

Nonreciprocal Mixtures in Suspension: The Role of Hydrodynamic Interactions

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The collective chasing dynamics of nonreciprocally coupled densities leads to stable traveling waves which can be mapped to a model for emergent flocking. In this Letter, we couple the nonreciprocal Cahn-Hilliard model to a fluid to minimally describe scalar active mixtures in a suspension, with the aim to explore the stability of the waves, i.e., the emergent flock in the presence of self-generated fluid flows. We show that the emergent polarity is linearly unstable to perturbations for a specific sign of the active stress recalling instabilities of orientational order in a fluid. Using numerical simulations, we find, however, that nonreciprocity stabilizes the waves against the linear instability in a large region of the phase space.

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Life at the microscale thrives under conditions of low Reynolds number hydrodynamics [1–8]. Motile and non-motile living agents create persistent long-ranged flows that dominate their interactions, resulting in striking dynamics as exemplified by dancing *Volvox* colonies [9], as well as active low Reynolds number turbulence above bacterial carpets [10,11] and in bacterial suspensions [12–14]. Solvent-mediated interactions have been known to substantially alter the collective behavior predicted by dry models in a variety of active matter systems [15,16]. For instance, active squirmers generate flows that can suppress motility-induced phase separation [17–20], whereas coupling to active fluids can also stabilize mixtures [21]. Moreover, polar and nematic order are known to become unstable due to hydrodynamic interactions [22,23]. While the active stresses generated by polar particles can lead to global spontaneous flows [24–26], in nematic systems they can lead to turbulence [27,28]. The prominent instability can be suppressed via confinement [24] or background elasticity and is thought to play a significant role in spindle self-assembly [29]. Hydrodynamic coupling can also lead to the emergence of order, as seen in the case of coordination between beating cilia and flagella [30–32], which facilitates mass transport [33,34] and leads to the emergence of metachronal waves [11,35–37].

Among different manifestations of nonequilibrium activity, a particularly interesting class comprises collective behavior emerging from action-reaction symmetry breaking due to naturally arising nonreciprocal interactions [38–40]. Nonreciprocity breaks time-reversal symmetry and gives rise to a dynamic chasing behavior between the species that serves as an effective propulsion mechanism of composite units [38]. At the collective level, this leads to the emergence of traveling waves and an accompanying global polar order [41–43], which are robust to sources of stochasticity in two dimensions and above [44]. Many commonly used experimental soft matter systems where the paradigm of nonreciprocity would be applicable, such as mixtures of Janus colloids [45] and Quincke rollers [46,47], are swimmers in suspensions. A natural question thus arises: how do hydrodynamic interactions impact the emergent traveling waves and the associated polar order that stems from nonreciprocity [see Fig. 1(a)]?

In this Letter, we address this question by developing a phenomenological model, which we call the hydrodynamic-nonreciprocal Cahn-Hilliard (h-NRCH) model for a non-reciprocally interacting binary scalar mixture suspended in a momentum-conserving fluid [see Fig. 1(b)]. The h-NRCH model is a modified version of the NRCH model incorporating the effect of solvent-mediated long-range hydrodynamic interactions. The central result of our Letter is that flow structures arising from active stresses lead to a wave-number-independent damping in the dynamics of the (otherwise-) slow modes. The damping is positive for $\Gamma > 0$ (a dimensionless scale of the active stress), implying that fluctuations decay with a finite timescale in this regime. It changes sign for $\Gamma < 0$, thereby triggering an instability reminiscent of the generic instability, which would destroy orientational order in the suspension [22]. We uncover another route to instability, a hydrodynamic counterpart of the diffusive Eckhaus instability that arises for any sign of Γ

