

Active Brownian Dynamics in Channels: First-Passage and Spatiotemporal Properties via Siegmund Duality

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Accumulation at boundaries represents a widely observed phenomenon in active systems with implications for microbial ecology and engineering applications. To rationalize the underlying physics, we study the first-passage properties and spatial distributions of an active Brownian particle (ABP) in a channel. Leveraging Siegmund duality, we establish a direct mapping between the propagators of ABPs with absorbing and hard-wall boundary conditions, yielding analytical results in both problems. We analyze the system across low and high activity regimes—quantifying persistent motion relative to diffusion—and show that active motion, together with a favorable initial orientation, typically lowers the mean first-passage time relative to passive diffusion. Notably, the full time-dependent propagator between hard walls approaches a wall-accumulated stationary state, given by the derivative of the splitting probability as a consequence of Siegmund duality.

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Active agents inevitably engage with boundaries: not only do the latter facilitate foraging and survival of microorganisms via surface-concentrated nutrients and biofilm formation, they also mediate their navigation by enabling persistent exploration in the absence of complex sensing behavior [1–8]. Boundaries are also the pivot point for functional tasks, where future microrobots are expected to translate their energy into mechanical work for performing therapeutic or engineering operations [9–14]. To guide the design of these applications and obtain a physical understanding of the microbiological phenomena, it is crucial to unravel two questions: first, how long does it take an active agent to reach a boundary; and second, how do boundaries alter their distributions?

To address these topics, a number of models—including the canonical active Brownian particle (ABP) [15–23], the active Ornstein-Uhlenbeck particle [24], and the run-and-tumble particle [25–31]—have shown how the intrinsic dynamics leverage activity to enable accelerated target reaching in comparison to passive particles. Confined dynamics also display characteristic phenomena: highly active

particles can accumulate near walls [32,33] without the need of hydrodynamics, while the time it takes agents to escape a wide, slowly varying channel has been linked to their distribution near the boundaries [34], reflecting the curvature-dependent pressure field of active systems [35–37]. The phenomenology of boundary interactions is thus rich, yet the coupling between translational and rotational degrees of freedom makes deriving analytical expressions for confined active dynamics challenging and requires new theoretical frameworks.

A particularly useful tool is Siegmund duality [38,39], which establishes an exact correspondence between stochastic dynamics with absorbing and hard boundaries (Fig. 1). Specifically, for two walls separated by L , the hard-wall propagator $p_H(z, t|z_0)$ (i.e., the probability density to be at position z at time t , given initial position z_0), follows from the absorbing-wall propagator p_A via

$$p_H(z, t|z_0) = \int_{z_0}^L dz' \frac{\partial}{\partial z} p_A(z', t|z). \quad (1)$$

A similar relation can be derived to compute p_A from p_H ; thus the solution for one boundary condition (BC) determines the solution for the other. Siegmund duality, first noted for Brownian motion [40,41] and later formalized in [38], constitutes a particular instance of a Markov duality [42–44]. At its core lies a time-reversal symmetry relating the forward Fokker-Planck operator of a process to the backward operator of its dual [39]. Constructing the dual of a stochastic process is, however, generally a nontrivial task. Only recently have explicit constructions been obtained for

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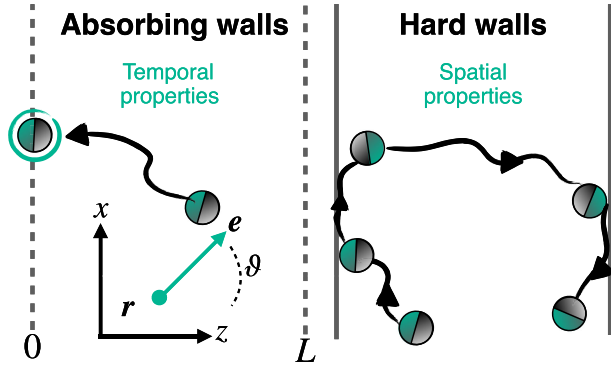


FIG. 1. Schematic of the duals: ABPs between absorbing (sticking) and hard (reflective) walls. The left side shows an ABP hitting an absorbing boundary, which gives access to first-passage properties. The right side illustrates the dynamics between hard walls, yielding particle distributions. The inset depicts the agent's position $\mathbf{r}$ and orientation θ .

continuous-time stochastic processes driven by an equilibrium Markov process, such as active particles, and discrete-time random walks with stationary time-reversal-symmetric increments, which may be correlated and non-Markovian [39,45–47].

In this Letter, we derive a systematic expansion for the absorbing-wall propagator in the low-activity regime and analyze the high-activity regime in the long-time limit, providing access to the first-passage statistics. We then use these analytical results to determine the hard-wall propagator via Eq. (1), which we also prove for ABPs. Our analytical results show that, depending on the initial position and orientation, activity can either enhance or hinder time efficiency. Moreover, the dynamics between hard walls relaxes to a wall-accumulated stationary state, akin to that found in many active systems [8,33,48–53].

