

Entangled States from Sparsely Coupled Spins for Metrology with Neutral Atoms

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Quantum states featuring extensive multipartite entanglement are a resource for quantum-enhanced metrology, with sensitivity up to the Heisenberg limit. However, robust generation of these states using unitary dynamics typically requires all-to-all interactions among particles. Here, we demonstrate that optimal states for quantum sensing can be generated with sparse interaction graphs featuring only a logarithmic number of couplings per particle. We show that specific sparse graphs with long-range interactions can approximate the dynamics of all-to-all spin models, such as the one-axis twisting model, even for large system sizes. The resulting sparse coupling graphs and protocol can also be efficiently implemented using dynamic reconfiguration of atoms in optical tweezers.

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Entangled resource states are the foundation of quantum metrology, enabling quantum systems to surpass classical limits of measurement precision. This has been extensively studied, beginning with realizations of squeezed states, which typically rely on symmetric all-to-all interactions [1–10]. Recently, it has been demonstrated that neutral atom arrays can be used to implement a wide range of long-range coupling graphs [11–15]. Beyond their role as a versatile platform for quantum computing [11–13,16–33] and simulation [34–39], these systems hold significant promise for next-generation atomic clocks and broader applications in metrology [10,15,40–43]. In this Letter, we show that sparse nonlocal coupling graphs produced by shuffling operations in neutral atom arrays can be used to prepare highly entangled states as a resource for quantum metrology, by emulating one-axis twisting with all-to-all connectivity with only $\sim \log_2(N)$ couplings per particle, where N is the system size.

The models we consider are inspired by sparse coupling graphs that have been explored in relation to quantum information scrambling both theoretically [44–49] and experimentally [12,50–53]. We compare and contrast two sparse coupling models with an all-to-all (A2A) and a nearest-neighbor (NN) coupling geometry. In the *powers of two* (PWR2) model, spins interact if and only if separated by a distance equal to an integer power of 2. This model's coupling graph has $\mathcal{E}_G = N \log_2(N) - N + 1$ edges for

N vertices. In contrast, for the *hypercube coupling graph*, spins reside on the vertices of an $m = \log_2(N)$ -dimensional hypercube, resulting in $\mathcal{E}_G = (N/2) \log_2(N)$ edges for N vertices. Illustrations of the respective coupling graphs for $N = 8$ are presented in Fig. 1.

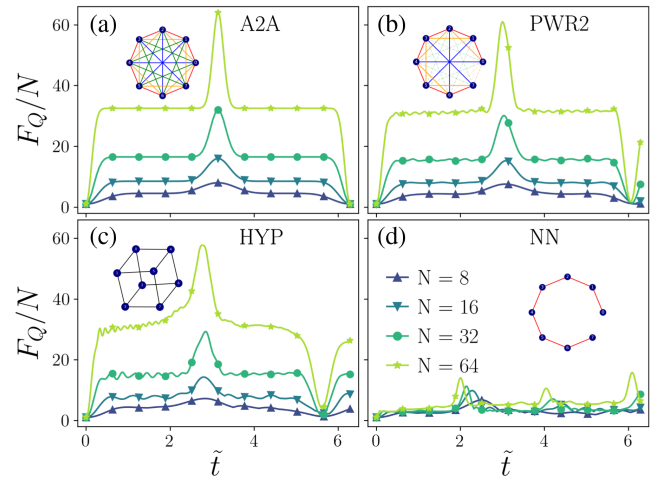


FIG. 1. (a)–(d) Evolution of the quantum Fisher information (QFI) density F_Q/N under the XY Hamiltonian given in Eq. (1) for different coupling graphs χ_{ij} and system sizes $N = 8, 16, 32, 64$ [see the legend in (d) for colors and markers]. The spins are initialized in an x -polarized state, and the time is normalized according to Eq. (2). The dynamics of the sparse coupling graphs PWR2 and hypercube closely resemble those of the A2A coupling graph, leading to a QFI density that scales with system size. On the other hand, the NN coupling graph leads to a significantly lower QFI. Insets in each panel display illustrations of the respective coupling graphs. The computations are performed with 1000 data points. For clarity of the figure, only a subset of the computed data points are shown.

