

Vorticity-Crystalline Order Coupling in Supersolids: Excitations and Reentrant Phases

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Rotation is a natural tool in ultracold gases to break time-reversal symmetry, yet its impact on the collective excitations of supersolids remains largely unexplored. We show theoretically that tuning the rotation frequency, rather than the interparticle interactions, can trigger the superfluid-to-supersolid transition in Bose-Einstein condensates. Computing excitation spectra in the presence of vortices and persistent currents, we uncover a vortex-driven *desoftening* mechanism whereby quantized vorticity elevates the gapless Goldstone mode to a finite-energy roton, restoring superfluidity. This effect results in reentrant supersolid phases as a function of rotation frequency, revealing a fundamental coupling between topological defects and crystalline order.

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Breaking time-reversal symmetry (TRS) underlies diverse phenomena from cosmological matter-antimatter asymmetry [1,2] to chiral superconductivity [3]. In condensed matter, magnetic fields break TRS and lift Kramers degeneracy [4], enabling the quantum Hall effect [5,6] and access to nontrivial topological states [7,8]. Additionally, the competition between TRS-breaking fields and internal interactions often drives complex phase topographies, such as reentrant superconductivity [9] and fractional quantum Hall transitions [10]. Analogously, in ultracold Bose and Fermi gases, external rotation breaks TRS by acting as a synthetic magnetic field [11,12]. This induces the well-known hallmarks of superfluidity such as quantized vortices [13,14] and persistent currents [15,16] (PCs), offering a highly controllable platform for exploring TRS-broken effects [17,18].

Externally applied rotating magnetic fields have made it possible to nucleate vortices in dipolar Bose-Einstein condensates (dBECs) in the superfluid (SF) as well as the emergent supersolid (SS) phases [19,20]. The SS phase [21–25], defined by the simultaneous breaking of global gauge and continuous translational symmetries, has now been realized in several ultracold platforms [26–28] including lanthanide atomic gases [29–32], light-mediated cavity systems [33], and spin-orbit-coupled Bose gases [34]. Microscopically, the transition is driven by the softening of a finite-momentum roton mode [35–41], i.e., the

vanishing of the roton gap, which signals the shift from a homogeneous SF to a density-modulated SS state.

While such softening could be induced by flow velocities exceeding the Landau critical limit in superfluid ⁴He [42], current dipolar experiments [29–31,40,41,43–45] typically access the roton instability through the precise tuning of anisotropic interparticle interactions instead of hydrodynamic flow. The resulting SS state comes with a rich hierarchy of collective excitations [46–53], including Higgs and Goldstone modes associated with spontaneous symmetry breaking [40,47]. While these modes can reveal key system properties such as the order of the phase transition [54], they have so far been explored almost exclusively within the manifold of time-reversal-invariant states.

In this Letter, we theoretically uncover spectral signatures of TRS breaking in rotating dBECs, revealing how the interplay between rotation and long-range interactions reconfigures a superfluid phase. Combining analytical insight with numerical calculations of excitation spectra in geometries hosting vortices and PC, we show that rotation can induce supersolidity in otherwise SF regimes. Crucially, vortex nucleation in the ensuing SS state can effectively *desoften* the Goldstone mode by lifting it back to a finite-energy roton excitation which restores the SF state. This novel mechanism yields a sequence of reentrant supersolid pockets as a function of rotation frequency, where the discrete entry of vortices periodically suppresses and restores crystalline order. Our results open an experimentally feasible route to probe the impact of TRS breaking on dipolar supersolids, revealing a hitherto unknown fundamental coupling between quantized vorticity and density modulation.

Theoretical model—Let us consider a dilute, weakly interacting gas of ultracold dipolar atoms confined in a

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rotationally symmetric potential $V_{\text{trap}}(\mathbf{r})$ and subjected to an external rotation with angular velocity Ω . The short-range contact and long-range dipolar potential are modeled by $U_c = g\delta(\mathbf{r})$ with $g = 4\pi\hbar^2 a_s/M$ and $U_{\text{dd}} = (3\hbar^2 a_{\text{dd}}/M)[1 - 3\cos^2\Theta/r^3]$, respectively. Here, M is the atomic mass, a_s is the tunable [55,56] s -wave scattering length, and a_{dd} denotes the dipolar length. Θ is the angle between $\mathbf{r}$ and the polarization direction, taken along z .

To determine the ground state condensate wave function $\Psi(\mathbf{r}, t)$ for a given Ω , we minimize the rotating-frame energy functional $E' = E - \Omega L_z$ by solving the extended Gross-Pitaevskii equation (eGPE) in imaginary time. The eGPE includes the Lee-Huang-Yang (LHY) beyond-mean-field correction within the local-density approximation [57–59], characterized by the coefficient g_{QF} (see Appendix A for details).

