

Complete Gravitational-Wave Spectrum of the Sun

Camilo García-Cely¹ and Andreas Ringwald²

¹*Instituto de Física Corpuscular (IFIC), Universitat de València—CSIC, Parc Científic UV, Carrer del Catedrático José Beltrán 2, E-46980 Paterna, Spain*

²*Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, 22607 Hamburg, Germany*

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The high-temperature plasma in the solar interior generates stochastic gravitational waves (GWs). Owing to its significance as the primary source of high-frequency GWs in the Solar System, we reexamine this phenomenon by highlighting some physical processes, including the contribution of macroscopic hydrodynamic fluctuations. Our analysis builds upon several studies of axion emission from the Sun, particularly in relation to the treatment of plasma effects. The resulting GW spectrum is comparable to many well-motivated early-Universe signals, yet orders of magnitude below the current sensitivities of axion helioscopes such as (Baby)IAXO.

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Introduction—While current observational efforts [1–6] to detect gravitational waves (GWs) have predominantly focused on frequencies below a few kilohertz, a growing community is seriously considering higher frequencies with the hope of detecting early Universe signals [7]. This prompts the identification of GW backgrounds associated with the standard model (SM) at this frequency range. The dominant source of such GWs is the Sun.

A seminal work by Weinberg [8] estimated a total power of approximately 6×10^7 W. Although this figure turns out to be an excellent approximation (we find 1.3×10^8 W), his spectrum was calculated only for soft gravitons from proton and electron collisions and neglected radial dependences—particularly of plasma effects [9,10]—as well as additional contributions such as that of photoproduction [11,12]. Employing a state-of-the-art solar model, we revisit the computation of the solar GW spectrum. For this we rely on several studies of axion emission from the Sun and show that they can be adapted to solar GWs, particularly those related to the treatment of plasma effects. Additionally, we point out the contribution from hydrodynamical fluctuations, which was previously overlooked but is essential for completing the spectrum in its low-frequency range. Our prediction of the complete solar GW power spectrum is summarized in Fig. 1.

We organize the discussion as follows. We start by reviewing properties of the solar plasma that will be

relevant throughout, then we compile all the contributions to the GW spectrum of the Sun and clarify what has been omitted in the literature. Subsequently, we discuss implications for axion helioscopes. In the Supplemental Material [13], we provide the details of our calculations. Throughout, we adopt Heaviside units, take $\hbar = c = 1$, and utilize the Minkowski metric $\eta_{\mu\nu} = \text{diag}(+ - - -)$.

The solar plasma—We model the Sun as a nonrelativistic plasma composed of electrons (e) and nucleons (Z) following a Maxwell-Boltzmann distribution. We take

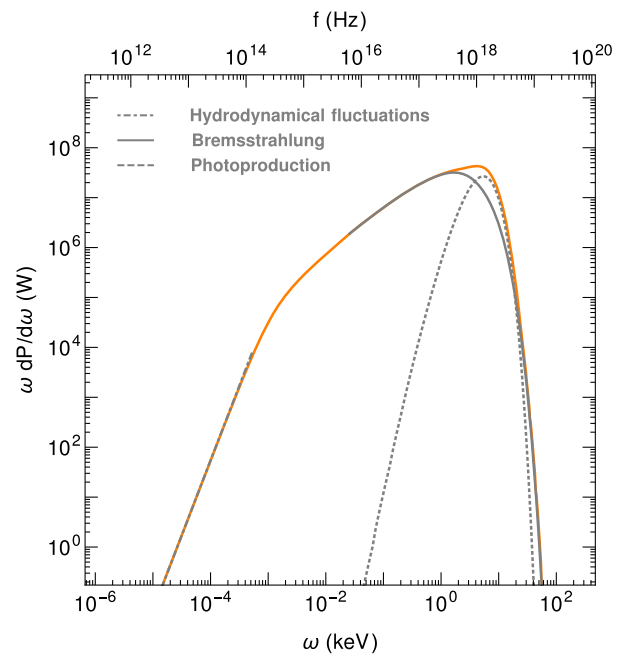


FIG. 1. GW power per unit logarithmic frequency originating from the solar thermal plasma as a function of the frequency, $\omega = 2\pi f$.

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the corresponding temperature and number density distributions from the B16-GS98 solar model [14] and estimate the electron density assuming full ionization. Elements heavier than helium are negligible and are excluded from the analysis.

GWs originate through microscopic and macroscopic mechanisms, respectively corresponding to particle collisions emitting one graviton, h [15], and GW emission sourced by hydrodynamic fluctuations. The former corresponds to frequencies much larger than that of collisions in the solar plasma, so there is sufficient time for them not to interfere with each other [17]. Below all collision frequencies of the plasma, fluctuations are large compared with the characteristic microscopic dimensions, allowing a hydrodynamical approach to calculate GW emission [18,19].

