

^{208}Pb Nuclear Charge Radius Revisited: Closing the Fine-Structure-Anomaly GapZewen Sun¹, Konstantin A. Beyer¹, Zoia A. Mandrykina¹, Igor A. Valuev¹,
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A comprehensive reevaluation of the root-mean-square nuclear charge radius is presented for the doubly magic ^{208}Pb extracted from muonic spectroscopy measurements. By integrating rigorous theoretical quantum electrodynamics calculations, state-of-the-art numerical methods, and a systematic reanalysis of the uncertainties, we reduced the long-standing muonic fine-structure anomaly and improved the goodness of fit by a factor of 20. The resulting value of 5.5062(5) fm for a Fermi distribution is fairly consistent with the previously reported muonic spectroscopy value, and 3 standard deviations larger than the commonly used compilation data, which indicates that the current value and its uncertainty could be significantly underestimated. Attributing the remaining discrepancy to theory errors which cannot be rigorously calculated, we suggest the rms charge radius with reduced model dependence to be 5.5062(17) fm. This Letter sets an improved benchmark for charge radius extraction in heavy nuclei and paves a path for systematic reevaluations across the nuclear chart.

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Introduction—Nuclear root-mean-square (rms) charge radii serve as fundamental benchmarks bridging various fields of physics. They are indispensable input parameters for nuclear-, atomic-, and molecular-physics calculations. Reliable rms radii are crucial for precision tests of quantum electrodynamics (QED) and for the determination of fundamental constants. Additionally, many searches for physics beyond the Standard Model rely on high-precision rms charge radii.

Accurate calculations of nuclear parameters from first principles are particularly challenging for heavy nuclei. Since direct measurements of nuclear charge radii are not possible, indirect extraction with significant theory input is necessary. Muonic atoms are highly sensitive to these nuclear parameters, such that the most accurate method to measure rms charge radii is based on muonic atom spectroscopy [1,2]. Motivated by the success of light muonic atom spectroscopy [3–5] and new measurements of heavy muonic atoms with microtargets [6], we started the development of a significantly more accurate theory for the rms radii extraction of heavy nuclei.

While muonic atom spectroscopy provides the most precise determinations of rms charge radii, its application to

heavy nuclei has revealed a persistent discrepancy, namely the muonic fine-structure anomaly [7–10], first observed nearly 50 years ago but never resolved. The theoretically predicted spectra of lead, tin, and zirconium exhibit small but systematic deviations from the experimentally measured values, for any value of the assumed nuclear rms charge radius. This discrepancy was attributed to the least understood and most complicated of all contributions: nuclear polarization. However, our rigorous microscopic calculations of the nuclear polarization have shown that it is most likely not solely responsible for the anomaly [11]. This aligns with the poor fit quality addressed above and hints at the uncertainty of the nuclear radius of double-magic ^{208}Pb being significantly underestimated. The same situation applies to other muonic atoms, as the methods for rms charge radius extraction have been the same.

In this Letter, we report a theoretical and computational breakthrough toward resolving this anomaly for ^{208}Pb by combining fully rigorous QED calculations with a modern statistical treatment of uncertainties. We reanalyzed the spectrum of muonic ^{208}Pb and found a best-fit rms of 5.5062(5) fm for a two-parameter Fermi (2pF) nuclear charge distribution, which is $3\sigma - 4\sigma$ larger than the tabulated value [12,13]; see Fig. 1. In contrast to the previously reported and tabulated data, our value is based on fully rigorous *ab initio* theory and improves the goodness of fit by a factor of 20. This significantly reduces the muonic fine-structure anomaly to the point where its existence should be questioned. Therefore, we are confident that the here presented rms charge radius of ^{208}Pb should be adopted as the new standard as extracted from muonic spectroscopy. Our analysis strongly suggests that

