\omega_{*pi}}{\omega_0} \right) (k_z^2 - k_{\perp 0}^2) \delta \phi_z \delta \phi_0. \quad (\text{C7})$$

Combining Eqs. (C5) and (C7) gives Eq. (7) of the main text.

*Appendix D: RSAE-ZF-MHD model derivation—*Combining Eqs. (4)–(7), the linear vorticity response can be written, after factoring out $ic^2 / (4\pi \omega_0 v_A^2)$, as

$$\nabla \cdot \delta \mathbf{J}_0^{(1)} = \nabla_{\perp} \cdot \left[\left(1 - \frac{\omega_{*pi}}{\omega_0} \right) (\omega_0^2 - \omega_{\text{gam}}^2) - k_{\parallel 0}^2 v_A^2 \right] \times \nabla_{\perp} \delta \phi_0 + \Lambda \delta \phi_0, \quad (\text{D1})$$

with Λ accounting for the favorable surface-averaged curvature, toroidal couplings, and, if needed, wave-particle interactions from both thermal and energetic particles [27,31].

Organizing the nonlinear terms according to derivatives of $\delta \phi_z$ and $\delta A_{\parallel z}$, the contribution from $\delta \phi_z$ becomes

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}(\delta \phi_z) = \frac{c}{B_0} \omega_0 k_{\theta 0} \left[\nabla_{\perp} \cdot \left(\frac{k_{\parallel 0}^2 v_A^2}{\omega_0^2} + 1 - \frac{\omega_{*pi}}{\omega_0} \right) \times \delta \phi'_z \nabla_{\perp} \delta \phi_0 + \left(\frac{k_{\parallel 0}^2 v_A^2}{\omega_0^2} - 1 + \frac{\omega_{*pi}}{\omega_0} \right) \times (\delta \phi_z''' \delta \phi_0 + \nabla_{\perp} \delta \phi'_z \cdot \nabla_{\perp} \delta \phi_0) \right], \quad (\text{D2})$$

while that from $\delta A_{\parallel z}$ is given by

$$\nabla \cdot \delta \mathbf{J}_0^{(2)}(\delta A_{\parallel z}) = -\frac{2c}{B_0} \omega_0 k_{\theta 0} \nabla_{\perp} \cdot \left[\frac{k_{\parallel 0} v_A}{\omega_0} \left(\frac{v_A \delta A'_{\parallel z}}{c} \right) \right] \nabla_{\perp} \delta \phi_0. \quad (\text{D3})$$

Summing these contributions, the eigenmode equation for the “RSAE-ZF-MHD” model can then be cast in the form given by Eq. (8).