The estimation of the collision frequencies for each species, $\omega_c^{(i)}$, is therefore of utmost importance for this Letter. By following the approach detailed next, we obtain the results presented in Fig. 2. We first calculate the viscosity cross section, σ_V . For instance, for eZ ,

$$\sigma_V^{(eZ)} = \int \frac{d\sigma^{(eZ)}}{d\Omega} \sin^2\theta d\Omega \simeq \frac{2\pi\alpha^2 Z^2}{E_i^2} L_{\text{Coulomb}}, \quad (1)$$

and we proceed similarly for ee . The collision frequencies are then estimated from the corresponding scattering rate as $\omega_c^{(e)} = n_e \sigma_V^{(ee)} v + \sum_Z n_Z \sigma_V^{(eZ)} v$.

To define L_{Coulomb} , we note the following. Screening gives rise, in the nonrelativistic limit, to an effective Yukawa potential with a range given by the Debye-Hückel screening scale, $\kappa = [4\pi\alpha(n_e + \sum_Z Z n_Z)/T]^{1/2}$ [9]; see Fig. 2. For instance, for eZ collisions,

$V_{\text{Coulomb}} = -Z\alpha/r \rightarrow -Zae^{-\kappa r}/r$. For nonrelativistic collisions, the scattering rate associated with these potentials may be nonperturbative. Nevertheless, in the Sun the scattering largely occurs either in the Born regime, where perturbation theory is applicable, or in the semiclassical regime, where classical physics can be employed. In these cases, the viscosity cross section is given by the last equality in Eq. (1), where the Coulomb logarithm, L_{Coulomb} , equals $\log 2p_i/\kappa$ and $\log E_i/(Z^2\alpha\kappa)$ in the Born and semiclassical approximations, respectively [20,21]. Throughout, p_i (p_f) denotes the initial (final) momentum of the colliding particles in the center-of-mass frame, and E_i is their total kinetic energy.

We note that the collision frequencies should be regarded as transition scales, particularly because Eq. (1) is valid only up to logarithmic accuracy [22], and different prescriptions exist for the velocity averaging [20]. Here we follow [17] and take v as equal to its thermal average. For further details, see the Supplemental Material [13].

GWs from hydrodynamical fluctuations—We closely follow Ref. [18] to calculate this contribution. In essence, GWs in this limit are sourced by tensor fluctuations of the energy-momentum tensor, whose Fourier transform is largely independent of the frequency and wave number. Being purely hydrodynamical, the tensor fluctuations are simply proportional to the shear viscosity, η , of the plasma. Plugging them into Einstein's equations, one obtains

$$\left. \frac{dP}{d\omega} \right|_{\text{hydrodynamics}} = \frac{16G\omega^2}{\pi} \int_{\text{Sun}} d^3\mathbf{r} \eta T. \quad (2)$$

While this emission has not been previously investigated for the Sun or stellar plasmas, it is known that it contributes to the GW emission from the early Universe plasma, that is, to the cosmic gravitational microwave background (CGMB) [18,23,24]. The only conceptual difference here is that viscosity must be computed differently because the solar plasma is nonrelativistic.

Owing to the significant mass ratio between protons and electrons, the primary source of shear viscosity may be attributed to momentum transfer involving protons. For this we note that proton-proton collisions in the Sun are semiclassical. Moreover, the proton component of the solar plasma is not strongly correlated, meaning that the average Coulomb potential energy is much smaller than the thermal energy (which follows from $\alpha n_p^{1/3}/T \sim 0.03\text{--}0.07 \ll 1$). This justifies employing the numerical fits to the state-of-the-art simulations reported by Ref. [25]—more precisely, its Eq. (31)—for the calculation of the viscosity. In practice, this is merely a slight modification of the Landau-Spitzer theory, which predicts $\eta \sim m_p v_p / \sigma_V^{(pp)}$ [22]. Let us remark that Eq. (2) loses its validity at very low frequencies—where large-scale solar phenomena must contribute to the GW signal [26]—or when $\omega \gtrsim \omega_c^{(p)}$ as sound waves

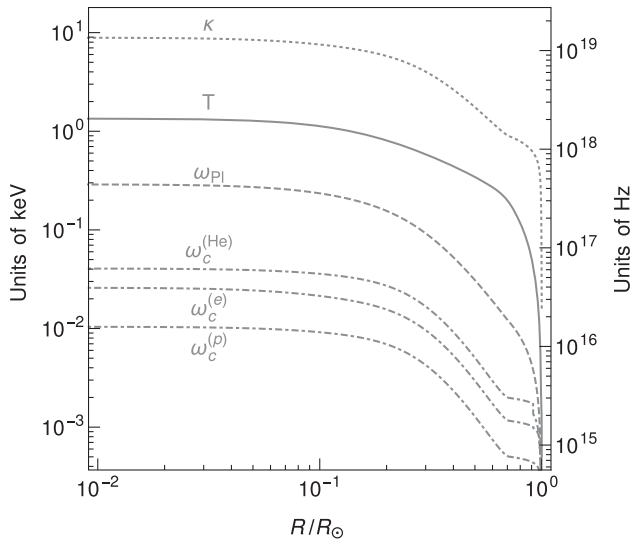


FIG. 2. Screening scale (dotted line), temperature (solid line), plasma frequency (dashed line), and collision frequencies (dot-dashed lines) as a function of distance from the center for the adopted solar model [14]. See the text for details.

associated with the hydrodynamical fluctuations are damped by viscosity [28].