The excitation spectrum is obtained by linearizing the eGPE around the rotational ground state Ψ_0 , $\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r})e^{-i\mu t/\hbar} + \delta\Psi(\mathbf{r}, t)$, using the Bogoliubov ansatz $\delta\Psi(\mathbf{r}, t) = [u_j(\mathbf{r})e^{-i\omega_j t} + v_j^*(\mathbf{r})e^{i\omega_j t}]e^{-i\mu t/\hbar}$, where μ is the chemical potential in the rotating frame. The linearization leads to the Bogoliubov-de Gennes (BdG) equations,

$$\begin{pmatrix} \hat{\mathcal{L}} & \hat{\mathcal{D}} \\ -\hat{\mathcal{D}}^* & -\hat{\mathcal{L}}^* \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \hbar\omega \begin{pmatrix} u \\ v \end{pmatrix}, \quad (1)$$

where $\hat{\mathcal{L}}u = [-(\hbar^2/2m)\nabla^2 + V_{\text{trap}}(\mathbf{r}) + 2g|\Psi_0|^2 + U_{\text{dd}} * |\Psi_0|^2 + \frac{5}{2}g_{\text{QF}}|\Psi_0|^3 - \mu - \Omega\hat{L}_z]u + \Psi_0 U_{\text{dd}} * (\Psi_0^* u)$ and $\hat{\mathcal{D}}u = [g\Psi_0^2 + \frac{3}{2}g_{\text{QF}}|\Psi_0|\Psi_0^2]u + \Psi_0 U_{\text{dd}} * (\Psi_0 u)$. Solving these equations yields the excitation frequencies ω_j , the corresponding eigenfunctions u_j and v_j , as well as the density fluctuations of the ground state $\delta n_j = 2\text{Re}[(\Psi_0^* u_j + \Psi_0 v_j)e^{i\omega_j t}]$. Thus the actual ground state is determined independently through energy minimization, and the excitation spectrum can identify dynamic instabilities of locally stationary states.

In the following, we consider parameters relevant to bosonic ^{162}Dy atoms ($a_{\text{dd}} = 130a_0$, with a_0 the Bohr radius) and focus on angular excitations. In our azimuthally symmetric geometry, where $[\hat{\mathcal{L}}, \hat{L}_z] = [\hat{\mathcal{D}}, \hat{L}_z] = 0$, SF modes are eigenstates of $\hat{L}_z$ with well-defined angular momentum m , dominating the low-energy spectrum. These excitations drive the emergence of supersolidity, quantified by the density contrast $\mathcal{C} = (n_{\text{max}} - n_{\text{min}})/(n_{\text{max}} + n_{\text{min}})$, where n_{max} and n_{min} denote local density extrema [26,60]. In the uniform SF phase $\mathcal{C} \rightarrow 0$, whereas $\mathcal{C} \rightarrow 1$ signals a fully modulated state. For clarity, we summarize the relevant angular velocities: (i) Ω , external rotation frequency; (ii) $\Omega_{\{\text{PC}, \text{R}, \text{Ph}\}}^{(q)}$, critical frequency for PC, roton, or phonon softening at winding number q ; (iii) $\Omega_{\text{GS}}^{(q)}$,

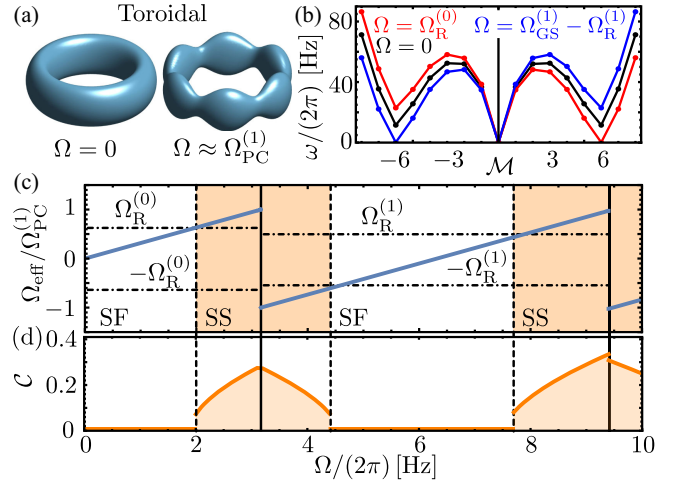


FIG. 1. Rotation-induced supersolidity. (a) Three-dimensional density isosurfaces (at 40% peak density) of the SF and SS states without and with external rotation $\Omega = 2\pi \times \Omega_{\text{PC}}^{(1)}$, respectively, at fixed scattering length $a_s = 92.8a_0$. The critical frequency $\Omega_{\text{PC}}^{(1)} \approx 3.2$ Hz marks nucleation of a singly quantized PC. (b) Dispersion relation in the SF phase for three values of Ω . (c) Effective flow $\Omega_{\text{eff}} = \Omega - \Omega_{\text{GS}}$ (solid blue) versus Ω , where Ω_{GS} is the ground-state angular velocity for winding number q . The dashed black curve $\Omega_{\text{R}}^{(q)}$ indicates the roton softening (Goldstone desoftening) threshold, and solid vertical black lines $\Omega_{\text{PC}}^{(q)}$ denote vortex nucleation reversing the effective flow, producing cyclic SF–SS alternation. (d) Ground-state density contrast $\mathcal{C}$ showing reentrant supersolid pockets.

ground-state angular velocity for charge q ; (iv) Ω_{nV} , critical frequency for nucleating an n -vortex(-lattice).