Model—We consider the motion of a two-dimensional ABP moving at a constant speed v along its instantaneous orientation $\mathbf{e}(t) = [\sin \theta(t), \cos \theta(t)]$, which is subject to rotational diffusion with diffusivity D_{rot} . The particle position also experiences translational diffusion with diffusivity D . The position $\mathbf{r}(t) = [x(t), z(t)]$ and orientation $\theta(t)$ are governed by the Langevin equations

$$\dot{\mathbf{r}} = v\mathbf{e} + \sqrt{2D}\boldsymbol{\eta} \quad \text{and} \quad \dot{\theta} = \sqrt{2D_{\text{rot}}}\xi, \quad (2)$$

where $\boldsymbol{\eta}(t)$ and $\xi(t)$ are independent, delta-correlated Gaussian white noises with zero mean. The ABP is confined between two walls located at $z = 0$ and $z = L > 0$ with two different BCs: absorbing (A) and hard (H) walls. Its dynamics are characterized by three timescales: the diffusive time $\tau = L^2/D$, the ballistic time $\tau_a = L/v$, and the rotational diffusion time $\tau_{\text{rot}} = 1/D_{\text{rot}}$. This entails two non-dimensional numbers: the Péclet number $\text{Pe} = \tau/\tau_a = vL/D$, measuring active motion relative to diffusion, and

$\gamma = \tau/\tau_{\text{rot}} = D_{\text{rot}}L^2/D$, reflecting the importance of rotational vs translational diffusion.

Since we are only interested in the dynamics perpendicular to the walls, we focus on the z coordinate. We denote the associated propagators, averaged over the equilibrium initial orientation, i.e., $p_{\text{eq}}(\theta_0) = 1/(2\pi)$, and integrated over the final orientation, by

$$p_{A,H}(z, t|z_0) = \int d\theta \int \frac{d\theta_0}{2\pi} p_{A,H}(z, \theta, t|z_0, \theta_0), \quad (3)$$

where $p_{A,H}(z, \theta, t|z_0, \theta_0)dzd\theta$ is the probability that the ABP is found in the interval $[z, z + dz]$ with orientation in $[\theta, \theta + d\theta]$ at time t , given the initial conditions $z(t=0) = z_0$ and $\theta(t=0) = \theta_0$. We first demonstrate Eq. (1) by adapting the main steps of Ref. [39]. This, in turn, enables us to derive analytical results for the time-dependent distribution between hard walls from the absorbing-wall propagator of ABPs.

Absorbing (sticking) walls—We first consider an ABP with sticking absorption at $z = 0$ and $z = L$: upon first hitting either wall, the particle remains (“sticks”) there for all later times. Accordingly, the propagator may contain Dirac masses at $z = 0$ and $z = L$ weighted by the time-integrated incoming boundary flux up to time t . It satisfies the backward Fokker-Planck equation (FPE)

$$\partial_t p_A(z, t|z_0, \theta_0) = \mathcal{L}_{z_0, \theta_0}^\dagger p_A(z, t|z_0, \theta_0), \quad (4)$$

where the backward operator $\mathcal{L}_{z_0, \theta_0}^\dagger := v \cos \theta_0 \partial_{z_0} + D \partial_{z_0}^2 + D_{\text{rot}} \partial_{\theta_0}^2$ acts on the *initial* variables (z_0, θ_0) , and we have averaged over the final orientation. We define the probability to find the ABP in $[\ell_A > 0, L]$ at time t with initial conditions (z_0, θ_0) :

$$Q_A(\ell_A, t|z_0, \theta_0) := \mathbb{P}_A[z(t) \geq \ell_A|z_0, \theta_0], \quad (5a)$$

$$= \int_{\ell_A}^L dz p_A(z, t|z_0, \theta_0). \quad (5b)$$

Since $\mathcal{L}^\dagger$ acts only on the initial variables, integrating Eq. (4) over $z \in [\ell_A, L]$ yields

$$\partial_t Q_A(\ell_A, t|z_0, \theta_0) = \mathcal{L}_{z_0, \theta_0}^\dagger Q_A(\ell_A, t|z_0, \theta_0), \quad (6)$$

subject to BCs $Q_A(\ell_A, t|0, \theta_0) = 0$, since a particle starting at $z = 0$ is absorbed immediately, and $Q_A(\ell_A, t|L, \theta_0) = 1$, as a particle starting at $z = L$ is absorbed and $z = L \geq \ell_A$. Moreover, from the initial condition $p_A(z, 0|z_0, \theta_0) = \delta(z - z_0)$, we obtain $Q_A(\ell_A, 0|z_0, \theta_0) = \Theta(z_0 - \ell_A)$ where $\Theta(x)$ is the Heaviside function.