The solar GW power spectrum from hydrodynamic fluctuations is given by the dash-dotted line in Fig. 1, where we conservatively put a cutoff for frequencies above $0.05\omega_c^{(p)}|_{\text{center}}$ in light of damping.

GWs from particle collisions—This contribution to the solar GW spectrum is obtained by thermally averaging individual graviton emission rates,

$$\frac{dP}{d\omega}\Big|_{\text{collisions}} = \int_{\text{Sun}} d^3\mathbf{r} \sum_i \omega \left\langle \frac{d\Gamma^{(i)}(\mathbf{r})}{d\omega dV} \right\rangle. \quad (3)$$

Here i runs over the processes detailed below, whose rates, to leading order in the screening scale, are summarized in Table 1 of the Supplemental Material [13]. These rates are calculated in the traceless-transverse gauge [29] by squaring the relevant amplitudes using CalcHEP [31] and FeynRules [32] and taking the nonrelativistic limit for electrons and nucleons (see also [33–35]). We assume spherical symmetry for the integration in the Sun.

Bremsstrahlung ($eZ \rightarrow eZh$ and $ee \rightarrow eeh$)—We account for plasma effects following the bremsstrahlung treatment of Ref. [10]. This entails modifying the propagator of the photon exchanged by the charged particles with an effective mass equal to κ , as suggested by the aforementioned Yukawa potential. In addition to corrections of order κ^2/p_i^2 , this introduces an infrared cutoff. This is essential because, in the absence of screening, the rates exhibit a logarithmic divergence for soft gravitons. A more rigorous formulation can be achieved using thermal field theory, wherein the inclusion of hard thermal loop self-energies in the propagators naturally regulates such divergences across a broad frequency range, as has been done for axions; see [36]. In this formalism, the screening scale squared κ^2 appears as a photon self-energy in the static limit ($\omega \rightarrow 0$) [37,38].

Notably, our explicit computations reveal that the bremsstrahlung rates depend on a common regulating logarithm, which for soft gravitons approaches the one entering the viscosity cross section of Eq. (1). This is not a coincidence: soft theorems [8] arising from basic principles of quantum field theory dictate that $\omega d\Gamma^{(eZ)}/d\omega dV|_{\text{soft}} = (32G/5\pi)E_i^2 \sin^2\theta n_e n_Z \sigma_V^{(eZ)} v$ for $\omega \ll E_i \sim T$, and there is a similar expression for ee with a factor $1/2$ [39]. Applying this to the center of the Sun is how Weinberg [8,17] originally found a flat power spectrum, $dP/d\omega$, with only a minor logarithmic dependence. Interestingly, despite its quantum-mechanical origin, this formula coincides [17,41,42] with the classical emission rate of GWs associated with the quadrupole of the eZ/ee hyperbolic orbit for sufficiently soft radiation [43]. Beyond the soft limit, our rate and the classical rate [46] differ, with the latter overestimating the spectrum because collisions in the

Sun’s interior are outside the classical regime [41]; see the Supplemental Material [13].

Weinberg’s calculation motivated studies of bremsstrahlung beyond the soft limit (see also [20,47,48]). References [49,50] reported rates associated with eZ (but not ee) using methods similar to ours but assuming an infinitely heavy nucleon and no screening. Likewise, bremsstrahlung rates neglecting screening were calculated while assuming that the emission comes from one electron moving in the Coulomb potential of a nucleon [51,52] or another electron [41]. Our results agree with these studies in the appropriate limit [53].

It is worth mentioning that Ref. [41] calculated ee bremsstrahlung rates in the nonrelativistic limit through the following clever insight: photon bremsstrahlung from ee results from their quadrupole, as their dipole vanishes by their indistinguishability, which allows the ee electromagnetic emission to be adapted to gravitational radiation, with the adjustment being a mere global factor. Our rates, obtained through a different approach, support this conclusion.

Finally, let us remark that our procedure relies on the validity of the Born approximation, which for ee and eZ holds throughout the Sun, except in the outermost regions, approximately the last 15% of the solar radius. Since bremsstrahlung is dominated by processes at the center, neglecting nonperturbative corrections to the graviton emission is justified. The solar GW power spectrum from bremsstrahlung is given by the solid line in Fig. 1, where we put a cutoff for frequencies below $\omega_c^{(e)}|_{\text{center}}$. We remark that for this we use the full rates reported in the Supplemental Material [13], where further details are provided.