Flow-induced asymmetry in the excitations—To gain analytical insight into the angular excitations for a SF under rotation, we consider a toroidal geometry of radius R under tight confinement ($R \gg \sqrt{\hbar/M\omega_r}$) such that the radial and angular motions are decoupled [54]. In this limit, one finds $\Omega_{\text{PC}}^{(q)} = \hbar(q - \frac{1}{2})/(MR^2)$, independent of the interactions, yielding the ground state angular velocity $\Omega_{\text{GS}} = \hbar q/(MR^2)$. The resulting dynamics are governed by the effective hydrodynamic flow, defined by the relative angular velocity $\Omega_{\text{eff}} = \Omega - \Omega_{\text{GS}}$. The corresponding excitation spectrum can be obtained by solving Eq. (1) around the ground state $\Psi_0 \sim e^{iq\phi}$. Because the mode momentum m couples to the PC, it is useful to introduce the quantum number $\mathcal{M} = m - q$, which is also the angular momentum of the density fluctuations δn . Then, the excitation spectrum separates into two contributions and can be written as

$$\omega_{\mathcal{M}}^{(q)}(\Omega) = \tilde{\omega}_{\mathcal{M}}^{(q)} - \mathcal{M}\Omega_{\text{eff}}. \quad (2)$$

The first term in Eq. (2) incorporates interaction effects, including the long-range dipolar interaction and LHY corrections, and is independent of Ω , thus preserving

momentum-reversal symmetry ($\mathcal{M} \rightarrow -\mathcal{M}$). In Fig. 1(b) (black curve), we show the corresponding dispersion of the nonrotating superfluid obtained from full three-dimensional BdG calculations. The second term introduces a Galilean shift proportional to the effective flow Ω_{eff} , which lifts the degeneracy between counterpropagating modes and effectively tilts the dispersion relation. This tilt is illustrated in Fig. 1(b) (red and blue curves), where the numerically obtained spectra are in excellent agreement with Eq. (2). As a consequence of this tilt, the spectral splitting is entirely governed by the effective flow, $\omega_{-\mathcal{M}}^{(q)}(\Omega) - \omega_{\mathcal{M}}^{(q)}(\Omega) = 2\mathcal{M}\Omega_{\text{eff}}$, providing a direct spectral signature of the PC. For $\Omega < \Omega_{\text{PC}}^{(1)}$, the ground state carries no circulation ($q = \Omega_{\text{GS}} = 0$), and the splitting follows the external rotation Ω . When $\Omega > \Omega_{\text{PC}}^{(1)}$, the system transitions into the $q = 1$ PC state, causing the sign of the effective flow Ω_{eff} to reverse (see the thick blue line in Fig. 1(c), which shows Ω_{eff} as a function of Ω). This leads to mode swapping: the excitation mode that was previously lower in energy becomes the higher-energy one, and vice versa (see also the Fig. 5 in Appendix B). Although the tight-ring approximation is not strictly satisfied in our full three-dimensional simulations of experimentally realistic traps, the physical mechanism captured by Eq. (2) remains robust and identifies the rotational instability of the SF toward the SS phase.

Roton instability and reentrant SS state—Equation (2) provides a direct criterion for identifying dynamical instabilities. We consider the regime where the long-range dipolar interactions induce strong momentum dependence, leading to a rotational instability driven by roton modes labeled by $m = m_R$. In accordance with Eq. (2), rotation induces a Galilean shift proportional to the relative flow, causing the roton branches to split: modes with m_R having the same sign as Ω_{eff} are lowered in energy (roton⁺), while those with the opposite sign are raised (roton[−]). As Ω increases, the roton⁺ energy softens and vanishes at $\omega_{m_R}^{(0)}(\Omega_R^{(0)}) = 0$, defining the critical angular velocity $\Omega_R^{(0)}$ for the roton instability in the absence of a PC [see red curve in Fig. 1(b)]. For $\Omega > \Omega_R^{(0)}$, the system enters a SS phase, where the roton⁺ becomes a gapless Goldstone mode and the roton[−] evolves into a gapped Higgs[−] mode.

Entering the PC state at $\Omega > \Omega_{\text{PC}}^{(1)}$ triggers mode swapping. The previously counterpropagating Higgs[−] mode becomes a Higgs⁺ mode that softens with increasing Ω , and once $\Omega_{\text{eff}} > -\Omega_R^{(1)}$, it converts into a roton⁺ mode. Simultaneously, the gapless Goldstone mode evolves into a gapped roton[−] mode [see blue curve in Fig. 1(b)]. This desoftening of the Goldstone mode drives the transition from the modulated SS back to the SF phase. As Ω approaches the $q = 2$ threshold $\Omega_{\text{PC}}^{(2)}$, the increased relative flow again induces roton softening and restores the SS modulation. Thus, sequential PC nucleation produces a