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Below, we explore how well the dynamics generated by these types of interactions approximate the one-axis twisting (OAT) Hamiltonian, a well-studied model which features uniform, infinite-range Ising interactions [1,3,54]. In the OAT Hamiltonian, all N spin-1/2 particles interact with each other, resulting in a dense, *all-to-all* connectivity with $\mathcal{E}_{G,A2A} = N(N-1)/2$ edges as shown in Fig. 1(a). While these can be engineered indirectly (e.g., via photon-mediated interactions between atoms in an optical cavity [55–59]), this degree of connectivity is challenging to realize with direct interactions in most physical setups. In the following, we show that the all-to-all dynamics can be well approximated by sparse coupling graphs and propose to use shuffling operations, interleaved with nearest-neighbor interactions and local rotations, to emulate the OAT dynamics efficiently.

Dynamical preparation of entangled states using sparse coupling graphs—We consider spins initially prepared in an x -polarized state $|+\rangle^{\otimes N}$ and allow it to evolve under the XY Hamiltonian

$$H_{XY} = \sum_{i,j} \chi_{ij} (S_i^x S_j^x + S_i^y S_j^y) \quad (1)$$

for different coupling graphs $\{\chi_{ij}\}$, where the spin operators in terms of Pauli operators are given by $S_i^\alpha = \sigma_i^\alpha/2$ ($\hbar = 1$) and $\alpha \in \{x, y, z\}$. The choice of the Hamiltonian is inspired by the studies conducted in [60,61]. To account for different numbers of bonds in the different coupling graphs considered here, we introduce the normalized time

$$\tilde{t} = t \frac{\mathcal{E}_G}{\mathcal{E}_{G,A2A}}, \quad (2)$$

where $\mathcal{E}_G$ is the number of edges in a given graph G and t is the physical time. For the A2A graph, $\chi_{ij} = \chi_0$ for $i \neq j$, and Eq. (1) is proportional to the OAT Hamiltonian $H_{\text{OAT}} = \chi_0 J_z^2$, where $J_z = \sum_i S_i^z$.

We quantify the generation of metrologically useful entanglement via the quantum Fisher information (QFI) [62–67] of a quantum state $|\psi\rangle$ which determines the optimal precision with which a system parameter θ can be estimated using $|\psi\rangle$ as a probe. For spin systems, the standard scenario corresponds to θ being a phase encoded by a small rotation of a pure probe state around a given axis. The QFI in such a scenario is

$$F_Q[\theta, J_\alpha] = 4(\Delta J_\alpha)^2, \quad (3)$$

where $J_\alpha = \sum_i S_i^\alpha$ is the generator of a rotation around axis α with $\alpha = x, y, z$ and $(\Delta J_\alpha)^2 = \langle J_\alpha^2 \rangle - \langle J_\alpha \rangle^2$ is its variance calculated over the state $|\psi\rangle$. For uncorrelated particles, the sensitivity to rotations is bounded by classical limits on measurement precision which yield $F_Q = N$ (standard quantum limit), where N is the number of particles in

the system. However, the presence of entanglement in the system pushes the sensitivity beyond this, indicating utility in phase estimation and metrology, with $F_Q = N^2$ being the fundamental Heisenberg limit in the absence of noise [63].

In Fig. 1, we show the evolution of the QFI for different coupling graphs and system sizes. The dynamics of the QFI for the PWR2 and hypercube coupling graphs [Figs. 1(b) and 1(c)] are reminiscent of the dynamics of the A2A coupling graph [Fig. 1(a)]. In all three cases, at $t \lesssim 1/\sqrt{N}$, there is an initial spin-squeezed region where the QFI rises steadily, followed by a plateau region, and a final rise to reach a state with maximum QFI, $F_Q \simeq N^2$, at $\tilde{t} = \pi$. To support this statement, we have calculated the squeezing parameter, which quantifies metrologically useful entanglement, at $\tilde{t} = 1/\sqrt{N}$, the QFI at the plateau region, and the overlap of the state with maximum QFI with the Greenberger-Horne-Zeilinger (GHZ) state, as detailed in Supplemental Material [68]. To understand the gain we obtain from using sparse interactions, it is instructive to compare the dynamics to the nearest-neighbor model [Fig. 1(d)], which shows much lower QFI throughout the dynamical evolution. Importantly, the QFI per particle fails to scale with system size, thus signifying no scaling gain with respect to the standard quantum limit. It is thus evident that the sparse models create states with significantly higher QFI and therein emulate the all-to-all OAT dynamics.