Photoproduction ($\gamma Z/e \rightarrow Z/eh$)—This is analogous to the Primakoff and Compton production of axions. Our results with $\kappa = \omega_{\text{pl}} = 0$ coincide with the differential rates for graviton emission reported in Ref. [11]. Nevertheless, from this it is impossible to make sense of the integrated rate, as it diverges due to the collinear divergence appearing at $\theta = 0$. To tackle this problem, we again employ insights from axion physics [10]. Concretely, we note that charged particles in the solar plasma mutually interact through their Coulomb fields, leading to a pair correlation function [10,54,55]. This gives rise to an effective form factor, $F(\theta)$, that depends on the screening scale κ and regularizes the total rate.

If one neglects such correlations and instead considers photons scattering off individual screened charges, one finds a rate with $|F(\theta)| \rightarrow |F(\theta)|^2$ [56]. In practice, this corresponds to modifying the photon propagator with an effective mass, a picture which is appropriate if the scattering process is so slow that the charged particles can move around and rearrange themselves [38]. This issue has been revisited recently for the Primakoff process of axions, which accounts for the velocity in the form factor [57]. From this study the difference between the two

approaches is expected to be negligible. See the Supplemental Material [13] for a detailed discussion.

Note also that the dispersion relation of the initial-state photon is influenced by the surrounding plasma, which imparts an effective mass to transverse photons equal to the plasma frequency, $\omega_{\text{pl}} = (4\pi n_e \alpha / m_e)^{1/2}$; temperature-dependent corrections are negligible, as the plasma is nonrelativistic and nondegenerate. Moreover, since the solar temperature is much greater than ω_{pl} (see Fig. 2), the kinetic energies of all particles are well above the plasma frequency, and the GW spectrum remains largely insensitive to the photon's effective mass, a fact we have verified explicitly. The main consequence is the phase-space factor p_i/ω , that provides a lower cutoff to the photoproduction contribution at each point of the Sun. We note that this is completely analogous to the Primakoff effect for axions, which we discuss in the Supplemental Material [13] for completeness. It is noteworthy that Ref. [12] calculated photoproduction rates using semiclassical methods and including the effect of the plasma frequency but did not account for screening, which is the dominant effect.

The resulting solar GW power spectrum from photoproduction is given by the dashed line in Fig. 1.

Transition regime and other processes—As it shifts from macroscopic to microscopic fluctuations, $dP/d\omega$ transitions from $\propto \omega^2$ to $\propto \omega^0$ within the range $\omega_c^{(p)} \lesssim \omega \lesssim \omega_c^{(e)}$. As in the case of the CGMB, a thorough examination of this regime necessitates a hydrodynamical simulation. Nevertheless, we postulate a double-broken power law, $\omega^2/(\omega^2 + \omega_0^2)$. The Landau-Spitzer viscosity for protons give $\omega_0 \approx (\omega_c^{(p)} \omega_c^{(e)})^{1/2} \propto 1/\eta$. Notably, the interpolating law with the predicted scale $\omega_0 \propto 1/\eta$ is also found in analytical studies of the energy-momentum tensor for relativistic plasmas in the so-called shear channel [18]. It is also worth mentioning that power shifts from $dP/d\omega \propto \eta \sim 1/\sigma_V$ to soft bremsstrahlung, where $dP/d\omega \propto \sigma_V$. Interestingly, the same behavior is found for the CGMB because $\sigma_V \sim \alpha^2/T^2$.

Additionally, we find that bremsstrahlung—also known as free-free emission—largely dominates over free-bound transitions—recombination of unbound charged particles into atomic states and a graviton—and bound-bound transitions—graviton emission from atomic transitions. In contrast, such processes are important for axions [58,59], giving the axion solar spectrum distinct small peaks depending on the solar model [57]. While a similar effect might be expected for GWs due to the analogy with axions, the quadrupolar nature of graviton emission renders free-bound and bound-bound contributions negligible, as detailed in Sec. III of the Supplemental Material [13], following [60–63]. Hence, the GW spectrum from the Sun lacks such small peaks.

The same applies to photoproduction from longitudinal photons at frequencies ω_{pl} , particularly in the presence of

the solar magnetic field [57,64–66]. This follows from the fact that gravitons couple uniquely to photons and fermions, leading to fixed relative contributions in the solar GW spectrum. In contrast, axions possess model-dependent couplings to photons and fermions, allowing scenarios where bremsstrahlung is subdominant compared to such photoproduction processes. In the light of this, the latter hardly contribute to the GW spectrum around $\omega_{\text{pl}} \sim 0.3$ keV or lower where bremsstrahlung dominates by more than 5 orders of magnitude.