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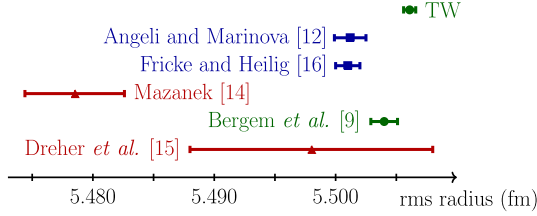


FIG. 1. Nuclear rms radius for ^{208}Pb extracted in this Letter compared to the literature values. The data based on muonic spectroscopy are marked with green [this work (TW) and Bergem *et al.* [9]], data based on the electron scattering are in red (Mazanek [14] (we actually found this thesis, read and digitalized it) and Dreher *et al.* [15]), and the combined values are in blue (Angeli and Marinova [12] and Fricke and Heilig [16]).

the currently accepted rms charge radius is too small and must be updated correspondingly.

Theory improvements—We calculated all individual contributions to the muonic spectrum which contribute $\gtrsim 10\%$ of the experimental precision. The leading contribution is the finite nuclear size (FNS) effect, obtained by numerically solving the Dirac equation with a nuclear potential derived from a 2pF nuclear charge model (see Supplemental Material [17] for details).

To the first order in the fine-structure constant α , there are two QED corrections: vacuum polarization (VP) and self-energy (SE). In contrast to electronic systems, where these two contributions have similar magnitude, VP dominates in muonic atoms. VP can be expressed as a sum of Uehling (Ue) and Wichmann-Kroll (WK) contributions, represented through their respective potentials. The dominant Ue corrections are calculated rigorously for a 2pF model using the method [30]. Since numerical precision is the only limiting factor, we ensured that the error is negligible compared to the experimental uncertainty. For the evaluation of the WK contribution including the FNS effect, two independent methods have been used [31,32]. Moreover, we have also included the Källén-Sabry (KS) contribution, which accounts for the second-order correction beyond the Uehling term, following the method described in Ref. [33].

In addition to the electron-positron loop, VP can also originate from any other particle-antiparticle (e.g., muon-antimuon) loop. However, these effects are exponentially suppressed by the mass of the virtual particles; thus, going beyond leading order is only necessary for electronic Ue at the current level of accuracy. The muon-antimuon loop gives the muonic Ue (μUe) correction, which can be calculated by a straightforward replacement of the virtual particle mass. For the hadronic Ue (HadUe) contribution, we employed the approach [34] derived from hadronic scattering data, and adapted it for muonic atoms [35].

The SE correction has been improved compared to Ref. [9] and calculated within a fully rigorous QED framework, following Ref. [36]. Furthermore, we investigated the model dependence of SE, which resulted in increased uncertainties

compared to the previous results. Similarly, for the relativistic recoil (RRecoil) correction, we adopted the approach of Ref. [37] and improved it by incorporating the 2pF model instead of the previously used exponential model.

Finally, the most cumbersome and challenging correction, the nuclear-polarization (NP) effect, has been calculated in the same manner as described in Ref. [11], now for all states appearing in the spectra. The uncertainties have been conservatively estimated as the spread of the results for the nine different Skyrme parametrizations that have been used in Ref. [11] and shown to cover a wide range in the parameter space. Special care had to be taken in the case of the $3d_{3/2}$ and $3d_{5/2}$ levels due to the resonances between the muonic transitions $3d_{3/2} \rightarrow 2p_{3/2}$, $3d_{5/2} \rightarrow 2p_{3/2}$, and $3d_{5/2} \rightarrow 2p_{1/2}$ and the lowest octupole nuclear excitation [38]; see Supplemental Material [17] for further information.

In our calculations, we also evaluated the screening effect of the interaction between the muon and the outer surrounding electrons following Ref. [39], assuming the lowest electronic shell is filled as $(1s)^2$. Despite its relatively large absolute impact on individual energy levels, the screening effect remains constant across the low part of the muonic spectrum. Therefore, the number of remaining electrons and the exact electronic configuration do not affect the spectral lines of heavy muonic systems relevant for the rms radius evaluation.