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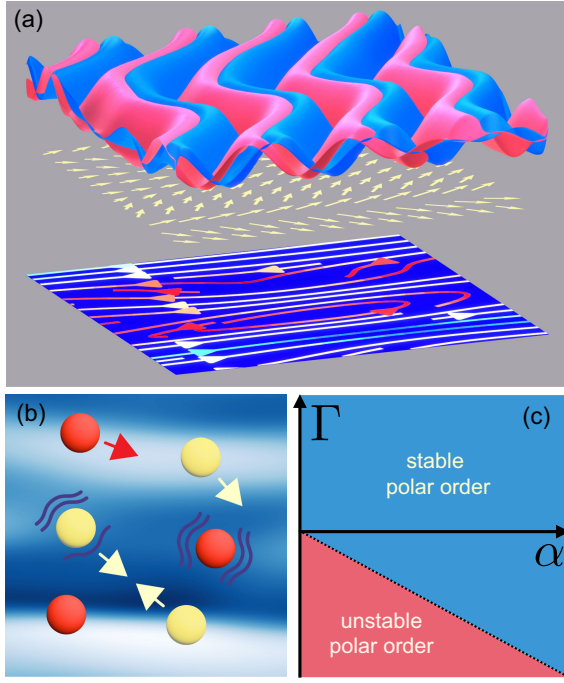


FIG. 1. Two species of nonreciprocally interacting active particles [depicted in (b)] are represented by the red and blue density profiles in panel (a). Nonreciprocity gives rise to a chasing pattern, whose emergent polar order is quantified by the yellow vector field. Densities are coupled with a background fluid [streamlines in panel (a)], which bends the densities layers and the corresponding orientational order. (c) Interplay between the strengths of nonreciprocity α and active stress Γ leads to enhanced stability for the polar order (traveling waves).

for waves with a wavelength below a threshold. These predictions are verified by numerical simulations, where we additionally find that polar order persists in large parts of the parameter space as represented in Fig. 1(c), in contrast to the results of the linear stability analysis.

The dynamics of the conserved densities $\phi_{1,2}$ of a nonreciprocally coupled binary mixture of scalar active particles is minimally described by the NRCH model [41,42]. The active particles produce active stresses $\mathbb{Q}$ [defined in Eq. (1)] in the fluid in which they are suspended, and they are in turn advected by the resulting hydrodynamic flow fields. The equations of motion for ϕ_a ($a \in \{1, 2\}$) that satisfy number conservation and the equations for the Stokesian incompressible velocity field $\mathbf{v}$ (i.e., $\nabla \cdot \mathbf{v} = 0$) that enforce momentum conservation in the solvent define the hydrodynamic NRCH (h-NRCH) model as

$$\begin{aligned} \partial_t \phi_a + \mathbf{v} \cdot \nabla \phi_a &= \nabla^2 [\mu_a - \alpha \varepsilon_{ab} \phi_b], \\ \eta \nabla^2 \mathbf{v} &= \nabla p + \nabla \cdot \mathbb{Q}, \\ \mathbb{Q} &= \kappa \left[\nabla \phi_a \nabla \phi_a - \frac{\mathbb{I}}{d} |\nabla \phi_a|^2 \right], \end{aligned} \quad (1)$$

where summation over repeated indices is implied. Here, ε_{ab} is the fully antisymmetric Levi-Civita tensor, α is the coefficient of nonreciprocal interaction, and η is the viscosity. The pressure p enforces incompressibility, and the chemical potential $\mu_a = \delta F / \delta \phi_a$ is derived from a free-energy $F = \int d^d r [-\frac{1}{2}(\phi_a \phi_a) + \frac{1}{4}(\phi_a \phi_a)^2 + \frac{1}{2}(\nabla \phi_a)^2]$. Note that we assume the same damping coefficient and stiffness for both species and absorb them in units of time and space. For $\alpha = 0$ and $\kappa = 1$, Eq. (1) reduces to the model-H in the Halperin-Hohenberg classification scheme [48]. In contrast, for nonequilibrium conditions, κ can take both positive and negative values. This is the case for a mixture of Janus colloids in suspension, where κ is shown to depend on the parameters controlling phoretic activity [49].