Discussion and outlook—Our interpolation of the solar GW power spectrum, including all processes, is presented in Fig. 1. In terms of power, the relative contribution of each process is as follows: 25% for photoproduction, 33% for ee bremsstrahlung, 24% for ep bremsstrahlung, and 17% for eHe bremsstrahlung, with the rest attributed to the hydrodynamical contribution. While the spectrum over several decades in frequency was not studied before, corrections of a few to the above percentages—and correspondingly to the overall spectrum—are expected, especially after accounting for degeneracy and effects beyond the Born approximation, as for solar axions [57].

Axion helioscopes [67] can be used as detectors of gravitational radiation exploiting the inverse Gertsenshtein effect [68,69], by which GWs convert to electromagnetic waves in the presence of an external magnetic field. Reference [70] presented an exclusion bound from CAST [71] and sensitivity prospects for IAXO [72] and then compared them to Weinberg's prediction of the solar GW spectrum. In Fig. 3 we present their findings in terms of the GW strain, $h_c \equiv 2(G dP/d\omega)^{1/2}/d_{\text{Sun}}\omega^{1/2}$, together with the complete spectrum as derived in Fig. 1. For comparison, we also display the CGMB, whose overall normalization depends upon the (unknown) maximum temperature of the primordial plasma [24]. In addition, we show the cosmic dark radiation bound, arising from the fact that GWs contribute to the energy budget of the Universe in the form of radiation and are thus limited by big bang nucleosynthesis and cosmic microwave background observations [7,73]. In addition, we derive prospects for BabyIA XO following Refs. [24,72]. The main challenge for GW detection in helioscopes lies in technological limitations related to magnetic fields and scalability. The strain sensitivity of a typical helioscope scales as $h_c \propto BLA^{1/2}$, where B is its magnetic field, L is its length, and A is its cross section. IAXO achieves $BLA^{1/2} \approx 90 \text{ T m}^2$ [24], yielding $h_c \sim 10^{-29}$ at 10^{18} Hz, which is still ~ 15 orders above the Sun spectrum; see Fig. 3. If these challenges are eventually overcome and the spectrum is measured with sufficient spatial and spectral resolution, this could provide a valuable window into the inner structure of the Sun, much like what has been envisioned for axions (see, e.g., [74]).

For further comparison, we include an estimate of the stochastic background associated with main-sequence stars

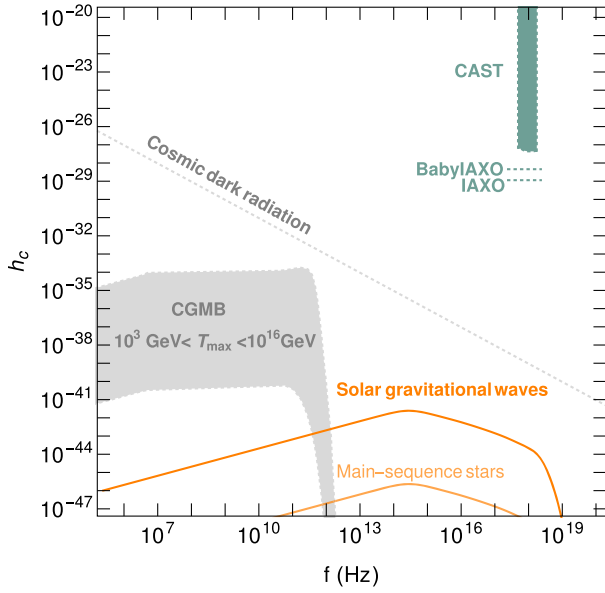


FIG. 3. Characteristic GW strain versus frequency. Exclusion bound and corresponding projected sensitivities of current (CAST) and future (BabyIAXO, IAXO) axion helioscopes, together with the corresponding predictions from the solar GW spectrum in Fig. 1, an estimate of the stochastic background from the main-sequence stars in the Milky Way, the CGMB spectrum, and the bound from the total radiation in the Universe.

in the Galaxy. Noting that these stars have masses similar to the Sun [75], we approximate their GW spectrum using that of the Sun, leading to $h_{c\text{stars}} = \sqrt{\int dV n_{\text{stars}} d_{\text{Sun}}^2 / d^2} h_{c\text{Sun}}$. A similar estimation has been performed in Ref. [76] for stellar neutrinos. Adapting their results for the ${}^7\text{Be}$ line, we find $h_{c\text{stars}} \sim 10^{-4} h_{c\text{Sun}}$. We note that this figure should be regarded as an order-of-magnitude estimate. Our calculation of solar GWs reveals a clear similarity between gravitons and axions. Expanding on this idea, we could consider the emission of massive spin-2 particles from the Sun, which could also constitute the dark matter of the Universe; see, e.g., [77–79]. Extending the methods discussed here to such particles, we find that the resulting spectrum is nearly identical to that of the solar GWs, differing only by an overall factor associated with the coupling of spin-2 particles to SM fields. In a separate publication [80], we will discuss this synergy and present observational prospects at helioscopes.