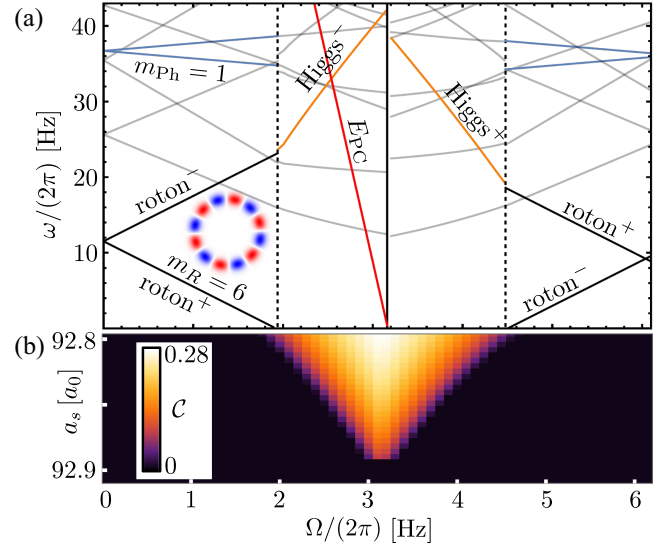


FIG. 2. Excitation spectrum and phase diagram in toroidal geometry. (a) BdG excitation energies vs rotational velocity Ω at $a_s = 92.8a_0$. Black, blue, and orange lines denote the lowest roton ($m_R = 6$), superfluid phonon ($m_{\text{ph}} = 1$), and Higgs modes, respectively. Vertical lines indicate the sequence of SF $\rightarrow$ SS, vortex nucleation, and SS $\rightarrow$ SF transitions. Signs $\pm$ indicate angular momentum orientation relative to Ω . The red curve shows the PC energy E_{PC} . (Inset) Bogoliubov mode profile f for the $m_R = 6$ roton at $\Omega = 0$ in the xy -plane [red (blue) indicates negative (positive) amplitude]. (b) Density contrast C in the (Ω, a_s) plane, delineating the SF and SS regimes.

cyclic phase evolution, yielding discrete supersolid pockets, as shown in Figs. 1(c) and 1(d), characterizing the reentrant SS state.

Numerical results for toroidal traps—We support the analytical insights by numerically solving the eGPE and BdG equations for the toroidal system introduced in Fig. 1. We consider the trapping potential $V_{\text{trap}}(\mathbf{r}) = M\omega_r^2(\sqrt{x^2 + y^2} - R)^2/2 + M\omega_z^2 z^2/2$, with $(\omega_r, \omega_z)/2\pi = (100, 120)$ Hz, radius $R = 2.8 \mu\text{m}$, and atom number $N = 32\,400$. Dipolar BECs and their supersolid phases in toroidal geometries have so far been studied only theoretically [54,61–68], although experimental realization is expected soon. Without rotation, the system undergoes a SF–SS transition at $a_s \approx 92.73a_0$, defining the quantum critical point. Slightly above this threshold, we present the excitation spectrum as a function of Ω at $a_s = 92.8a_0$ in Fig. 2(a).

At $\Omega = 0$, the spectrum exhibits a set of roton modes, along with higher-lying superfluid sound modes; see, for example, the profile ($f = u + v^*$) of the $m_R = 6$ roton mode in Fig. 2(a). Finite rotation lifts the degeneracy, causing the $m_R = 6$ roton⁺ (roton[−]) branch to soften (harden). At $\Omega_R^{(0)} \approx 2\pi \times 1.95$ Hz, the energy of the roton⁺ mode vanishes, signaling the spontaneous breaking of azimuthal symmetry and the formation of a SS with

$n_d = 6$ localized density sites; see also Fig. 1(a). Here, the roton⁺ mode evolves into a gapless Goldstone mode, while the roton⁻ becomes a gapped Higgs⁻ mode.

A central result of our work is the nature of the phase transition. As shown in Fig. 2(b), the finite Higgs-mode gap at the critical point indicates first-order behavior, which we explain within a Ginzburg-Landau framework [69,70]. Taking the superfluid fraction f_s as the order parameter and treating angular momentum classically, we approximate $L_z \approx I\Omega(1 - f_s)$ [71], where I is the classical moment of inertia. This introduces a term linear in f_s in the energy functional, so minimizing $E'(f_s)$ yields a discontinuous jump in f_s [and equivalently in $\mathcal{C}$, see Fig. 1(d)] at the transition. While interaction-driven supersolid transitions can be either first or second order [72], rotational breaking of TRS shifts the transition to first order.

The red line in Fig. 2 shows the energy difference $E_{\text{PC}} = E'(q=1) - E'(q=0)$, obtained from the eGPE, indicating that for $\Omega > \Omega_{\text{PC}}^{(1)} \approx 2\pi \times 3.2$ Hz the state with $q=1$ becomes energetically favorable. Correspondingly, we observe the swapping of angular momenta of modes predicted by Eq. (2) and the subsequent reduction of the energy of the Higgs⁺ mode. At $\Omega_{\text{R}}^{(1)} \approx 2\pi \times 4.47$ Hz, the system reenters the SF phase, evidenced by the desoftening of the Goldstone mode.

As Ω increases, the cycle repeats, producing a sequence of SF $\rightarrow$ SS $\rightarrow$ SF transitions [Fig. 1(d)]. This periodic recurrence, reentrant supersolidity, constitutes the central result of this work. While reentrance in ultracold gases has mostly been studied in optical lattices [73–78], the mechanism here is fundamentally different: it arises from discrete nucleation of topological charge, which alternately restores and suppresses crystalline order. Since the critical velocities for roton softening and Goldstone desoftening depend on the winding number q , the rotation-induced SS phase exists only within $|\Omega - \Omega_{\text{GS}}| < \Omega_{\text{R}}^{(q)}$. A phononic instability may also occur in the $m = m_{\text{ph}} = 1$ mode at $\Omega_{\text{ph}} = \tilde{\omega}_1$. However, rotation-induced supersolidity requires the roton instability to occur first, i.e., $\Omega_{\text{R}}^{(q)} < \Omega_{\text{ph}}^{(q)}$. This condition is fulfilled near the quantum-critical point, where roton energies lie below phononic ones [blue curve in Fig. 2(a)].