Hard (reflective) walls—We next consider an ABP between two hard walls at $z = 0$ and $z = L$ and denote by $p_H(z, \theta, t|z_0)$ the probability density averaged over the

initial angle. It satisfies the forward FPE

$$\partial_t p_H(z, \vartheta, t|\tilde{z}_0) = \mathcal{L}_{z,\vartheta} p_H(z, \vartheta, t|\tilde{z}_0), \quad (7)$$

where the forward operator $\mathcal{L}_{z,\vartheta} := -v \cos \vartheta \partial_z + D \partial_z^2 + D_{\text{rot}} \partial_\vartheta^2$ acts on the *final* variables (z, ϑ) . The system is subject to no-flux BCs: $v \cos \vartheta p_H - D \partial_z p_H = 0$ at $z = 0$ and L . We now define the conditional probability to find the ABP in $[0, \ell_H]$ at time t given the fixed final orientation $\vartheta(t) = \vartheta$ as

$$Q_H(\ell_H, t|\vartheta, \tilde{z}_0) := \mathbb{P}_H[z(t) \leq \ell_H|\vartheta, \tilde{z}_0], \quad (8a)$$

$$= \frac{\int_0^{\ell_H} dz p_H(z, \vartheta, t|\tilde{z}_0)}{p_{\text{eq}}(\vartheta)}. \quad (8b)$$

Here, Eq. (8b) follows from Bayes' rule: $p_H(z, t|\vartheta, \tilde{z}_0) = p_H(z, \vartheta, t|\tilde{z}_0)/p(\vartheta, t|\tilde{z}_0)$. As $\vartheta(t)$ evolves independently of $z(t)$ and is initialized at stationarity, $\vartheta(0) \sim p_{\text{eq}}(\vartheta)$, its marginal remains $p_{\text{eq}}(\vartheta)$ for all t [hence $p(\vartheta, t|\tilde{z}_0) = p_{\text{eq}}(\vartheta)$]. Further, since $p_{\text{eq}}(\vartheta) = 1/(2\pi)$ is ϑ independent, integrating Eq. (7) over $z \in [0, \ell_H]$ yields

$$\partial_t Q_H(\ell_H, t|\vartheta, \tilde{z}_0) = \mathcal{L}_{\ell_H, \vartheta} Q_H(\ell_H, t|\vartheta, \tilde{z}_0). \quad (9)$$

More generally, if $p_{\text{eq}}(\vartheta)$ is not constant, Eq. (9) remains valid provided the ϑ dynamics is reversible, independent of $z(t)$, and initialized at stationarity [39]. Translational Brownian noise prevents singular boundary masses. Therefore, the probability to lie in $[0, \ell_H]$ vanishes at $\ell_H = 0$, yielding the BC $Q_H(0, t|\vartheta, \tilde{z}_0) = 0$. Further, hard-wall confinement enforces $z(t) \in [0, L]$ for all t , hence $Q_H(L, t|\vartheta, \tilde{z}_0) = 1$, and the localized start $p_H(z, \vartheta, 0|\tilde{z}_0) = \delta(z - \tilde{z}_0)$ translates to $Q_H(\ell_H, 0|\vartheta, \tilde{z}_0) = \Theta(\ell_H - \tilde{z}_0)$.

Siegmund duality—Under the angle shift $\vartheta = \vartheta_0 + \pi$, which encodes the reversal of the active drift $-v \cos(\vartheta_0 + \pi) = v \cos \vartheta_0$, the operators match $\mathcal{L}_{z_0, \vartheta_0 + \pi} = \mathcal{L}_{z_0, \vartheta_0}^\dagger$, i.e., the hard-wall process at time t is conditioned on an orientation opposite to the absorbing process's initial one. Identifying $\ell_A = \tilde{z}_0$ in Eq. (5a) and $\ell_H = z_0$ in Eq. (8a), their initial conditions also coincide. Given these constraints, $Q_H(z_0, t|\vartheta(t) = \vartheta_0 + \pi, \tilde{z}_0)$ and $Q_A(\tilde{z}_0, t|z_0, \vartheta_0)$ satisfy the same differential equation with the same boundary and initial conditions, and are therefore equal. Averaging over $\vartheta_0 \sim p_{\text{eq}}$ and using the invariance of p_{eq} under $\vartheta_0 \mapsto \vartheta_0 + \pi$ yields the Siegmund duality relation [38,39]

$$\mathbb{P}_H[z(t) \leq z_0|\tilde{z}_0] = \mathbb{P}_A[z(t) \geq \tilde{z}_0|z_0]. \quad (10)$$

Hence, after averaging the initial orientations over $p_{\text{eq}}(\vartheta)$, the probability that the hard-wall ABP started at $\tilde{z}_0$ lies to the left of z_0 at time t equals the probability that the absorbing ABP started at z_0 lies to the right of $\tilde{z}_0$ at the same time.