Our study represents a step toward establishing a multi-frequency GW background from SM processes that is similar to that of neutrinos [81,82]. In fact, our approach can be extended to other stellar plasmas [83] and to the early stages of the Universe after nucleosynthesis, where plasmas are also nonrelativistic. Whether we can aspire to probe such strain sensitivities in the future remains an open question.

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- [1] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), GW170817: Observation of gravitational waves from a binary neutron star inspiral, *Phys. Rev. Lett.* **119**, 161101 (2017).
- [2] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
- [3] J. Antoniadis *et al.* (EPTA and InPTA Collaborations), The second data release from the European Pulsar Timing Array—III. Search for gravitational wave signals, *Astron. Astrophys.* **678**, A50 (2023).
- [4] D. J. Reardon *et al.*, Search for an isotropic gravitational-wave background with the Parkes pulsar timing array, *Astrophys. J. Lett.* **951**, L6 (2023).
- [5] G. Agazie *et al.* (NANOGrav Collaboration), The NANOGrav 15 yr data set: Evidence for a gravitational-wave background, *Astrophys. J. Lett.* **951**, L8 (2023).
- [6] H. Xu *et al.*, Searching for the nano-hertz stochastic gravitational wave background with the Chinese pulsar timing array data release I, *Res. Astron. Astrophys.* **23**, 075024 (2023).
- [7] N. Aggarwal *et al.*, Challenges and opportunities of gravitational wave searches above 10 kHz, [arXiv:2501.11723](https://arxiv.org/abs/2501.11723).
- [8] S. Weinberg, Infrared photons and gravitons, *Phys. Rev.* **140**, B516 (1965).
- [9] E. E. Salpeter, Electrons screening and thermonuclear reactions, *Aust. J. Phys.* **7**, 373 (1954).
- [10] G. G. Raffelt, Astrophysical axion bounds diminished by screening effects, *Phys. Rev. D* **33**, 897 (1986).
- [11] N. A. Voronov, Gravitational Compton effect and photo-production of gravitons by electrons, *Sov. Phys. JETP* **37**, 953 (1973).
- [12] D. V. Galtsov, Photocoulomb gravitons and gravitational emittance of the Sun, *Zh. Eksp. Teor. Fiz.* **67**, 425 (1974).
- [13] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/gtwg-pr41> for derivations supporting the emission rates of GWs from solar processes, including the cross sections used in collision rate computations (Sec. I), the Feynman rules for graviton emission (Sec. II), analytic expressions for bremsstrahlung and photoproduction rates (Sec. III), and a discussion of bound-bound and free-bound transitions (Sec. IV).
- [14] N. Vinyoles, A. M. Serenelli, F. L. Villante, S. Basu, J. Bergström, M. C. Gonzalez-Garcia, M. Maltoni,