Figure 2(b) maps the rotation-induced supersolid phase in the (Ω, a_s) plane via the density contrast $\mathcal{C}$. A finite $\mathcal{C}$, indicating SS formation, appears over a broader Ω range near the quantum critical point and a narrower one away from it. The maximum $\mathcal{C}$ occurs at $\Omega = \Omega_{\text{PC}}^{(1)} = 2\pi \times 3.2$ Hz, consistent with our analytical predictions. Since $\Omega_{\text{PC}}^{(1)}$ increases as the ring radius R decreases, this offers a route to stabilize the SS phase over a wider Ω interval.

Numerical results for oblate harmonic traps—To show that rotation-induced supersolidity is not geometry specific

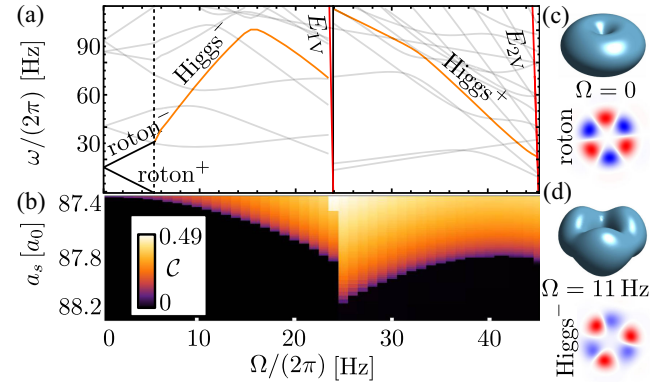


FIG. 3. Excitation spectrum and phase diagram in an oblate harmonic trap. (a) Roton (black), Higgs (orange), and other (gray) excitation frequencies versus rotation Ω at $a_s = 87.4a_0$. Dashed and solid vertical lines mark $\Omega_{\text{R}}^{(0)}$ and $\Omega_{1\text{V}}$, respectively; red lines show the energy differences $E_{1\text{V}}$ and $E_{2\text{V}}$ relative to the ground state. (b) Density contrast $\mathcal{C}$ versus Ω and a_s . (c) Density isosurface of the SF (at 40% peak density) and corresponding roton ($m_{\text{R}} = 3$) density fluctuations δn . (d) Density isosurface of the SS and density fluctuations of the Higgs⁻.

and to connect with established platforms [20,79,80], we consider an oblate harmonic trap with $R = 0$ and trapping frequencies $(\omega_r, \omega_z) = 2\pi \times (125, 250)$ Hz for $N = 2 \times 10^4$ atoms [50]. The corresponding density isosurfaces of the nonrotating SF and rotating SS ground states are shown in Fig. 1(b).

At $\Omega = 0$, the SF exhibits a central density minimum (the “red-blood cell” configuration) due to tight in-plane confinement and repulsive interactions in the xy plane [81]. This azimuthally symmetric state supports angular roton modes as the lowest-energy excitations, analogous to the toroidal case. Under rotation, the degeneracy of these modes is lifted, leading to branch splitting and softening. In particular, the $m_{\text{R}} = 3$ roton drives the transition to a SS state with three density maxima. The transition is signaled by the emergence of zero-energy Goldstone and Higgs modes, as shown in Fig. 3(a), where only the roton and Higgs modes are highlighted and the remaining spectrum is plotted in gray.

The finite central density in the oblate harmonic trap makes vortex nucleation more costly than in the multiply-connected geometries [20,82–84], shifting the critical vortex frequencies to higher values [red lines in Fig. 3(a)] and enlarging the parameter space for rotation-induced supersolidity [Fig. 3(b)]. For $a_s \lesssim 87.8a_0$, $\Omega_{\text{R}}^{(1)} < \Omega_{1\text{V}}$, so roton softening precedes vortex entry, whereas for $a_s \gtrsim 87.8a_0$ the ordering reverses. In the narrow range $87.8a_0 \lesssim a_s \lesssim 88.1a_0$, the roton shows characteristic desoftening. In the oblate trap, the reentrant supersolid forms a single pocket, as the two-vortex state, comprising two singly quantized vortices, already breaks angular symmetry and modifies the density modulation. Although mode

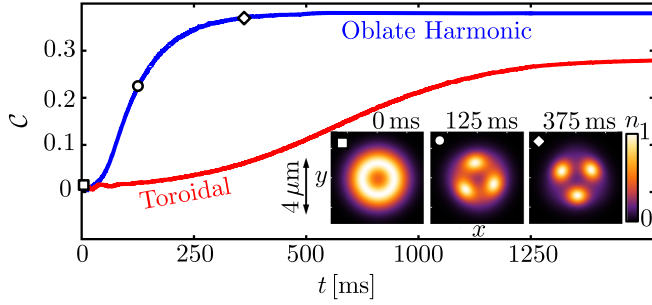


FIG. 4. Dynamical onset of supersolid order. Time evolution of the density contrast C for toroidal (red) and oblate harmonic (blue) traps. Following a sudden quench of rotation to $\Omega/2\pi = 3$ Hz (toroidal) and 23 Hz (oblate harmonic), the nonrotating SF ground state undergoes spontaneous density modulation. (Insets) Snapshots of the integrated density $n(x, y)$ in the oblate harmonic trap at specific time intervals (see markers), refer to Ref. [85] for video of the dynamics. Scattering lengths are $a_s = 92.8a_0$ (toroidal) and $87.4a_0$ (oblate harmonic).

coupling makes the excitation spectrum more intricate than in the toroidal case, rotation-induced roton softening and Goldstone desoftening remain robust (Fig. 3).