In Fig. 2, we analyze how the maximum QFI scales with system size. Specifically, we propose an ansatz $F_Q(t^*) \propto N^\beta$, where t^* is the physical time at which the QFI reaches its maximum, and extract β numerically.

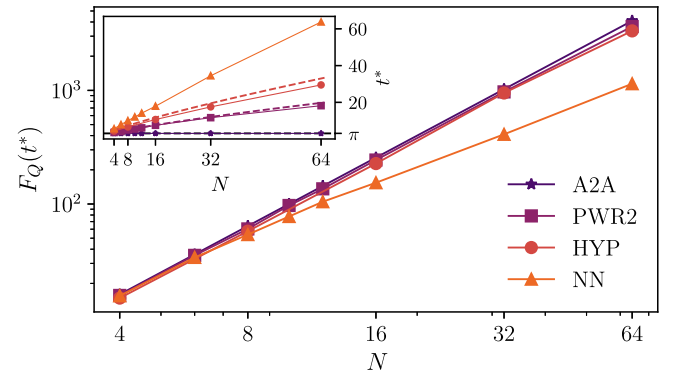


FIG. 2. The maximum value of the QFI under the dynamics generated by the XY Hamiltonian in Eq. (1) on an initially x polarized state is shown as a function of system size N for different coupling graphs. We extract the scaling coefficients $F_Q \propto N^\beta$ for different coupling graphs, yielding $\beta = 2.00$ for A2A coupling, $\beta = 1.96$ for the hypercube, and $\beta = 1.97$ for the PWR2 graph. The NN coupling shows a much weaker scaling of $\beta = 1.51$. Inset: physical time t^* to reach the maximum QFI as a function of N . Markers show numerically obtained values, while dashed lines indicate the expected scaling based on the mean-field model $t^* \approx \pi(\mathcal{E}_{G,A2A}/\mathcal{E}_G)$.

We find that the PWR2 and hypercube geometries lead to states that follow $F_Q \propto N^{1.97}$ and $F_Q \propto N^{1.96}$, respectively. While we employ exact diagonalization for small system sizes, all results for $N > 16$ are obtained using the time-dependent variational principle approach with matrix product states (MPS) [69–74]. The main text shows the results for bond dimension $D = 256$ for $N = 32$ and $D = 512$ for $N = 64$, truncation error $\epsilon = 10^{-13}$, and the time step $\chi_0 dt = 10^{-2}$. We refer the reader to Supplemental Material [68] for a detailed analysis of the numerical convergence. The inset in Fig. 2 illustrates the dependence of t^* on the system size N . For the all-to-all coupling graph, this time is known to be constant at $t^* = \pi$ [62]. For the NN model, as expected, it increases linearly with the system size, whereas we observe that $t^* \sim N/\log_2(N)$ in the sparse models. This scaling is consistent with the mean-field prediction given by Eq. (2), which is shown as dashed lines in the figure.

Sparse coupling graphs and collective behavior—We can analyze the results obtained from the sparse coupling graphs by studying how robust the OAT dynamics are to perturbations that break the permutational invariance of the Hamiltonian. To this end, it is instructive to rewrite the XY Hamiltonian in Eq. (1) for a given coupling graph $\{\chi_{ij}\}$ as the sum of

$$H_{\text{gOAT}} = \chi_0 J_z^2 - \sum_{ij} \chi_{ij} \vec{S}_i \cdot \vec{S}_j \quad (4)$$