In Table I, we present all corrections, for simplicity calculated perturbatively (see Supplemental Material [17] for details) on the wave functions with FNS from 2pF, to illustrate the hierarchy of the effects. For the fitting procedure, all VP effects have been included in the Hamiltonian and, therefore, treated nonperturbatively.

Experimental data—For our current evaluation, we have used the experimental measurements of muonic ^{208}Pb reported in Ref. [9] (see Table II) and recently confirmed them to be reliable [40]. The data are split in three runs, which correspond to the high-, medium-, and low-energy parts of the spectrum. This split was arranged to allow for specific tuning of the detector. We performed the fit with respect to all transitions, but also analyzed the results from individual runs.

Finally, it is important to note that the most recent spectral measurement of muonic lead was performed almost 40 years ago, and there continues to exist tension between theory and the experiment. With our rigorous QED calculation we aim to prompt a new experimental campaign to measure the spectrum of muonic lead with a focus on state-of-the-art uncertainty analysis explicitly splitting statistical and systematical errors.

Fitting procedure—We calculated the transition energies as a function of 2pF parameters a and c , including all corrections described above. The best-fit parameters are obtained by minimizing χ^2 between the experimental data and the theory prediction. The theoretical uncertainties

TABLE I. Corrections to muonic ^{208}Pb binding energies (keV) calculated perturbatively with FNS wave functions from the 2pF model for the best-fit values $a = 0.5283$ fm, $c = 6.6409$ fm, rms radius = 5.5062 fm. A positive correction refers to an increase in binding energy.

State	2pF binding energy	Ue	WK	KS	μUe	HadUe	SE	NP	RRecoil	Screening
$1s_{1/2}$	10525.905	67.084	-0.504	0.552	0.238	0.159	-3.345(50)	5.712(603)	-2.555(66)	5.555
$2s_{1/2}$	3581.659	19.336	-0.250	0.149	0.041	0.027	-0.653(6)	1.063(156)	-1.317(27)	5.536
$2p_{1/2}$	4783.784	32.290	-0.356	0.251	0.044	0.030	-0.440(20)	1.923(263)	-2.114(4)	5.548
$2p_{3/2}$	4601.766	29.761	-0.344	0.229	0.033	0.023	-0.745(10)	1.864(256)	-2.172(2)	5.547
$3p_{1/2}$	2129.324	10.759	-0.167	0.081	0.014	0.009	-0.180(15)	0.584(99)	-1.003(3)	5.512
$3p_{3/2}$	2082.436	10.230	-0.163	0.077	0.011	0.008	-0.245(10)	0.619(96)	-1.025(0)	5.510
$3d_{3/2}$	2163.565	10.502	-0.192	0.076	0.001	0.001	0.020(2)	0.238(56)	-1.162(1)	5.525
$3d_{5/2}$	2121.477	9.848	-0.185	0.071	0.001	0.000	-0.070(5)	0.037(37)	-1.143(0)	5.523

arise from a combination of model dependencies, deviations between different numerical methods, and empirical estimations of unaccounted higher-order contributions, and are fully taken into account. Experimental uncertainties are assumed to be uncorrelated as shown in Table II. For more detailed information, see Supplemental Material [17].

Results and discussion—The best-fit 2pF parameters c and a , together with the corresponding rms charge radius, are shown in Table III. Figure 2 depicts the likelihood contours for the 2pF parameters up to the 5σ level. The fit was performed for each of the three runs separately and the final value was extracted from a combined fit to all the experimental data. The individual fits are represented by different colors: purple, green, and blue correspond to runs I, II, and III, respectively. The full fit is depicted in orange. The insets show the residual transition-energy differences between best-fit theory and measurement for each of the fits. The ellipses corresponding to the three individual runs are in mild tension with each other which is ultimately responsible for the poor fit quality of the full fit ($\chi^2/\text{d.o.f.} = 9.5$). Interestingly, the best-fit values for the individual runs still fall closely to a line of constant rms charge radius. Note that the figure does not show the best-fit value for run I as it is far off the other two, but can be found in Table III.