For strong nonreciprocity, or large α , the system breaks parity and time-reversal symmetry and organizes in stripelike patterns where one species chases the other. This transition to a novel nonequilibrium phase is easily characterized by the polar order parameter $\mathbf{J} \equiv \phi_1 \nabla \phi_2 - \phi_2 \nabla \phi_1$, which measures the local phase difference between the two density fields. The spatial average of $\mathbf{J}$ vanishes for bulk phase separation [41] or for defect-riddled states [50,51], but has a finite value in the moving phase where $\mathbf{J}$ is parallel to the wave vector of the wave [41,44,50]. Following Ref. [44], we recast the h-NRCH model (1) in terms of $\mathbf{J}$ and the amplitude $\rho = \sqrt{\phi_1^2 + \phi_2^2}$. The equation of motion for $\mathbf{J}$ is

$$\begin{aligned} \partial_t \mathbf{J} + (\mathbf{v} + 2\alpha \rho^{-2} \mathbf{J}) \cdot \nabla \mathbf{J} + 2\alpha \rho^{-2} \mathbf{J} (\nabla \cdot \mathbf{J}) \\ = -\nabla \cdot \mathbf{J} + (r - u|\mathbf{J}|^2)|\mathbf{J}|^2 \mathbf{J} + D_J \nabla (\nabla \cdot \mathbf{J}), \end{aligned} \quad (2)$$

where $V_{ij} = \partial_i v_j$ is the velocity gradient tensor. $\mathbf{J}$ advects itself with a coefficient proportional to the nonreciprocity α , underscoring its similarity to the polar order parameter in flocking dynamics [52]. The velocity field enters the dynamics in two ways: first it advects $\mathbf{J}$ [the second term on the lhs of Eq. (2)], and second through the velocity gradient tensor, which shears (symmetric part) and rotates (antisymmetric part) $\mathbf{J}$ [53]. The diffusion coefficient $D_J = 1 - \rho^2 - 2m$ [with $m = \rho^4(r - u|\mathbf{J}|^2)/2$] and the coefficients $r = 2(1 - \rho^2)/\rho^4$ and $u = 2/\rho^8$ of the Mexican-hat potential in the rhs of Eq. (2) depend on ρ , which evolves as

$$\partial_t \rho + \mathbf{v} \cdot \nabla \rho + \alpha (\nabla \cdot \mathbf{J}) / \rho = m|\mathbf{J}|^2 / \rho^3 + D_\rho \nabla^2 \rho, \quad (3)$$

where $D_\rho = 5 - 3\rho^2 + 2m$. The active stress in terms of $\mathbf{J}$ and ρ is

$$\mathbb{Q}_{ij} = \kappa \left[\frac{J_i J_j}{\rho^2} - \frac{\delta_{ij}}{\rho^2 d} |\mathbf{J}|^2 + \partial_i \rho \partial_j \rho - \frac{\delta_{ij}}{d} |\nabla \rho|^2 \right]. \quad (4)$$

The first two terms in the expression for $\mathbb{Q}$ are similar in form to the active stress generated in suspensions of active

agents irrespective of whether they are polar (and thus self-propelled) or apolar [22,56].

Traveling waves $\phi_{10} = \rho_0 \cos(\mathbf{q}_0 \cdot \mathbf{x} - \omega_0 t)$ and $\phi_{20} = \rho_0 \sin(\mathbf{q}_0 \cdot \mathbf{x} - \omega_0 t)$, propagating with angular frequency $\omega_0 = \alpha q_0^2$ and amplitude $\rho_0^2 = 1 - q_0^2$, proposed first for the dry case [43,44,50], are also exact solutions of the h-NRCH equations in Eq. (1). From Eqs. (2) and (3), we observe that $\mathbf{v} = 0$, $\mathbf{J} = J_0 \hat{\mathbf{q}}_0$, and any constants ρ_0 are homogeneous solutions of the dynamics. The magnitude $J_0 = \rho_0^2 \sqrt{1 - \rho_0^2}$ is determined by the minima of the Mexican-hat potential in Eq. (2), such that $m(J_0, \rho_0) = 0$.