and

$$V_{\text{pert}} = -\sum_{ij} (\chi_0 - \chi_{ij}) S_i^z S_j^z. \quad (5)$$

Here, H_{gOAT} is a generalized OAT model (introduced in Ref. [60]) composed of the usual OAT twisting term and the isotropic Heisenberg Hamiltonian. Both of these commute with the collective spin operator $\mathbf{J}^2 = J_x^2 + J_y^2 + J_z^2$ for any coupling graph, thus restricting the dynamics of initial spin coherent states to the subspace of permutationally symmetric states. The last term acts as a perturbation that will couple states of different total $\mathbf{J}$, causing the state to leave the symmetric subspace and deviate from the OAT dynamics. Perturbation theory indicates that such deviation will be more prominent if the eigenstates of H_{gOAT} corresponding to different values of $\mathbf{J}$ are separated by a small gap. Thus, here we study the scaling of the spectral gap Δ in the gOAT Hamiltonian with system size for different coupling graphs. In Supplemental Material [68], we construct the excitation spectra for the two sparse coupling graphs considered here (PWR2 and hypercube) using spin-wave theory [75–77]. For comparison, we also include results for the well-known case of algebraically decaying (AD) interactions where $\chi_{ij} = \chi_0 |i - j|^{-\alpha}$ in 1D (with $\alpha = 0$ corresponding to the A2A case and $\alpha \rightarrow \infty$ to the NN case). In Fig. 3, we show the behavior of the spectral gap Δ as a function of system size N for various cases of interest. We observe a power-law

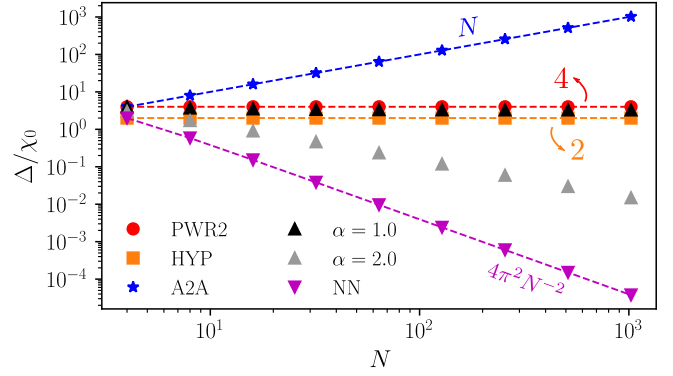


FIG. 3. Spectral gap Δ/χ_0 of the isotropic Heisenberg Hamiltonian in Eq. (4) as a function of system size N for different coupling graphs with various levels of connectivity. Systems with asymptotically vanishing gaps deviate from the collective OAT dynamics, while systems with nonvanishing gaps in the large- N limit display robust collective dynamics that can generate metrologically useful states. Examples of the former are the NN and the power-law coupling graph with $\alpha > 1$. Meanwhile, the latter includes the A2A and power-law couplings with $\alpha \leq 1$ (for 1D graphs), as well as the sparse coupling graphs PWR2 and hypercube studied here, which display a constant gap as a function of system size. In all cases, symbols indicate numerical evaluation of the gap from an exact expression (see Supplemental Material [68]), while dashed lines indicate closed-form analytical approximations, indicated for each case in the figure.

scaling $\Delta \sim N^\gamma$ for all coupling graphs, consistent with previous studies. Crucially, we show analytically in Supplemental Material [68] that the sparse graphs lead to constant gaps, i.e., $\Delta/\chi_0 = 2, 4$ for the hypercube and PWR2, respectively. This implies that the collective dynamics generated by these models are well protected from the noncollective perturbations, thus explaining why these models generate metrologically useful states. Furthermore, since the gap remains exactly constant (instead of increasing with N), we argue that this level of connectivity is optimal for producing such behavior. It closely mimics the behavior of the known optimal case of AD couplings with $\alpha = 1$ for 1D graphs.

Implementing sparse graphs in tweezer arrays—With an analytical understanding of why sparse models preserve collective spin behavior and are able to generate metrologically relevant resource states, we now shift focus to proposing experimental implementations. Specifically, we focus on neutral atom arrays employing tweezer-assisted shuffling operations. This choice is due to its good scalability, coherent control of each atom, and flexibility in atom movement. To realize the hypercube connectivity, these rearrangement operations can be chosen to execute a “Faro shuffle” [78,79], which moves the particle originally located in site i to site i' according to the following rule:

$$i' = \mathcal{R}(i = b_m \dots b_2 b_1) = b_1 b_m \dots b_2. \quad (6)$$

Hereby, the binary representation of site $i = b_m \dots b_2 b_1$ is reversed, such that the least significant bit b_1 of i becomes the most significant bit of $i' = \mathcal{R}(i)$. In [68], we propose an alternative shuffling sequence which yields the PWR2 geometry. To realize the spin-exchange Hamiltonian, we leverage the hypercube geometry native to this Faro shuffle and a sequence of global rotations and zz (Ising) interactions:

$$R_\alpha = e^{-i\pi/4 \sum_i \sigma_i^\alpha}, \quad \alpha \in \{x, y\}, \quad (7)$$

$$H_{zz} = 2\chi_0 \sum_\nu S_{2\nu-1}^z S_{2\nu}^z, \quad (8)$$

where ν indicates the atomic position, rather than the spin index. While spin-exchange Hamiltonians can be implemented natively in Rydberg arrays [14], this requires encoding in highly excited states, which are prone to decay. Our approach, based on hyperfine qubits and shuffling operations, enables robust entanglement generation using spins encoded in long-lived atomic ground states except during short pulsed gate operations. In [68], we provide details about how to implement this procedure in practice. During each time step of duration $dt = t^*/M$, we apply $m \in \{1, \dots, \log_2(N)\}$ Faro shuffles to build up the hypercube coupling graph. Before each shuffling operation, we evolve the system according to

$$U_m(dt) = [R_x e^{-iH_{zz}dt/2} R_x^\dagger] [R_y e^{-iH_{zz}dt/2} R_y^\dagger], \quad (9)$$

which effectively realizes a first-order Trotter decomposition, such that the full evolution during one time step takes the following form:

$$U(t, t+dt) = \prod_m \mathcal{R} U_m(dt), \quad (10)$$

which is schematically depicted in Fig. 4(a). In Fig. 4(b), we show the value of the maximum QFI F_Q achieved by the stroboscopic protocol as a function of the number of iterations M , for different system sizes. For large enough M , the protocol reaches Heisenberg scaling $F_Q \sim N^2$, as expected from the fine-grained Trotterization. For small M , we observe a transition toward smaller values of QFI closer to the standard quantum limit. In this regime, the unitary in Eq. (9) fails to approximate the continuous evolution, and previous studies [80,81] suggest the existence of a chaotic regime, leading to highly scrambled states with low QFI. We further analyze this behavior by calculating the expectation value of the collective spin operator $\langle \mathbf{J}^2 \rangle$, shown in the inset in Fig. 4(b) for $N = 16$. For collective spin states, $\langle \mathbf{J}^2 \rangle = j(j+1)$, where $j = N/2$, and we see how this collective behavior is lost for $M < 10$ iterations.

In addition to these quantities, we also calculate the tripartite mutual information (TMI) $\mathcal{I}_3$, which has been