TABLE II. Experimentally measured transitions with their error bars [9].

Set	Transition	Energy (keV)
Run I	$2p_{3/2} - 1s_{1/2}$	5962.854(90)
	$2p_{1/2} - 1s_{1/2}$	5778.058(100)
Run II	$3d_{3/2} - 2p_{1/2}$	2642.332(30)
	$3d_{5/2} - 2p_{3/2}$	2500.590(30)
	$3d_{3/2} - 2p_{3/2}$	2457.569(70)
Run III	$3p_{3/2} - 2s_{1/2}$	1507.754(50)
	$3p_{1/2} - 2s_{1/2}$	1460.558(32)
	$2s_{1/2} - 2p_{1/2}$	1215.330(30)
	$2s_{1/2} - 2p_{3/2}$	1030.543(27)

The likelihood contours for all the fits are well aligned with the lines of constant radius, which explains the relatively small uncertainty on the rms charge radius compared to the individual 2pF parameters. Interestingly, run I, shown in purple, has a narrow but very extended likelihood surface, which results in a comparatively larger error of the rms charge radius than for the lower-energy runs. This is opposite to the usual notion that the high-energy $2p - 1s$ transitions are most sensitive to the rms charge radius and can be explained by the large theory uncertainty for these low-lying states. We therefore believe that all experimental data should be utilized for the fitting procedure.

To understand the implications of the present analysis it is important to remember the three basic assumptions which underlie the present Letter: (i) the experimental values including their uncertainties are correct, (ii) we have correctly accounted for every relevant QED effect and the theory uncertainties, and (iii) the nuclear charge distribution is close enough to a 2pF model.

We lack the expertise and raw data to comment on the validity of (i). We have assessed assumption (ii) to the best of our knowledge. It is well known that muonic spectroscopy is very sensitive to nuclear properties, and thus we see inaccuracy of (iii) as the most likely explanation for the remaining discrepancy between individual runs.

Even though the theory uncertainties have been estimated to the best of our ability, the quality of those estimates cannot be checked directly. Assuming that the experimental data are correct and the spectrum is fully described within QED,

TABLE III. Nuclear parameters of ^{208}Pb determined from the fits to muonic transition energies, in fm.

	Run I	Run II	Run III	Total	Ref. [9]
c	6.55(17)	6.685(66)	6.639(7)	6.6409(25)	6.6447(5)
a	0.59(11)	0.505(34)	0.531(5)	0.5283(18)	0.5249(3)
rms radius	5.525(46)	5.508(5)	5.509(2)	5.5062(5)	5.5040(11)
$\chi^2/\text{d.o.f.}$		0.0001	2.9	9.5	187