- C. Peña Garay, and N. Song, A new generation of standard solar models, *Astrophys. J.* **835**, 202 (2017).
- [15] The emission of multiple gravitons becomes relevant only at Planck-scale temperatures [16].
- [16] J. Ghiglieri, M. Laine, J. Schütte-Engel, and E. Speranza, Double-graviton production from standard model plasma, *J. Cosmol. Astropart. Phys.* **04** (2024) 062.
- [17] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, New York, 1972), Sec. 10.4.
- [18] J. Ghiglieri and M. Laine, Gravitational wave background from standard model physics: Qualitative features, *J. Cosmol. Astropart. Phys.* **07** (2015) 022.
- [19] E. Lifshitz and L. Pitaevskii, *Statistical Physics, Part 2* (Butterworth-Heinemann, Oxford, 1980), pp. 88 and 89.
- [20] S. Weinberg, Soft bremsstrahlung, *Phys. Rev. D* **99**, 076018 (2019).
- [21] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and S. K. Zhdanov, Scattering in the attractive Yukawa potential in the limit of strong interaction, *Phys. Rev. Lett.* **90**, 225002 (2003).
- [22] L. Pitaevskii and E. Lifshitz, *Physical Kinetics*, Vol. 10 (Elsevier Science, New York, 2012), p. 41.
- [23] J. Ghiglieri, G. Jackson, M. Laine, and Y. Zhu, Gravitational wave background from standard model physics: Complete leading order, *J. High Energy Phys.* **07** (2020) 092.
- [24] A. Ringwald, J. Schütte-Engel, and C. Tamarit, Gravitational waves as a big bang thermometer, *J. Cosmol. Astropart. Phys.* **03** (2021) 054.
- [25] J. Daligault, K. O. Rasmussen, and S. D. Baalrud, Determination of the shear viscosity of the one-component plasma, *Phys. Rev. E* **90**, 033105 (2014).
- [26] High-Reynolds-number turbulence from stellar convection has been claimed to contribute at micro-Hertz scales a GW power $\omega dP/d\omega \approx 4 \times 10^{-12}$ W [27].
- [27] M. F. Bennett and A. Melatos, Stochastic microhertz gravitational radiation from stellar convection, *Astrophys. J.* **792**, 55 (2014).
- [28] P. Klose, M. Laine, and S. Procacci, Gravitational wave background from non-Abelian reheating after axion-like inflation, *J. Cosmol. Astropart. Phys.* **05** (2022) 021.
- [29] At these frequencies, the Sun is effectively in free fall and thus at rest in this gauge, resulting in no medium response; see, e.g., [30].
- [30] W. Ratzinger, S. Schenk, and P. Schwaller, A coordinate-independent formalism for detecting high-frequency gravitational waves, *J. High Energy Phys.* **08** (2024) 195.
- [31] A. Belyaev, N. D. Christensen, and A. Pukhov, CalcHEP 3.4 for collider physics within and beyond the standard model, *Comput. Phys. Commun.* **184**, 1729 (2013).
- [32] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, FeynRules 2.0—A complete toolbox for tree-level phenomenology, *Comput. Phys. Commun.* **185**, 2250 (2014).
- [33] S. Y. Choi, J. S. Shim, and H. S. Song, Factorization and polarization in linearized gravity, *Phys. Rev. D* **51**, 2751 (1995).
- [34] J. F. Donoghue, General relativity as an effective field theory: The leading quantum corrections, *Phys. Rev. D* **50**, 3874 (1994).
- [35] H. H. Patel, Package-X 2.0: A Mathematica package for the analytic calculation of one-loop integrals, *Comput. Phys. Commun.* **218**, 66 (2017).
- [36] T. Altherr and U. Kraemmer, Gauge field theory methods for ultradegenerate and ultrarelativistic plasmas, *Astropart. Phys.* **1**, 133 (1992).
- [37] J. I. Kapusta, *Finite Temperature Field Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1989).
- [38] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics: The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles* (University of Chicago Press, Chicago, 1996).
- [39] Although Refs. [8,17] seemingly omitted it, this symmetrization factor is explicitly included in the pp chain [40] and in axion bremsstrahlung from ee [10].
- [40] H. A. Bethe and C. L. Critchfield, The formation of deuterium by proton combination, *Phys. Rev.* **54**, 248 (1938).
- [41] R. J. Gould, The graviton luminosity of the Sun and other stars, *Astrophys. J.* **288**, 789 (1985).
- [42] A. M. Steane, Gravitational bremsstrahlung in plasmas and clusters, *Phys. Rev. D* **109**, 063032 (2024).
- [43] Note that this is in agreement with the arguments presented in [44] concerning the treatment of a classical GW as a collection of individual gravitons (see also [45]).
- [44] D. Carney, V. Domcke, and N. L. Rodd, Graviton detection and the quantization of gravity, *Phys. Rev. D* **109**, 044009 (2024).
- [45] F. Dyson, Is a graviton detectable?, *Int. J. Mod. Phys. A* **28**, 1330041 (2013).
- [46] M. Carmeli, Gravitational bremsstrahlung in nonrelativistic collisions, *Phys. Rev.* **158**, 1243 (1967).
- [47] J. Pradler and L. Semmelrock, Accurate Gaunt factors for nonrelativistic quadrupole bremsstrahlung, *Astrophys. J.* **916**, 105 (2021).
- [48] R. Flauger and S. Weinberg, Absorption of gravitational waves from distant sources, *Phys. Rev. D* **99**, 123030 (2019).
- [49] B. M. Barker, S. N. Gupta, and J. Kaskas, Graviton bremsstrahlung and infrared divergence, *Phys. Rev.* **182**, 1391 (1969).
- [50] G. Papini and S. R. Valluri, Gravitons in Minkowski space-time. Interactions and results of astrophysical interest, *Phys. Rep.* **33**, 51 (1977).
- [51] D. V. Gal'tsov and Y. V. Grats, Gravitational radiation from Coulomb collisions, *Sov. Phys. J.* **17**, 1713 (1974).
- [52] R. J. Gould, Quadrupole bremsstrahlung in the scattering of identical charged bosons and fermions, *Phys. Rev. A* **23**, 2851 (1981).
- [53] A recent attempt to interpolate between the external field approximation and the semiclassical approach was given in Ref. [42]. Our results with no screening do not match theirs as they neglect electrons' indistinguishability.
- [54] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1* (Butterworth-Heinemann, Oxford, 1980), p. 79.
- [55] G. G. Raffelt, Plasmon decay into low mass bosons in stars, *Phys. Rev. D* **37**, 1356 (1988).
- [56] W. K. De Logi and A. R. Mickelson, Electrogravitational conversion cross-sections in static electromagnetic fields, *Phys. Rev. D* **16**, 2915 (1977).