To test the stability of the ordered state, we perturb the fields around their stationary values, substituting $\rho(\mathbf{x}, t) = \rho_0 + \delta\rho(\mathbf{x}, t)$ and $\mathbf{J}(\mathbf{x}, t) = J_0 \hat{\mathbf{q}}_0 + \delta\mathbf{J}_\parallel(\mathbf{x}, t) \hat{\mathbf{q}}_0 + \delta\mathbf{J}_\perp$ into Eqs. (2) and (3), and probe the evolution of the fluctuations in space and time. For the vector order parameter $\mathbf{J}$, we have decomposed the fluctuations as parallel and perpendicular with respect to the direction of propagation, i.e., $\delta\mathbf{J}_\perp \cdot \hat{\mathbf{q}}_0 = 0$. We retain terms until linear order in the perturbations to obtain the dynamical matrix for $\delta\mathbf{J}_\perp$ and $\delta\rho$, whose eigenvalues determine the stability of the polar order. To reach this point we eliminate the pressure field using the condition of incompressibility and express $\mathbf{v}$, which we likewise decompose into parallel and perpendicular components, in Fourier space in terms of $\delta\mathbf{J}_\perp$ and $\delta\rho$ [53]. To the lowest order, we find

$$\delta v_\parallel = -\frac{i\Gamma q_0}{q} \left[\frac{(\mathbf{q} \cdot \delta\mathbf{J}_\perp)}{q} + 2q_0 \rho_0 \cos\psi \sin^2\psi \delta\rho \right], \quad (5)$$

where ψ is the angle between $\mathbf{q}$ and $\mathbf{q}_0$, namely, $\psi = \cos^{-1}(\hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_0)$, and $\Gamma = \kappa/\eta$ is the only relevant dimensionless parameter. In $d = 2$, we can readily substitute this expression in the dynamics of Eqs. (2) and (3) and focus on the modes $\delta\mathbf{J}_\perp(\mathbf{q}, t)$, whose linearized dynamics reads as

$$\partial_t \delta\mathbf{J}_\perp = [\lambda_0(\psi) - iqv_g(\psi) \cos\psi - q^2 D(\psi)] \delta\mathbf{J}_\perp. \quad (6)$$

Here, λ_0 is a wave-number-independent growth rate given as

$$\lambda_0(\psi) = -\Gamma q_0^2 \rho_0^2 \sin^2\psi \left(\gamma + \frac{2q_0^2}{\rho_0^2} \sin^2\psi \right), \quad (7)$$

where

$$\gamma = \frac{1 - 3q_0^2}{1 - q_0^2}. \quad (8)$$

The generation of a wave-number-independent and anisotropic growth rate λ_0 is a key result of this Letter. λ_0 arises either from the advection of scalar densities by self-generated long-range hydrodynamic flows or, analogously, from shear stresses deforming the polarity. The

perturbations propagate with the velocity v_g and diffuse with the coefficient D , both of which are anisotropic for $\Gamma \neq 0$. D stabilizes the instability that arises at zero wave number due to λ_0 at small enough system sizes; see the End Matter for the expressions for v_g and D and [53] for further discussion. If the sign of λ_0 is positive for any ψ , fluctuations in that direction will grow exponentially and eventually destroy the ordered state. From Eq. (7), we note that the sign of λ_0 depends on ψ and on the wave number of the reference state q_0 , especially through γ . This quantity changes sign from positive to negative at $q_0 = q_E = 1/\sqrt{3}$, a threshold that matches the Eckhaus instability criterion for layered densities [57].