Differentiating Eq. (10) with respect to z_0 , and renaming variables, straightforwardly gives the announced result in Eq. (1).

The duality relation [Eq. (10)] also allows deriving the survival probability from the hard-wall propagator via (see End Matter)

$$S(z_0, t) = \int_0^{z_0} dz [p_H(z, t|0) - p_H(z, t|L)], \quad (11)$$

and relates the mean-first-passage time (MFPT) from z_0 to either wall to the local time spent by ABPs in $[0, z_0]$ between hard walls:

$$\langle T \rangle = \int_0^{z_0} dz \int_0^\infty dt [p_H(z, t|0) - p_H(z, t|L)]. \quad (12)$$

Interestingly, Eqs. (11) and (12) are reminiscent of the relations derived in Ref. [54], where first-passage observables are connected to time integrals of propagator differences in complementary geometries.

First-passage properties—We decompose the propagator of an ABP between absorbing walls $p_A(z, t|z_0)$ as

$$p_A = P_A + E_0 \delta(z) + E_L \delta(z - L), \quad (13)$$

where $P_A(z, t|z_0)$ denotes the probability density of an ABP between absorbing nonsticking walls, i.e., boundaries for which the particle disappears upon reaching a wall [$P_A(z = 0, t) = P_A(z = L, t) = 0$], while boundary masses are encoded in the exit probabilities:

$$E_{0,L}(z_0, t) = \pm D \int_0^t dt' \partial_z P_A(z, t'|z_0)|_{z=0,L}. \quad (14)$$

In the low activity regime ($\text{Pe} \ll 1$), where translational diffusion dominates over active motion, we compute $P_A(z, t|z_0) = \int d\vartheta \int d\vartheta_0 P_A(z, \vartheta, t|z_0, \vartheta_0)/(2\pi)$ by solving the forward FPE [Eq. (7)] with initial condition $P_A(z, \vartheta, t = 0|z_0, \vartheta_0) = \delta(z - z_0)\delta(\vartheta - \vartheta_0)$. The time-dependent behavior can be obtained by first moving to Laplace space [$\hat{f}(s) = \int_0^\infty f(t)e^{-st}dt$] and inserting the Péclet number expansion $P_A = \hat{P}_A^{(0)} + \text{Pe}\hat{P}_A^{(1)} + \text{Pe}^2\hat{P}_A^{(2)} + O(\text{Pe}^3)$ into Eq. (7). Here, $\hat{P}_A^{(0)}$ corresponds to the propagator of a diffusive particle (with rotational diffusion) between two absorbing walls and can be readily computed using the method of reflections (see End Matter). This procedure yields a recursive relation

$$\begin{aligned} \hat{P}_A^{(i+1)}(z, \vartheta, s|z_0, \vartheta_0) &= \int_0^L dz' \int_0^{2\pi} d\vartheta' \hat{P}_A^{(i)}(z, \vartheta, s|z', \vartheta') \\ &\times \left[-(D/L) \cos(\vartheta') \partial_{z'} \hat{P}_A^{(i)}(z', \vartheta', s|z_0, \vartheta_0) \right]. \end{aligned} \quad (15)$$