Dynamical emergence of SS under rotation—To establish the experimental feasibility of our results, we perform real-time simulations incorporating phenomenological damping γ and noise in the initial state. Starting from a nonrotating SF ground state, we suddenly quench the rotation at $t = 0$ to $\Omega/2\pi = 23$ Hz for the oblate harmonic trap. This value sits slightly below the critical frequency $\Omega_{IV}/2\pi = 24$ Hz and precludes vortex nucleation while inducing pronounced azimuthal symmetry breaking. The SF state becomes metastable after the quench, see also Appendix C for further information. Figure 4 illustrates the time evolution of the density contrast C (blue line), which grows from zero and saturates at $C \approx 0.38$, signaling the formation of a stable SS phase. The insets in Fig. 4 display snapshots of the integrated 2D density profiles, $n(x, y) = \int |\Psi|^2 dz$, at selected time intervals (indicated by markers), capturing the emergence of the crystalline structure. An analogous dynamical evolution occurs for the toroidal geometry (red line in Fig. 4), demonstrating the robustness of the rotationally induced SS across different trapping potentials.

Conclusions and outlook—We have investigated the Bogoliubov excitation spectra of rotating dipolar condensates. In geometries supporting angular excitations, such as toroidal and oblate harmonic traps, the breaking of TRS lifts roton degeneracy and triggers a first-order SF-SS transition. We uncover a nontrivial interplay between crystalline order and topological excitations, namely, PCs and quantized vortices, which drives a periodic sequence of SF $\leftrightarrow$ SS transitions. This behavior is mediated by the desoftening of the gapless Goldstone mode and the simultaneous suppression of density modulations, giving rise to reentrant

SS phases, a phenomenon widely encountered in diverse systems such as liquid crystals [86], spin glasses [87], quantum hall states [88], and superconductivity [89].

Experimentally, the transition can be detected via *in situ* imaging of density modulation [29–32], Bragg spectroscopy of the roton gap [40,41], and phase-contrast observation of vortex nucleation under controlled rotation ramps [19,20]. These phenomena could be realized across diverse platforms, including dipolar molecules [90], exciton-polariton condensates [91], and other lanthanide elements [29,92], provided that (i) the Hamiltonian supports low-energy angular excitations, (ii) the spectrum is partially concave, and (iii) the roton softening threshold $\Omega_R^{(q)}$ is the system’s lowest critical velocity. By employing rotation as a control parameter, our work enables supersolidity in regimes where comparatively weak long-range interactions would otherwise suppress it. Immediate extensions include finite-temperature effects [93] and further exploration of collective modes such as sound excitations [94], Josephson oscillations [95,96], and shear waves [97].

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

Appendix A—In this section, we describe the numerical methods employed to obtain the ground states and dynamical properties presented in the main text. All simulations are performed at zero temperature within the mean-field framework of the eGPE [43,45,59,98].

The time evolution of the condensate wave function $\Psi(\mathbf{r}, t)$ in the rotating frame is governed by

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = \left(\hat{\mathcal{H}} - \Omega \hat{L}_z \right) \Psi(\mathbf{r}, t), \quad (\text{A1})$$

where the eGPE operator $\hat{\mathcal{H}}$ is defined as

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2M}\nabla^2 + V_{\text{trap}}(\mathbf{r}) + g|\Psi|^2 + g_{\text{QF}}|\Psi|^3 + \int d^3\mathbf{r}' U_{\text{dd}}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2. \quad (\text{A2})$$

The external potential $V_{\text{trap}}(\mathbf{r})$ accounts for the static confinement, which in this work is either a toroidal or an oblate harmonic potential. Quantum fluctuations are incorporated via the LHY correction with the coefficient $g_{\text{QF}} = \frac{32}{3}g\sqrt{a_s^3/\pi}(1 + \frac{3}{2}\epsilon_{\text{dd}}^2)$, where $\epsilon_{\text{dd}} = a_{\text{dd}}/a_s$.