We will now discuss the impact of all four possible combinations of the signs of Γ and γ on the stability of the emergent order. For $q_0 < q_E$, γ is positive and the sign of λ_0 is simply opposite to that of Γ . For $\Gamma > 0$, λ_0 is negative, ensuring the existence of a region of parameter space where traveling waves persist even when coupled to a Stokesian fluid. Conversely, for $\Gamma < 0$ orientational order is linearly unstable, and the system transits to a new nonequilibrium steady state [Fig. 2(a)]. As the instability destroys the emergent flock, we call the instability in this sector the h-polar instability; see Fig. 2(c). Yet another route to an instability appears when γ reverses sign, signaling an instability for ψ around zero for $\Gamma > 0$ and around $\psi = \pi/2$ for $\Gamma < 0$; see Fig. 2(b). We call these the h-Eckhaus and the h-polar + Eckhaus instabilities. The standard diffusive Eckhaus instability of a patterned density happens due to the advection of amplitude fluctuations in the direction of propagation. The hydrodynamic Eckhaus instability occurs due to the same mechanism entering via the second term in $v_\parallel$ in Eq. (5).

To confirm our analytical predictions we run numerical simulations of the h-NRCH Eq. (1) using parameters where we expect traveling waves for $\Gamma = 0$ [50] and explore all four regimes of Fig. 2(c). Additional data for the h-Eckhaus and h-polar + Eckhaus are shown in [53]. All simulations are initialized with $\mathbf{v} = 0$ and a traveling wave, i.e., $\phi_a = \phi_{a0}$, with random uniform perturbations [53]. In the stable regime, the traveling waves are stable and perturbations decay away in finite time. In the region where the h-polar instability is expected, for large enough $|\Gamma|$, the waves are destroyed and the steady state is one of structureless spatiotemporal chaos. The snapshots of Fig. 2(e) show the evolution of the layered structure at the onset of the instability, where it is apparent that the hydrodynamic instability happens through deformation of the densities layers; see Supplemental Material (SM) [53] for a quantification of the large deformations. These pictures are reminiscent of the buckling instability of smectic A under dilative stress [58,59], with the important difference that here the mechanical stresses are internally generated by the fluid flow [Fig. 2(d)]. This flow stretches the density layers, inducing undulations transverse to the