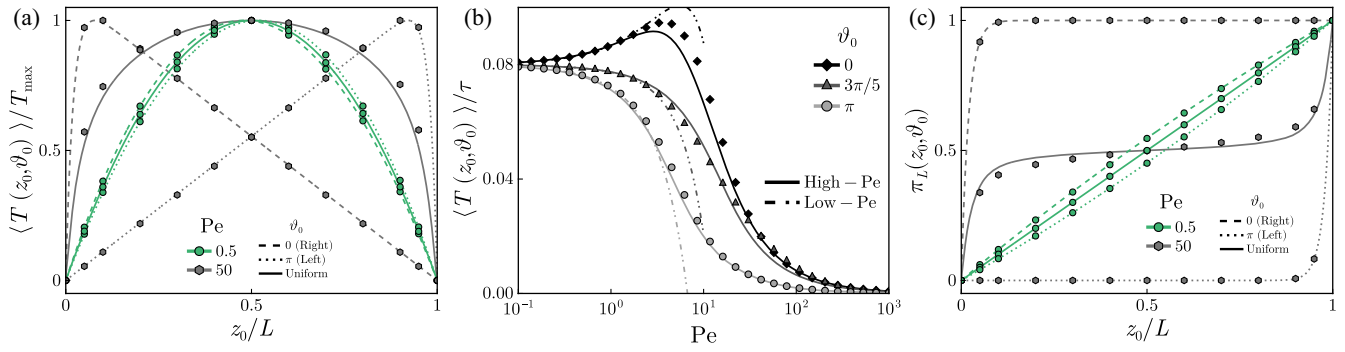


FIG. 2. First-passage properties. (a) Mean-first-passage time $\langle T \rangle$ normalized by its maximum $T_{\max}$, as a function of the initial position z_0 . (b) Mean-first-passage time $\langle T \rangle$ as a function of the Péclet number and for $z_0/L = 0.2$. (c) Splitting probability π_L at the right wall as a function of z_0 . In (a)–(c) we set $\gamma = 3$. In (b), dash-dotted and solid lines denote the low-Pe expansion up to $O(Pe^2)$ and the high-activity result [Eq. (16)], respectively. Symbols correspond to simulation results.

We then use the survival probability $\hat{S}(s|z_0, \vartheta_0) = \int_0^{2\pi} \int_0^L \hat{P}_A(z, \vartheta, s|z_0, \vartheta_0) dz d\vartheta$ to compute the MFPT via $\langle T \rangle = \hat{S}(s=0|z_0, \vartheta_0)$ [Fig. 2].

Analytical progress is also possible in the high-activity regime, where persistent motion dominates over orientational diffusion $\tau_a \ll \tau_{\text{rot}}$ (i.e., $\gamma \ll Pe$). Therefore, we introduce $\epsilon = \gamma/Pe$ and first look into the boundary layer of width $O(L\epsilon)$. Rescaling the equation for the MFPT, $\mathcal{L}_{z_0, \vartheta_0}^\dagger \langle T \rangle = -1$, with the stretched coordinate $y = (z_0 - L)/(L\epsilon)$ shows that (close to the walls) rotational diffusion can be neglected, given $\gamma \ll Pe^{2/3}$ (see End Matter). Neglecting rotational diffusion reduces the high-activity limit to a standard one-dimensional biased-diffusion problem. Taking the BCs $\langle T(0, \vartheta_0) \rangle = \langle T(L, \vartheta_0) \rangle = 0$, we obtain (for $\cos \vartheta_0 \neq 0$) [55]

$$\langle T \rangle_{\gamma/Pe \rightarrow 0} = \frac{\tau}{Pe \cos \vartheta_0} \left[\frac{1 - e^{-Pe \cos(\vartheta_0) z_0/L}}{1 - e^{-Pe \cos(\vartheta_0)}} - \frac{z_0}{L} \right]. \quad (16)$$

Our results [Fig. 2(a)] show that randomly oriented, active particles take the longest when starting at the channel's midpoint, with a symmetric MFPT with respect to the initial position z_0 . This picture changes for particles initially oriented towards the right wall ($z/L = 1$): At low activity ($Pe = 0.5$), the maximal MFPT shifts to $z_0/L \lesssim 0.5$ and becomes asymmetric, because active motion helps the particle to reach the right wall (towards which it is oriented) faster. For large Péclet numbers ($Pe = 50$), the peak of the MFPT shifts towards $z_0 \gtrsim 0$ and decays linearly thereafter, which reflects the typical time taken by a persistent particle to cover the remaining distance at constant speed: $\langle T \rangle \simeq (L - z_0)/v$. The picture reverses when the particle initially travels towards the left wall.

We further investigate the MFPT as a function of Pe , which decreases monotonically for particles starting at $z_0 = 0.2L$ and oriented towards the closer wall at $z = 0$ ($\vartheta_0 = 3\pi/5, \pi$) [Fig. 2(b)], showing that active motion speeds up their wall arrival. Interestingly, the MFPT

becomes nonmonotonic for particles starting close to a wall and departing towards the opposite one ($\vartheta_0 = 0$). While the agent reaches the closer boundary through diffusion at $Pe \lesssim 1$ with $\langle T \rangle \simeq z_0(L - z_0)/(2D)$, it reaches the opposite boundary almost instantaneously for $Pe \gtrsim 100$. At intermediate Pe the MFPT increases, which can be understood from the small-Pe expansion of Eq. (16): $\langle T \rangle / \tau = z_0(L - z_0)/(2L^2) + C(z_0) \cos \vartheta_0 Pe + O(Pe^2)$, where $C(z_0 = 0.2L) > 0$. Thus, the MFPT increases for particles oriented towards the opposite wall $|\vartheta_0| < \pi/2$, leading to an overall nonmonotonic behavior as the MFPT decreases as $\sim 1/Pe$ at large Pe . Physically, this behavior results from the fact that active motion takes the particle away from the closer wall and thus increases the time to reach the latter via translational diffusion. We note that Eq. (16) agrees with our simulations at large $Pe \gg \gamma^{3/2} \approx 5.2$ and low Pe , while discrepancies appear at moderate Pe and at larger γ , where rotational diffusion becomes important. Our low-Pe expansion captures the simulations irrespective of γ and notably until $Pe \approx 10$ for selected cases, as it accounts for the rotational-translational coupling [see End Matter, Figs. 4(a) and 4(b)].