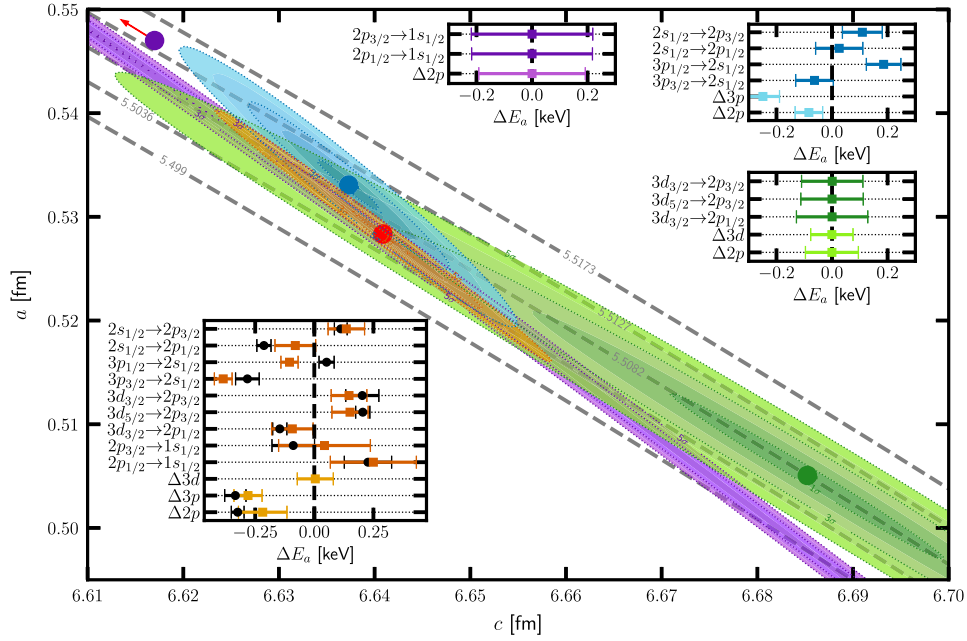


FIG. 2. The fit of the spectrum of muonic ^{208}Pb to a 2pF distribution. The colored regions show the 5σ likelihood for the 2pF parameters and the dotted contours highlight the 5σ , 3σ , and 1σ steps. Each color corresponds to a fit of different groups of transitions: purple only fits run I, green corresponds to run II, blue to run III, and orange is the combined fit of all the transitions. The insets show the resulting residual transition energies in the same colors, with measured transitions in darker shade and resulting fine-structure splittings in lighter color. The previous fit as taken from Ref. [9] is shown in black wherever available, and the error bars include experimental errors only as the theory uncertainty was not explicitly quoted. The dashed lines correspond to curves of constant radius. The colored circles indicate the best-fit parameters and the red circle the final values for the overall fit. Note that the best-fit value for run I lies outside of the plotted parameter range, as indicated by the red arrow.

we attribute the remaining discrepancy to unaccounted uncertainties like, e.g., model dependence. By normalizing the error such that the $\chi^2/\text{d.o.f.}$ becomes unity, and ensuring that the spectrum is reproduced within the accuracy of the calculation, we arrive at a conservative, reduced model-dependent rms charge radius of 5.5062(17) fm.

Comparison with previous data—Comparison of our results with the most recent full extraction of the rms charge radius of lead in [9] is straightforward. The underlying data are the same; however, we have updated QED theory and error analysis. We find a best-fit rms charge radius which is in mild tension with their previously reported muonic spectroscopy value; see Table III.

The results of Bergem *et al.* [9] were combined with electron scattering data and optical isotope-shift measurements in the compilations of Fricke *et al.* [13] and Angeli and Marinova [12]. This combination of datasets is performed to empirically correct for the assumption of a nuclear 2pF distribution. Taken at face value, their final number is in strong tension with our result. We discuss below that the systematic error introduced through the exact shape of the nuclear charge density cannot be quantified exactly; however, we still believe that the tabulated rms charge radius of lead requires updating.

Model dependence—Based on the recent indication that ^{208}Pb is slightly deformed [41], we investigated the effect

on the rms radius value. We calculated the corresponding deformation parameter β , following the methods described in Supplemental Material [17], and performed the fit for the deformed Fermi model. The resulting rms radii of the two models are the same, up to all significant figures reported in Table III. The difference in $\chi^2/\text{d.o.f.}$ between the two models is also negligible.

As a basic test of model dependence we also considered a 3pF model with additional free parameter w (see Supplemental Material [17]). The resulting fit is consistent with the 2pF model and the reduced $\chi^2/\text{d.o.f.}$ does not suggest preference for this modification. We would like to stress that a full analysis of the model dependence would require a parametrization of all possible density functions and is therefore not feasible until new, *ab initio* nuclear densities are available.