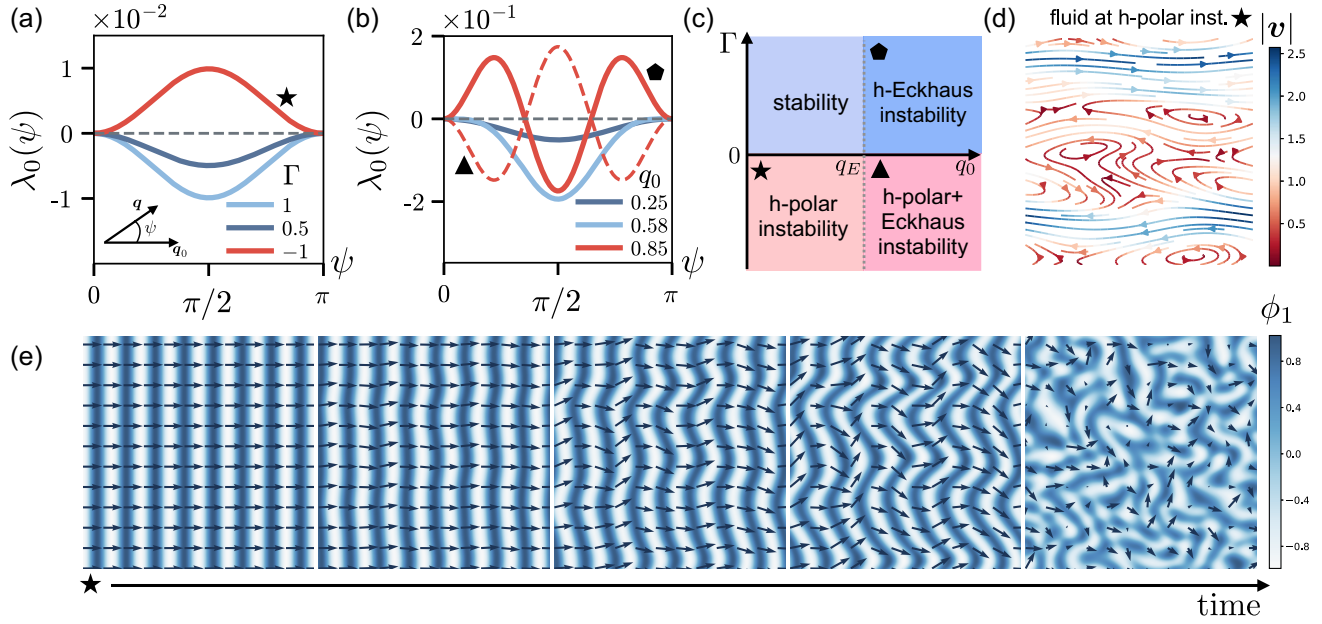


FIG. 2. Linear stability of the ordered state. (a) Wave-number-independent contribution ($q = 0$) of the eigenvalue determining the evolution of the transverse fluctuations. The pattern wave number is fixed at $q_0 = 0.1$, and we vary the coefficient of the active stress Γ . (b) Same plot for a fixed $|\Gamma| = 1$ (positive for all except the dashed line) and different initial wave numbers q_0 . (c) Stability diagram in the $\Gamma - q_0$ plane showing three different routes to linear instability as identified from $\lambda_0(\psi)$. (d) Streamlines of fluid velocity $\mathbf{v}$ at the onset of the h-polar instability; the colors indicate the local fluid speed. (e) Time evolution of the density ϕ_1 undergoing the h-polar instability. The arrows on top represent the corresponding polar order parameter $\mathbf{J}$. The snapshots are from numerical simulations on a square domain of side $L = 32\pi$ discretized over 512^2 grid points with $\alpha = 1$, $q_0 = 0.5$, and $\Gamma = -1$.

direction of order, and ultimately destabilizes the structure [Fig. 2(e)]. For $q_0 > q_E$ the wave is destroyed due to both the hydrodynamic [see Eq. (7)] and the conserved Eckhaus instabilities that appear in the diffusion coefficient D [see Eq. (6)]. The final state of the system is determined by the sign of Γ . For $\Gamma > 0$, the upper right quadrant in Fig. 2(c), the system settles to a new pattern with a stable wavelength smaller than q_E and selected by the mass conservation law [44]. For $\Gamma < 0$, we observe spatiotemporal chaos at long times, without prominent length or time scales [53].

The role played by α has so far been to determine the reference state for the stability analysis. Focusing on $\Gamma < 0$, we now show that α plays a pivotal role in stabilizing the polar order that it creates. We perform simulations for varying α , Γ , and the box size L to obtain the steady state phase diagram summarized in Fig. 3(c). We find two distinct categories of dynamics: a traveling band phase and a disordered phase, which contains a transitional region consisting of well-defined density layers with many dislocations, shown in representative snapshots of field ϕ_1 in Fig. 3(a) and Supplemental Material, Movies 1–3. The phases are characterized by analyzing the circularly averaged structure factor $S(q)$ [53] [see Fig. 3(b)] and the average polar order $\mathbf{J}$ in the steady state in Fig. 3(c). For the disordered phase, $S(q, t) \sim q^{1+\delta}$ for $q < q_m$ with $\delta \ll 1$, which implies that the order parameter is uncorrelated at large length scales. Here $q_m < 1$ is the wave number at

which $S(q)$ peaks. Consequently, the dissipation spectrum $\mathcal{E}(q, t)$ behaves similar to that of a randomly stirred Stokes fluid, and we obtain $\mathcal{E}(q, t) \sim q^{-1}$ for $q < q_m$ [53]. $S(q)$ for the different cases are shown in Fig. 3(b), from which we confirm that the spectral properties of the transitional and strongly disordered cases are similar. Therefore, we observe that, within the linearly unstable region, global polar order persists for $|\alpha| \gg |\Gamma|$ (see Supplemental Material, Movies 2 and 3), as we will discuss below.