Another important first-passage quantity is the splitting probability [55–58], i.e., the probability that the agent reaches the right wall before the left one. For small Pe , it can be computed as the long-time limit of the exit probability $\pi_L(z_0) = E_L(z_0, t \rightarrow \infty)$ [Eq. (14)]. For high activity, where the dynamics reduce to biased diffusion, we employ the same approach as for $\langle T \rangle$ [with BCs $\pi_L(0, \vartheta_0) = 0$, $\pi_L(L, \vartheta_0) = 1$], leading to [55]

$$\pi_L(z_0, \vartheta_0)_{\gamma/Pe \rightarrow 0} = \frac{1 - e^{-Pe \cos(\vartheta_0) z_0/L}}{1 - e^{-Pe \cos(\vartheta_0)}}. \quad (17)$$

See also Ref. [58] for a related high-activity boundary-layer analysis of the splitting probability. For randomly oriented agents, the splitting probability at the right wall π_L increases almost linearly with z_0 at low activity, where translational diffusion dominates [Fig. 2(c)]. Our results

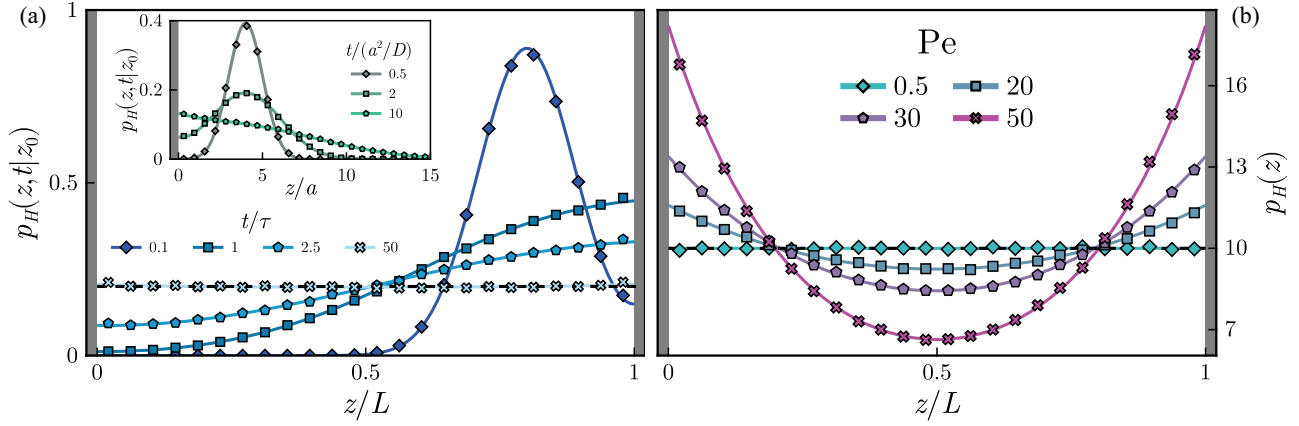


FIG. 3. Probability densities of an ABP between hard walls. (a) Probability density $p_H(z, t|z_0)$ as a function of the position z for different times t . Here, $z_0/L = 0.8$ and $Pe = 0.5$. Lines correspond to the low- Pe [up to $O(Pe^2)$] expansion and symbols are simulation results. (Inset) Probability density $p_H(z, t|z_0)$ for the case of a single wall at $z = 0$. We set $z_0/a = 4$ and $Pe = 0.5$, and rescale length scales with the particle's hydrodynamic radius $a = \sqrt{3D/(4D_{\text{rot}})}$. (b) Stationary distribution $p_H(z)$ for different Péclet numbers. Lines for $Pe \geq 20$ correspond to the high- Pe solution [Eq. (18)] and symbols are simulation results. In (a) and (b), vertical gray lines indicate the walls.

further show that particles initially oriented towards the right wall have a higher probability of reaching it than those oriented away from it. The difference in probabilities becomes smaller at the boundaries, where translational diffusion outperforms active motion. At high activity, however, π_L approaches a plateau at ≈ 0.5 for most z_0 , reflecting the ballistic motion of the particle towards either wall with equal probability. For a fixed initial orientation, the behavior drastically changes: π_L jumps to 1 when ϑ_0 points towards the wall at $z = L$, and to 0 when it points towards the opposite one.