In the absence of “better” test distributions, rooted in physical principles, the exact level of accuracy at which the deviation of the nuclear charge distribution from 2pF affects the rms charge radius can only be estimated. A conservative limit would be to compare the 2pF specific value from muonic spectroscopy with values extracted from electron-scattering experiments. The latter are, in principle, sensitive to the shape of the charge distribution; however, the subtraction of QED effects faces challenges. As we lack the expertise to comment on the procedure,

we simply highlight that the rms charge radii tend to deviate on the level of ~ 0.001 fm.

Conclusions—We performed fully rigorous, state-of-the-art QED calculations for the spectrum of muonic lead to extract the nuclear rms charge radius from a fit to experimental measurements. Using a two-parameter Fermi distribution, we determine the nuclear rms radius value of $5.5062(5)$ fm, achieving a 20-fold improvement in fit quality over previous analyses. The remaining overall high $\chi^2/\text{d.o.f.} = 9.5$ we traced back to an inconsistency between the fits of groupings of the transitions by energy.

Our analysis reduces the previously reported fine-structure anomaly in $\Delta 2p$ from 6σ to 2σ , suggesting that the apparent anomaly arose primarily from underestimated uncertainties in earlier analyses. Contrary to conventional understanding, our analysis demonstrates that transitions involving $2s$, $2p$, and $3p$ states impose at least equally stringent constraints on radius extraction as traditionally emphasized $2p - 1s$ transitions. This observation may influence future experimental strategies.

To reduce residual model dependence, we conservatively normalize uncertainties to achieve $\chi^2/\text{d.o.f.} = 1$, yielding a reduced model-dependent estimate of $5.5062(17)$ fm. This value is consistent with the previous work in Ref. [9], which analyzed the same muonic spectral data, but our Letter incorporates complete QED corrections for the first time. Our result is significantly larger than the currently accepted standard based on combined muonic and electron-scattering data.