For a given Γ , there exists a critical $\alpha_c(L)$ where we observe the transition from the disordered phase to the traveling bands. From our simulations, we observe that α_c increases with Γ ; thus, we assume the simple scaling relation $\alpha_c \sim |\Gamma|$ to quantify the transition in terms of rescaled parameter $h = \alpha/\alpha_c \sim \alpha/|\Gamma|$ [60]. In Fig. 3(c), we plot the average polar order $\bar{J} \equiv |\langle \mathbf{J} \rangle|/J_0$ versus h for different system sizes. The magnitude of $\mathbf{J}$ depends on the dominant length scales of the polar patterns, and we normalize it by J_0 , which is estimated from $S(q)$ [53]. This ensures that, for a perfect traveling wave, $\bar{J} = 1$ irrespective of the wave number, while $\bar{J} = 0$ for the disordered case. Consistent with the snapshots, we find a disorder-order transition with increasing h for all values of L . $\bar{J}$ vanishes for disordered states ($h \sim 1$), and in the transitional state ($h \lesssim h_c$), we find noisy partial order. For $h > h_c$, global polar order sets in, and $\bar{J} \sim 1$ is close to its expected value for perfect polar order. Thus, even in the

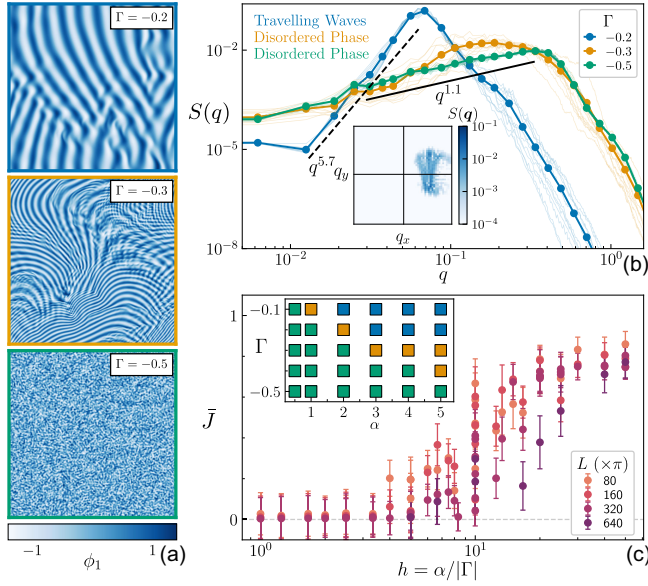


FIG. 3. Order and disorder in the full nonlinear theory. (a) Snapshots show the two distinct phases that we observe in the $\alpha - \Gamma$ parameter space: traveling waves (blue) and disordered phase, as shown by snapshots in the transitional state (orange) and deep in the disordered phase (green). (b) Shell-averaged structure factor $S(q)$ in different states for a fixed α and different Γ marked by the same colors as the borders. For traveling bands, $S(q)$ shows a peak at the dominant wave number. For the crossover and disordered states, we observe a large $\sim q^1$ power law, with an additional contribution at larger wave numbers for the transitional state. In both (a) and (b), we show the data for $\alpha = 4$. Inset: in the polar phase, structure factor $S(q)$ is anisotropic, consistent with the traveling bands. (c) Average polar order $\bar{J}$ versus $h = \alpha/|\Gamma|$ shows a disorder-order transition with increasing h for all system sizes. Inset: phase diagram in the $\alpha - \Gamma$ space for $L = 320\pi$ obtained by visual inspection of the snapshots and the structure factor.

presence of a small q linear instability, the system shows global polar order for $h > h_c$. From our simulations, we infer $h_c \sim 10$.