Confined particles—Leveraging Siegmund duality [Eq. (1)] and Eq. (15), we compute the time-dependent propagator in the presence of hard walls $p_H(z, t|z_0)$. Setting $\bar{z}_0 = L$ in Eq. (10) and taking $t \rightarrow \infty$ shows that the stationary distribution $p_H(z) = p_H(z, t \rightarrow \infty|z_0)$ is a simple derivative of the splitting probability [39,45] (see End Matter). In the high-activity regime, it can thus be computed from Eq. (17), without solving the stationary FPE directly. It reads

$$p_H(z) = \frac{\partial \pi_L}{\partial z} = \frac{Pe}{2\pi L} \int_0^{2\pi} \cos(\vartheta_0) \frac{e^{-Pe \cos(\vartheta_0) z/L}}{1 - e^{-Pe \cos(\vartheta_0) L}} d\vartheta_0, \quad (18)$$

thereby recovering the result of Ref. [33]. Figure 3(a) shows how, starting from a localized initial condition, the probability density spreads over the channel width and eventually reaches a stationary distribution. The latter evolves from an almost uniform profile at low activity ($Pe = 0.5$) to an increasingly pronounced U-shaped profile as activity increases [Fig. 3(b)], indicating enhanced wall accumulation. In our model, this accumulation results solely from the interactions of active agents with hard walls, as active particles remain near the boundary until rotational diffusion allows them to reorient and escape.

This finding is in qualitative agreement with experimental [48–50] and computational [33] observations, yet the importance of steric vs hydrodynamic interactions [50], yielding similar wall-accumulated distributions, may depend on the specific systems.

The time-dependent propagator with hard walls (in the low- Pe regime) provides direct access to the moments of the position $\langle z^n(t) \rangle = \int_0^L p_H(z, t|z_0) z^n dz$. As suggested by the stationary distribution, the mean position converges to the channel's midpoint, $\langle z(t) \rangle_{t \rightarrow \infty} = L/2$, while the fluctuations around it are perturbed $\langle [z(t) - \langle z(t) \rangle]^2 \rangle_{t \rightarrow \infty} = (L^2/12) + A Pe^2 + O(Pe^4)$ with $A > 0$, showing that activity enhances the uniform Brownian result.

Finally, our model recovers the one-wall problem in the limit of infinite channel width, $L \rightarrow \infty$ [Fig. 3(a) (inset)]. In this case, the MFPT diverges [16] and no stationary distribution exists: without the right boundary, the particle can escape to infinity, leading to a probability density that broadens continuously in space over time.

Conclusions—We have analytically characterized active Brownian dynamics between two walls. A systematic low-Péclet expansion derived for the absorbing problem can be transferred through Siegmund duality to the time-dependent probability density between hard walls. We thereby quantified the evolution towards a stationary wall-accumulated state, thus providing a theoretical footing for experimental and computational observations of active agents. Our work adds to our understanding of active Brownian transport in channels, by unraveling how the interplay of active motion and diffusion dictates first-passage statistics of active agents between two walls.

The formalism employed here offers a systematic approach for accessing both time-related efficiencies and spatial distribution properties in a class of problems that are

otherwise difficult to address. It admits several extensions; these include alternative agent dynamics, such as run-and-tumble motion [45,59–62]; stochastic resetting processes [39,63–66]; non-Markovian processes [39,54,67]; higher-dimensional settings that could introduce nontrivial modifications [68]; more complex geometries, e.g., structured walls [34,58,69,70] or concentric confinement; and partially absorbing boundaries [71–73]. The latter problems may prove more tractable in one setting—absorbing or reflecting—whose solution can be systematically transferred to its dual counterpart. Beyond confining boundaries, this framework may be applied to study the impact of potential barriers on escape problems [29,74]. Overall, it may thus enable progress in quantifying biological and soft-matter systems, where confinement strongly regulates transport and reaction efficiency, while systematic theoretical approaches remain comparatively scarce.

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Data availability—There are no publicly available data or software supporting this manuscript. We provide a Mathematica Script for analytical formulas on Zenodo: [75]. Requests for further information or data should be sent to the authors.