- [57] S. Hoof, J. Jaeckel, and L. J. Thormaehlen, Quantifying uncertainties in the solar axion flux and their impact on determining axion model parameters, *J. Cosmol. Astropart. Phys.* **09** (2021) 006.
- [58] J. Redondo, Solar axion flux from the axion-electron coupling, *J. Cosmol. Astropart. Phys.* **12** (2013) 008.
- [59] M. Pospelov, A. Ritz, and M. B. Voloshin, Bosonic super-WIMPs as keV-scale dark matter, *Phys. Rev. D* **78**, 115012 (2008).
- [60] S. Boughn and T. Rothman, Aspects of graviton detection: Graviton emission and absorption by atomic hydrogen, *Classical Quantum Gravity* **23**, 5839 (2006).
- [61] T. Rothman and S. Boughn, Can gravitons be detected?, *Found. Phys.* **36**, 1801 (2006).
- [62] V. Berestetskii, E. Lifshitz, and L. Pitaevskii, *Quantum Electrodynamics, Vol. 4*, Course of Theoretical Physics (Elsevier Science, New York, 1982).
- [63] V. A. Baturin, S. V. Ayukov, A. V. Oreshina, A. B. Gorshkov, V. K. Gryaznov, I. L. Iosilevskiy, and W. Däppen, Hydrogen ionization inside the Sun, *Solphys.* **300**, 3 (2025).
- [64] A. Caputo, A. J. Millar, and E. Vitagliano, Revisiting longitudinal plasmon-axion conversion in external magnetic fields, *Phys. Rev. D* **101**, 123004 (2020).
- [65] C. A. J. O'Hare, A. Caputo, A. J. Millar, and E. Vitagliano, Axion helioscopes as solar magnetometers, *Phys. Rev. D* **102**, 043019 (2020).
- [66] E. Guarini, P. Carenza, J. Galan, M. Giannotti, and A. Mirizzi, Production of axionlike particles from photon conversions in large-scale solar magnetic fields, *Phys. Rev. D* **102**, 123024 (2020).
- [67] P. Sikivie, Experimental tests of the invisible axion, *Phys. Rev. Lett.* **51**, 1415 (1983); **52**, 695(E) (1984).
- [68] M. E. Gertsenshtein, Wave resonance of light and gravitational waves, *Sov. Phys. JETP* **14**, 84 (1962).
- [69] D. Boccaletti, V. De Sabbata, P. Fortini, and C. Gualdi, Conversion of photons into gravitons and vice versa in a static electromagnetic field, *Nuovo Cimento B* **70**, 129 (1970).
- [70] A. Ejlli, D. Ejlli, A. M. Cruise, G. Pisano, and H. Grote, Upper limits on the amplitude of ultra-high-frequency gravitational waves from graviton to photon conversion, *Eur. Phys. J. C* **79**, 1032 (2019).
- [71] V. Anastassopoulos *et al.* (CAST Collaboration), New CAST limit on the axion-photon interaction, *Nat. Phys.* **13**, 584 (2017).
- [72] E. Armengaud *et al.* (IAXO Collaboration), Physics potential of the International Axion Observatory (IAXO), *J. Cosmol. Astropart. Phys.* **06** (2019) 047.
- [73] N. Aggarwal *et al.*, Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies, *Living Rev. Relativity* **24**, 4 (2021).
- [74] S. Hoof, J. Jaeckel, and L. J. Thormaehlen, Axion helioscopes as solar thermometers, *J. Cosmol. Astropart. Phys.* **10** (2023) 024.
- [75] J. Bovy, Stellar inventory of the solar neighbourhood using Gaia DR1, *Mon. Not. R. Astron. Soc.* **470**, 1360 (2017).
- [76] E. Brocato, V. Castellani, S. Degl'Innocenti, G. Fiorentini, and G. Raimondo, Stars as galactic neutrino sources, *Astron. Astrophys.* **333**, 910 (1998).
- [77] E. Babichev, L. Marzola, M. Raidal, A. Schmidt-May, F. Urban, H. Veermäe, and M. von Strauss, Bigravitational origin of dark matter, *Phys. Rev. D* **94**, 084055 (2016).
- [78] E. Babichev, L. Marzola, M. Raidal, A. Schmidt-May, F. Urban, H. Veermäe, and M. von Strauss, Heavy spin-2 dark matter, *J. Cosmol. Astropart. Phys.* **09** (2016) 016.
- [79] X. Chu and C. Garcia-Cely, Self-interacting spin-2 dark matter, *Phys. Rev. D* **96**, 103519 (2017).
- [80] J. Galán, C. García-Cely, and A. Ringwald (to be published).
- [81] E. Vitagliano, J. Redondo, and G. Raffelt, Solar neutrino flux at keV energies, *J. Cosmol. Astropart. Phys.* **12** (2017) 010.
- [82] E. Vitagliano, I. Tamborra, and G. Raffelt, Grand unified neutrino spectrum at Earth: Sources and spectral components, *Rev. Mod. Phys.* **92**, 045006 (2020).
- [83] See [41] for a discussion of white dwarfs and neutron stars.