We are convinced that the current standard rms charge radius of ^{208}Pb may be underestimated and requires reconsideration in light of this Letter. More broadly, our results strongly indicate the necessity of a systematic, theory-driven reevaluation of the rms charge radii across the periodic table along the avenue demonstrated here using muonic spectroscopy and modern QED theory.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] J. A. Wheeler, Some consequences of the electromagnetic interaction between μ^- -mesons and nuclei, *Rev. Mod. Phys.* **21**, 133 (1949).
- [2] E. Borie and G. A. Rinker, The energy levels of muonic atoms, *Rev. Mod. Phys.* **54**, 67 (1982).
- [3] C. G. Parthey, A. Matveev, J. Alnis, R. Pohl, T. Udem, U. D. Jentschura, N. Kolachevsky, and T. W. Hänsch, Precision measurement of the hydrogen-deuterium $1s - 2s$ isotope shift, *Phys. Rev. Lett.* **104**, 233001 (2010).
- [4] R. Pohl, F. Nez, G. S. King, A. Antognini, F. Biraben, A. P. Brandt, F. D. Colsher, M. D. Deslattes, A. E. Eilbeck, C. R. Gentry *et al.*, Laser spectroscopy of muonic deuterium, *Science* **330**, 1509 (2010).
- [5] K. Schuhmann, L. M. P. Fernandes, F. Nez, M. A. Ahmed, F. D. Amaro, P. Amaro, F. Biraben, T.-L. Chen, D. S. Covita, A. J. Dax *et al.*, The helion charge radius from laser spectroscopy of muonic helium-3 ions, *Science* **388**, 854 (2025).
- [6] A. Antognini, N. Berger, T. E. Cocolios, R. Dressler, R. Eichler, A. Eggenberger, P. Indelicato, K. Jungmann, C. H. Keitel, K. Kirch *et al.*, Measurement of the quadrupole moment of ^{185}Re and ^{187}Re from the hyperfine structure of muonic x rays, *Phys. Rev. C* **101**, 054313 (2020).
- [7] Y. Yamazaki, H. D. Wohlfahrt, E. B. Shera, M. V. Hoehn, and R. M. Steffen, Discrepancy between theory and experiment in nuclear polarization corrections of muonic ^{208}Pb , *Phys. Rev. Lett.* **42**, 1470 (1979).
- [8] T. Q. Phan, P. Bergem, A. Rüetschi, L. A. Schaller, and L. Schellenberg, Nuclear polarization in muonic ^{90}Zr , *Phys. Rev. C* **32**, 609 (1985).
- [9] P. Bergem, G. Piller, A. Rueetschi, L. A. Schaller, L. Schellenberg, and H. Schneuwly, Nuclear polarization and charge moments of ^{208}Pb from muonic x rays, *Phys. Rev. C* **37**, 2821 (1988).
- [10] C. Piller, C. Gugler, R. Jacot-Guillarmod, L. A. Schaller, L. Schellenberg, H. Schneuwly, G. Fricke, T. Hennemann, and J. Herberz, Nuclear charge radii of the tin isotopes from muonic atoms, *Phys. Rev. C* **42**, 182 (1990).
- [11] I. A. Valuev, G. Colò, X. Roca-Maza, C. H. Keitel, and N. S. Oreshkina, Evidence against nuclear polarization as source of fine-structure anomalies in muonic atoms, *Phys. Rev. Lett.* **128**, 203001 (2022).
- [12] I. Angeli and K. Marinova, Table of experimental nuclear ground state charge radii: An update, *At. Data Nucl. Data Tables* **99**, 69 (2013).
- [13] G. Fricke, C. Bernhardt, K. Heilig, L. Schaller, L. Schellenberg, E. Shera, and C. Dejager, Nuclear ground state charge radii from electromagnetic interactions, *At. Data Nucl. Data Tables* **60**, 177 (1995).
- [14] P. Mazanek, Gemeinsame Auswertung von Messungen myonischer Atome, optischer Isotopieverschiebungen und elastischer Elektronenstreuung zur Festlegung von Radien stabiler und instabiler Kerne unterhalb von $N = 126$ sowie Bestimmung von Kernradien aus μ -Atom-Messungen für Erbium- und Wolframisotope, Ph.D. thesis, Johannes Gutenberg University, 1992.
- [15] B. Dreher, J. Friedrich, K. Merle, H. Rothhaas, and G. Lührs, The determination of the nuclear ground state and