For a binary nonreciprocal mixture it is useful to rewrite the dynamics of the fields as the coupled dynamics for a vector order parameter and an amplitude [44]. With this transformation, the hydrodynamic stress resembles the active stress that appears in theories for flocking in a fluid [22,56]. Fluctuations around the ordered state drive velocity fluctuations that shear the polarity to either stabilize it or destabilize it to produce featureless spatiotemporal chaos. The order-disorder transition in a suspension is *stabilized* by nonreciprocity, and a new state with multiple dislocations appears in the regime where the transition happens. The instabilities of the h-NRCH model bear similarities to the generic instability in active matter as regards the form of the active stress term and the mechanism of the instability leading to the generation of a wave-number-independent damping. The important difference is that h-polar instability

is not generic as it does not destabilize the polar order for both signs of Γ in the Eckhaus stable regime $q < q_E$. Our findings highlight the complex interplay between nonreciprocal interactions and hydrodynamics in determining collective behavior in active systems and offers new insights into pattern formation in a suspension. These results are particularly relevant for experimental realization of large-scale nonreciprocal systems involving suspension of active particles, such as droplets [61,62], chemically active colloids [63,64], and mixtures of bacteria in wet environments [65]. In future work, we plan to explore the role of geometric confinement [66] or inertia [67–69] on the emergent polar order. Our Letter highlights the stabilizing effect of nonreciprocity, in line with recent reports of similar stabilizing effects in other contexts [50,70].

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G. P. performed analytical calculations and numerical simulations to support the analytical predictions. N. R. carried out large-scale pseudospectral simulations to further investigate the nonlinear effects. All authors contributed to the design of research, discussion and interpretation of the results, and writing of the manuscript. S. S. and R. G. supervised the project.

Data availability—The data supporting the main findings of this study are available in the paper and its Supplementary Material. Any additional data can be made available upon request.

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End Matter

Appendix: Group velocity and effective diffusion coefficient—Here we report the explicit expressions for the group velocity and the diffusion coefficient for $\delta \mathbf{J}_\perp$; see Eq. (6). These expressions are derived assuming small Γ ; for details see the SM [53]. For $\Gamma < 2$ we find that the phase propagates anisotropically with the modified speed $v_g(\psi)$ given by

$$v_g(\psi) = q_0 \alpha \left[2 - \Gamma \sin^2 \psi \left(\gamma + \frac{2q_0^2}{\rho_0^2} \sin^2 \psi \right) \right]. \quad (\text{A1})$$

The anisotropic diffusion coefficient is

$$D(\psi) = \gamma q_0^2 \cos^2 \psi + q_0^2 \sin^2 \psi + D_m(\psi), \quad (\text{A2})$$

where D_m is a cross-diffusion term (i.e., it vanishes for $\psi = 0, \pi/2$) proportional to Γ generated by the coupling to the fluid. The Eckhaus factor γ defined in Eq. (8) determines the sign of the longitudinal diffusivity $D(0)$ in Eq. (A2). This triggers a conserved version of the instability that occurs in the complex Landau-Ginzburg dynamics [71] and appears in NRCH without the fluid [43,44]. The full expression for D_m is

$$D_m(\psi) = -\frac{\Gamma}{4\rho_0^2} \sin^2(2\psi) \left[\alpha^2 \left(\gamma + 2\frac{q_0^2}{\rho_0^2} \sin^2 \psi \right) + q_0^2 (2 + 4q_0^2 \sin^2 \psi - 7q_0^2) \right]. \quad (\text{A3})$$

For $\Gamma < 0$, D_m is positive in large parts of the parameter space. The diffusive instability appearing in this regime due to a change of sign of D_m is also controlled by γ . Notice that the term proportional to $\alpha^2 \Gamma$ in Eq. (A3) is the same as the one that appears in λ_0 in Eq. (7).

Where the h-polar instability occurs, D determines the threshold system size above which the h-polar instability sets in; see SM [53] for details. It competes with λ_0 in determining the system size $L_I = (\sqrt{D/\lambda_0})$ below which the h-polar instability is suppressed. From Fig. 2(a), λ_0 is maximum at $\psi = \pi/2$, an estimate for $L_I \approx 2\pi/|\Gamma|\rho_0^2$, i.e., the larger the coupling $|\Gamma|$ is, the smaller the length scale for which the instability becomes manifest is. For perturbations along $\psi = \pi/2 + \epsilon$, α dependent D_m determines L_I , where we find that, for a given Γ , L_I increases rapidly with α implying that it contributes to stabilize the flock; see SM [53] for a detailed report.