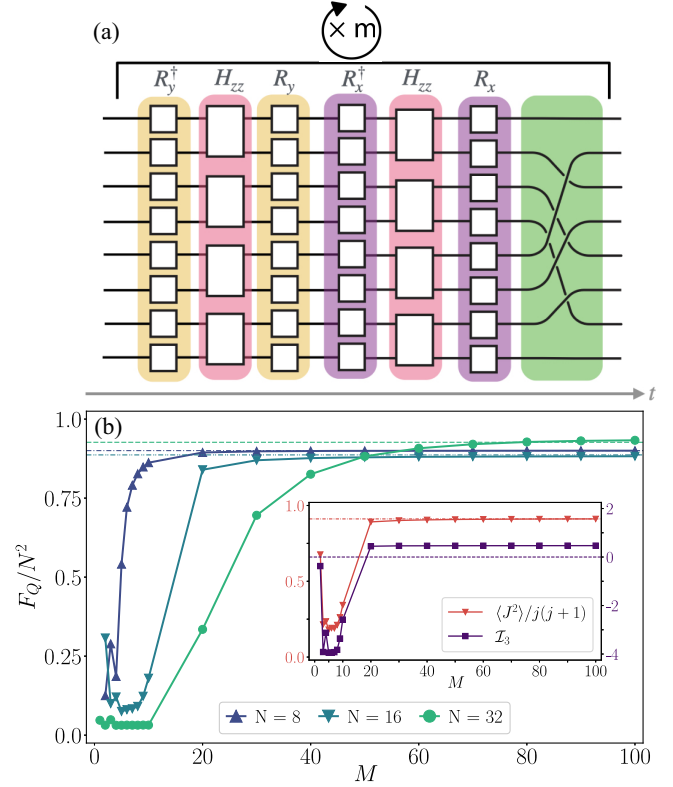


FIG. 4. (a) Our circuit implementation consists of single-qubit rotations Eq. (7) (purple for R_x and yellow for R_y) followed by zz interactions between nearest-neighbor spins according to Hamiltonian in Eq. (8) (pink for H_{zz}) and inverse Faro shuffles $\mathcal{R}$ (green). The application of H_{zz} and R_α is done in $m = \log_2(N)$ steps and leads to a first-order Trotter decomposition of the spin-interaction Hamiltonian according to Eq. (10). (b) The normalized QFI F_Q/N^2 as a function of the number of iterations M , where we evolve the initial x -polarized state up to t^* for the hypercube geometry. The dashed lines of corresponding colors represent the values of F_Q/N^2 at $t = t^*$ for $N = \{8, 16, 32\}$, respectively, extracted from the continuous time evolution. Inset: the expectation values of the collective spin operator $\langle \mathbf{J}^2 \rangle$ normalized to $j(j+1)$, with $j = N/2$ (red, left axis) and $\mathcal{I}_3$ (violet, right axis), are shown as a function of M for $N = 16$. For small values of M , there is a strong deviation from the collective spin behavior and a loss of permutational symmetry, indicating a scrambled state characterized by negative $\mathcal{I}_3$. The horizontal dash-dotted (red) and dashed (violet) lines represent $\max\{\langle \mathbf{J}^2 \rangle\}$ and $\mathcal{I}_3 = 0$, respectively.

extensively studied in the context of information scrambling. The TMI $\mathcal{I}_3$ between three regions A , B , and C is defined as

$$I(A:B:C) = \mathcal{I}_3 = I(A;B) + I(A;C) - I(A;BC), \quad (11)$$

where $I(A;B) = S_A + S_B - S_{AB}$ is the mutual information between subregions A and B and S_A is the Von Neumann entropy. A negative $\mathcal{I}_3$ signifies that the combined region BC contains more information about subregion A than the

subregions B and C individually. Thus, negativity of the tripartite mutual information indicates many-body entanglement in the system [48,82–84]. However, for permutationally symmetric states (such as the collective states considered here), the tripartite mutual information is positive [85]. A simple example is the GHZ state that has $\mathcal{I}_3 = \ln 2$. For large M , we obtain that the tripartite mutual information is positive as shown in the inset in Fig. 4(b).

We observe the presence of a sharp threshold as a function of the number of iterations M that separates permutationally symmetric states at larger M from scrambled states at smaller M characterized by negative $\mathcal{I}_3$. The minimum number of iterations required to attain Heisenberg-limited states increases with system size. While the actual scaling is challenging to analyze numerically, because usual MPS methods stop being efficient for scrambled states, we observe a sublinear scaling of M with the size of the system. Moreover, in Supplemental Material [68], we show that achieving states featuring Heisenberg scaling (i.e., $F_Q \propto N^2$ rather than $F_Q = N^2$) is less demanding, since shorter evolution times suffice.

Summary and outlook—We demonstrated the generation of resource states for quantum-enhanced metrology from sparse coupling graphs by showing how they effectively emulate one-axis twisting dynamics with fewer resources. We also proposed a stroboscopic protocol, implementable on current neutral atom arrays, which requires only tweezer shuffling, nearest-neighbor Ising interactions, and local rotations to prepare metrologically relevant states exhibiting Heisenberg scaling. While sparse nonlocal graphs had previously been shown to lead to fast information scrambling, our findings demonstrate that tasks requiring states with a specific entanglement structure, such as quantum sensing, can also be improved by the use of this tool. With advancements in atom shuffling using tweezer arrays [11,12,51], squeezing generated by these sparse graphs could potentially be applied in optical tweezer clocks, paving the way for entanglement-enhanced optical clocks [15,86,87]. Our results pave the way for further exploration of the utility of sparse coupling graphs for other tasks in quantum computing and quantum simulation. In particular, combining sparse nonlocal interactions within the framework of variational circuits has the potential to open new avenues for the generation of useful entangled states for quantum optimization and quantum machine learning [88]. Moreover, extending these techniques to digital circuit models could open new avenues for fault-tolerant preparation of metrologically relevant states.

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Data availability—The data that support the findings of this Letter are openly available [89].

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