- transition charge density from measured electron scattering data, *Nucl. Phys.* **A235**, 219 (1974).
- [16] G. Fricke and K. Heilig, *Nuclear Charge Radii* (Springer-Verlag, Berlin Heidelberg, 2004).
- [17] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/h3xz-xdxx> for details of the calculations and fitting procedure, which includes Refs. [18–29].
- [18] Z. Sun, I. A. Valuev, and N. S. Oreshkina, Nuclear deformation effects in the spectra of highly charged ions, *Phys. Rev. Res.* **6**, 023327 (2024).
- [19] P. J. Mohr, E. Tiesinga, D. B. Newell, and B. N. Taylor, Codata Internationally recommended 2022 values of the fundamental physical constants, https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=958002 (2024).
- [20] M. J. Martin, Nuclear data sheets for $A = 208$, *Nucl. Data Sheets* **108**, 1583 (2007).
- [21] G. Audi, A. H. Wapstra, and C. Thibault, The Ame2003 atomic mass evaluation: (II). Tables, graphs and references, *Nucl. Phys.* **A729**, 337 (2003).
- [22] National Nuclear Data Center, Reduced transition probabilities or $B(E2; 2^+ \rightarrow 0^+)$ values (National Nuclear Data Center, 2025), <https://www.nndc.bnl.gov/be2/help.html> (Accessed: 2025-03-18).
- [23] J. B. Bellicard and K. J. van Oostrum, Elastic electron scattering from lead-208 at 175 and 250 MeV, *Phys. Rev. Lett.* **19**, 242 (1967).
- [24] J. Heisenberg, R. Hofstadter, J. S. McCarthy, I. Sick, B. C. Clark, R. Herman, and D. G. Ravenhall, Elastic electron scattering by Pb^{208} and new information about the nuclear charge distribution, *Phys. Rev. Lett.* **23**, 1402 (1969).
- [25] I. Sick, The charge density difference between ^{208}Pb and ^{209}Bi , *Nucl. Phys.*, **A208**, 557 (1973).
- [26] H. Euteneuer, J. Friedrich, and N. Voegler, Charge distribution of ^{208}Pb and the difference in $\rho(r)$ for Pb and Tl investigated by elastic electron scattering, *Phys. Rev. Lett.* **36**, 129 (1976).
- [27] H. Euteneuer, J. Friedrich, and N. Voegler, The charge-distribution differences of ^{209}Pb , $^{208,207,206,204}\text{Pb}$, $^{205,203}\text{Tl}$ investigated by elastic electron scattering and muonic x-ray data, *Nucl. Phys.* **A298**, 452 (1978).
- [28] H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, R. D. Barton, and D. Kessler, Precise measurement of the muonic x-rays in the lead isotopes, *Phys. Rev.* **187**, 1565 (1969).
- [29] D. Kessler, H. Mes, A. C. Thompson, H. L. Anderson, M. S. Dixit, C. K. Hargrove, and R. J. McKee, Muonic x rays in lead isotopes, *Phys. Rev. C* **11**, 1719 (1975).
- [30] S. Klarsfeld, Analytical expressions for the evaluation of vacuum-polarization potentials in muonic atoms, *Phys. Lett.* **66B**, 86 (1977).
- [31] V. K. Ivanov, S. S. Baturin, D. A. Glazov, and A. V. Volotka, Vacuum-polarization Wichmann-Kroll correction in the finite-basis-set approach, *Phys. Rev. A* **110**, 032815 (2024).
- [32] Z. Mandrykina, Z. Sun, and N. S. Oreshkina, Wichmann-Kroll correction including finite-nuclear-size effect for muonic atoms (to be published).
- [33] P. Indelicato, Nonperturbative evaluation of some QED contributions to the muonic hydrogen $n = 2$ Lamb shift and hyperfine structure, *Phys. Rev. A* **87**, 022501 (2013).
- [34] S. Breidenbach, E. Dizer, H. Cakir, and Z. Harman, Hadronic vacuum polarization correction to atomic energy levels, *Phys. Rev. A* **106**, 042805 (2022).
- [35] Z. A. Mandrykina, M. Thierfeld, and N. S. Oreshkina, Hadronic vacuum polarization effect in muonic atoms, [arXiv:2509.01311](https://arxiv.org/abs/2509.01311).
- [36] N. S. Oreshkina, Self-energy correction to the energy levels of heavy muonic atoms, *Phys. Rev. Res.* **4**, L042040 (2022).
- [37] V. A. Yerokhin and N. S. Oreshkina, QED calculations of the nuclear recoil effect in muonic atoms, *Phys. Rev. A* **108**, 052824 (2023).
- [38] C. M. Shakin and M. S. Weiss, Nuclear polarization and the muonic x-ray spectrum of Pb, *Phys. Rev. C* **8**, 411 (1973).
- [39] N. Michel, N. S. Oreshkina, and C. H. Keitel, Theoretical prediction of the fine and hyperfine structure of heavy muonic atoms, *Phys. Rev. A* **96**, 032510 (2017).
- [40] A. Knecht (private communication).
- [41] J. Henderson, J. Heery, M. Rocchini, M. Siciliano, N. Sensharma, A. D. Ayangeakaa, R. V. F. Janssens, T. M. Kowalewski, Abhishek, P. D. Stevenson *et al.*, Deformation and collectivity in doubly magic ^{208}Pb , *Phys. Rev. Lett.* **134**, 062502 (2025).