The corresponding energy functional associated with the eGPE, the minimization of which yields the stationary states, is given by $E'[\Psi] = E[\Psi] - \Omega L_z$ with

$$E[\Psi] = \int d^3\mathbf{r} \left[\frac{\hbar^2}{2M} |\nabla\Psi|^2 + V_{\text{trap}}(\mathbf{r})|\Psi|^2 + \frac{g}{2}|\Psi|^4 + \frac{2}{5}g_{\text{QF}}|\Psi|^5 \right] + \frac{1}{2} \iint d^3\mathbf{r} d^3\mathbf{r}' \times |\Psi(\mathbf{r})|^2 U_{\text{dd}}(\mathbf{r}-\mathbf{r}') |\Psi(\mathbf{r}')|^2. \quad (\text{A3})$$

For numerical implementation, the eGPE is rescaled into a dimensionless form using a characteristic length scale $l_s = 10^{-6}$ m and time scale $t_s = Ml_s^2/\hbar$. Ground states are obtained via imaginary-time propagation using a split-step Fourier spectral method. To analyze the dynamical response and excitation spectra, we perform subsequent real-time propagation with a time step of $\Delta t = 10^{-4}t_s$ and a convergence criterion for the energy of $|E[\Psi(\mathbf{r}, t + \Delta t)] - E[\Psi(\mathbf{r}, t)]|/E[\Psi(\mathbf{r}, t)] < 10^{-14}$. The non-local dipolar term is evaluated efficiently as a convolution in momentum space, utilizing a spherical cutoff for the interaction kernel to eliminate aliasing effects and spurious interactions between periodic images.

Simulations are conducted in a three-dimensional box with dimensions $L_x = L_y = L_z = 22 \mu\text{m}$. We confirmed numerical convergence by comparing results across grid resolutions of $128 \times 128 \times 64$ and $256 \times 256 \times 128$, noting that the accuracy of the BdG excitation frequencies depends primarily on the physical box size. Solving the eigenvalue problem in Eq. (1) is numerically challenging due to the long-range interactions, which make it impossible to store the full matrix. We therefore employ a matrix-free implementation of the Arnoldi method, as provided by MATLAB's `eigs` function, to compute the lowest eigenvalues and corresponding eigenvectors.

To model the dynamics of the rotation-induced super-solid state, we include a phenomenological damping term in the eGPE:

$$(i - \gamma)\hbar \frac{\partial \Psi}{\partial t} = (\hat{\mathcal{H}} - \Omega \hat{L}_z)\Psi, \quad (\text{A4})$$

where $\gamma \ll 1$ is a small dimensionless damping parameter. This term accounts for dissipative processes and facilitates the system's relaxation toward stable dynamical configurations. For the real-time dynamics discussed in the main text, we have used $\gamma = 0.035$ for the toroidal trap and $\gamma = 0.02$ for the oblate harmonic trap. At $t = 0$, we also introduce a small perturbation to account for experimental inhomogeneities and to break the angular symmetry [19].

Appendix B—To gain analytical insight into the excitation spectrum under rotation, we consider a superfluid confined to a narrow ring of radius R satisfying $R \gg \sqrt{\hbar/M\omega_r}$. We derive the excitation energies as a function of the external rotation Ω and the ground-state winding number q by assuming the solutions for the nonrotating case ($\Omega = q = 0$) are known. This reference set of solutions is denoted by

$$(\tilde{\omega}_m, \tilde{u}_m e^{im\phi}, \tilde{v}_m e^{im\phi}), \quad (\text{B1})$$

where $\tilde{u}_m$ and $\tilde{v}_m$ represent the quasiparticle amplitudes in the transversal (r, z) directions.

We now demonstrate that the vector

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \tilde{u}_{\mathcal{M}} e^{im\phi} \\ \tilde{v}_{\mathcal{M}} e^{i(m-2q)\phi} \end{pmatrix} \quad (\text{B2})$$

is a valid solution to the BdG equations and leads to the dispersion relation utilized in the main text. We begin by employing the tight-confinement approximation for the gradient operator, $\nabla_\phi \approx (1/R)[\partial/\partial\phi]$. Furthermore, we approximate the ground-state wave function as

$$\Psi(\mathbf{r}) \approx |\tilde{\Psi}(r, z)| e^{iq\phi}, \quad (\text{B3})$$

assuming that the radial density profile is approximately independent of the circulation q . Under these conditions, the chemical potential can be expressed as

$$\mu = \tilde{\mu} + \frac{q^2 \hbar^2}{2MR^2} - \Omega \hbar q, \quad (\text{B4})$$

where $\tilde{\mu}$ is the chemical potential for the nonrotating system ($\Omega = q = 0$).

Inserting these expressions into the BdG equations and performing straightforward algebraic manipulation yields the following expression for the dispersion relation $\omega_{\mathcal{M}}^{(q)}$:

$$\omega_{\mathcal{M}}^{(q)}(\Omega) = \tilde{\omega}_{\mathcal{M}} + \frac{mq\hbar}{MR^2} - \frac{q^2\hbar}{MR^2} + \Omega(q - m). \quad (\text{B5})$$

By introducing the fluctuation momentum $\mathcal{M} = m - q$, we recover the Galilean-shifted dispersion relation:

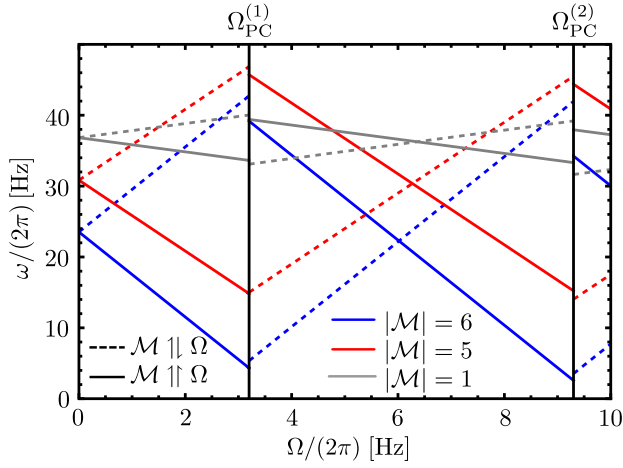


FIG. 5. Mode swapping in a rotating toroidal superfluid. The energies of the six lowest excitations ($\mathcal{M} = \pm 6, \pm 5, \pm 1$) for the system from Fig. 2 in the main text with $a_s = 93a_0$. The modes propagating in the direction of Ω are shown as solid lines, whereas we indicate modes propagating antiparallel to Ω as dashed lines. The vertical black lines indicate the critical rotation frequencies at which PCs are generated, corresponding to mode swapping.

$$\omega_{\mathcal{M}}^{(q)}(\Omega) = \tilde{\omega}_{\mathcal{M}} - \mathcal{M} \left(\Omega - \frac{q\hbar}{MR^2} \right) = \tilde{\omega}_{\mathcal{M}} - \mathcal{M}\Omega_{\text{eff}}. \quad (\text{B6})$$

It is important to note that the derivation above strictly holds when the radial and angular motions are decoupled ($R \gg \sqrt{\hbar/M\omega_r}$). In our numerical simulations, this condition is not perfectly satisfied, and the particle density $|\Psi|^2$ acquires a slight q dependence due to radial centrifugal shifts, see Fig. 5. Because of the radial coupling, the energies are not continuous, $\omega_{\mathcal{M}}^{(q-1)}(\Omega_{\text{PC}}^{(q)}) \neq \omega_{-\mathcal{M}}^q(\Omega_{\text{PC}}^{(q)})$. However, we find that the general form of the dispersion remains robust if one replaces the intrinsic frequency with a q -dependent one, $\tilde{\omega}_{\mathcal{M}} \rightarrow \tilde{\omega}_{\mathcal{M}}^{(q)}$. This replacement allows us to construct the full spectrum by combining the numerically calculated excitation energies at different q values with the analytical Doppler shift, rather than solving the full BdG problem for every value of Ω .

This method remains valid as long as the radial trapping frequency dominates the rotation ($\omega_r \gg \Omega$). If this condition is violated, centrifugal effects would cause the density to depend explicitly on Ω . Our numerical analysis confirms that such effects are negligible for the parameter regimes studied in the main text. Nevertheless, all results presented in the main text are cross-verified by solving the full, nondecoupled BdG problem to ensure that all rotational and interaction effects are rigorously captured.

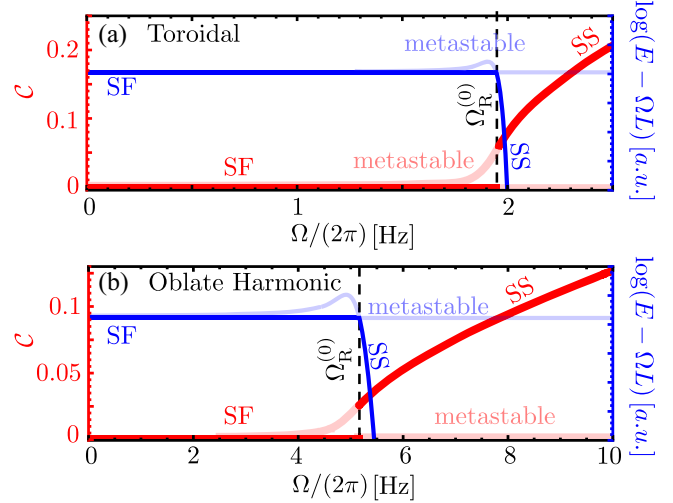


FIG. 6. Bistability of SF and SS close to the phase transition. The energy (blue) and the contrast (red) for the toroidal (a) and oblate harmonic (b) trap. The thick solid lines denote the ground state, while the translucent curves indicate the metastable state. The scattering lengths are $92.8a_0$ (a) and $87.4a_0$ (b).

Appendix C—In the main text, we have shown from general arguments that the SF-to-SS phase transition is of first order when triggered by rotation. One property of first-order phase transitions is the existence of metastable states close to the transition. We can identify these metastable states by choosing initial conditions in the imaginary-time propagation that lie close to them. The true ground state can then be determined by comparing the corresponding energies in the rotating frame. In the main text, we have carried out this analysis carefully by choosing various initial conditions. All data shown in Figs. 1–3 correspond to true ground states. Here, instead, we focus on the bistability of the SF-to-SS transition close to the phase boundary, i.e., the coexistence of a SF and SS state for the same set of parameters. In Fig. 6, we show the contrast (red curves) and the energy (blue curves) as functions of the rotation frequency Ω for the toroidal (a) and oblate harmonic (b) geometries. The ground state is shown as a thick line and, as discussed in the main text, the first-order nature of the phase transition is evident from the sudden jump in the contrast at $\Omega = \Omega_R^{(0)}$. Metastable states are shown by translucent curves, confirming the bistability of both phases. While the SF is metastable over a broad range of rotation frequencies, the metastability of the SS is restricted to a narrower window.