- [1] R. Di Leonardo, L. Angelani, D. Dell’Arciprete, G. Ruocco, V. Iebba, S. Schippa, M. P. Conte, F. Mecarini, F. De Angelis, and E. Di Fabrizio, Bacterial ratchet motors, *Proc. Natl. Acad. Sci. U.S.A.* **107**, 9541 (2010).
- [2] D. Ray, C. Reichhardt, and C. J. O. Reichhardt, Casimir effect in active matter systems, *Phys. Rev. E* **90**, 013019 (2014).
- [3] P. Galajda, J. Keymer, P. Chaikin, and R. Austin, A wall of funnels concentrates swimming bacteria, *J. Bacteriol.* **189**, 8704 (2007).
- [4] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, Active particles in complex and crowded environments, *Rev. Mod. Phys.* **88**, 045006 (2016).
- [5] J. Simmchen, J. Katuri, W. E. Uspal, M. N. Popescu, M. Tasinkevych, and S. Sánchez, Topographical pathways guide chemical microswimmers, *Nat. Commun.* **7**, 10598 (2016).
- [6] G. C. L. Wong *et al.*, Roadmap on emerging concepts in the physical biology of bacterial biofilms: From surface sensing to community formation, *Phys. Biol.* **18**, 051501 (2021).
- [7] O. Hallatschek, S. S. Datta, K. Drescher, J. Dunkel, J. Elgeti, B. Waclaw, and N. S. Wingreen, Proliferating active matter, *Nat. Rev. Phys.* **5**, 407 (2023).
- [8] P. Denissenko, V. Kantsler, D. J. Smith, and J. Kirkman-Brown, Human spermatozoa migration in microchannels reveals boundary-following navigation, *Proc. Natl. Acad. Sci. U.S.A.* **109**, 8007 (2012).
- [9] G. Gompper *et al.*, The 2025 motile active matter roadmap, *J. Phys. Condens. Matter* **37**, 143501 (2025).
- [10] V. A. Baulin *et al.*, Intelligent soft matter: Towards embodied intelligence, *Soft Matter* **21**, 4129 (2025).
- [11] G. Volpe *et al.*, Roadmap for animate matter, *J. Phys. Condens. Matter* **37**, 333501 (2025).
- [12] P. Erkoc, I. C. Yasa, H. Ceylan, O. Yasa, Y. Alapan, and M. Sitti, Mobile microrobots for active therapeutic delivery, *Adv. Ther.* **2**, 1800064 (2019).
- [13] Y. Alapan, O. Yasa, B. Yigit, I. C. Yasa, P. Erkoc, and M. Sitti, Microrobotics and microorganisms: Biohybrid autonomous cellular robots, *Annu. Rev. Control Rob. Auton. Syst.* **2**, 205 (2019).
- [14] R. Nauber, S. R. Gouda, M. Goeckenjan, M. Bornhäuser, C. Ribeiro, and M. Medina-Sánchez, Medical microrobots in reproductive medicine from the bench to the clinic, *Nat. Commun.* **14**, 728 (2023).
- [15] F. Di Trapani, T. Franosch, and M. Caraglio, Active Brownian particles in a circular disk with an absorbing boundary, *Phys. Rev. E* **107**, 064123 (2023).
- [16] Y. Baouche, M. Le Goff, C. Kurzthaler, and T. Franosch, First-passage-time statistics of active Brownian particles: A perturbative approach, *Phys. Rev. E* **111**, 054113 (2025).
- [17] S. A. Iyaniwura and Z. Peng, Mean first passage time of active Brownian particles in two dimensions, *New J. Phys.* **27**, 104401 (2025).
- [18] U. Basu, S. N. Majumdar, A. Rosso, and G. Schehr, Active Brownian motion in two dimensions, *Phys. Rev. E* **98**, 062121 (2018).
- [19] U. Basu, S. N. Majumdar, A. Rosso, and G. Schehr, Long-time position distribution of an active Brownian particle in two dimensions, *Phys. Rev. E* **100**, 062116 (2019).
- [20] A. Poncet, O. Bénichou, V. Démery, and D. Nishiguchi, Pair correlation of dilute active Brownian particles: From low-activity dipolar correction to high-activity algebraic depletion wings, *Phys. Rev. E* **103**, 012605 (2021).
- [21] M. Caraglio, Two-dimensional active Brownian particles crossing a parabolic barrier: Finite rectangular domain with absorbing boundary conditions, *J. Stat. Mech. Theory Exp.* **2026**, 023205 (2026).
- [22] I. Santra, U. Basu, and S. Sabhapandit, Active Brownian motion with directional reversals, *Phys. Rev. E* **104**, L012601 (2021).
- [23] E. Q. Z. Moen, K. S. Olsen, J. Rønning, and L. Angheluta, Trapping of active Brownian and run-and-tumble particles: A first-passage time approach, *Phys. Rev. Res.* **4**, 043012 (2022).
- [24] E. Woillez, Y. Kafri, and V. Lecomte, Nonlocal stationary probability distributions and escape rates for an active Ornstein–Uhlenbeck particle, *J. Stat. Mech. Theory Exp.* **2020**, 063204 (2020).
- [25] L. Angelani, R. Di Leonardo, and M. Paoluzzi, First-passage time of run-and-tumble particles, *Eur. Phys. J. E* **37**, 59 (2014).
- [26] A. Dhar, A. Kundu, S. N. Majumdar, S. Sabhapandit, and G. Schehr, Run-and-tumble particle in one-dimensional confining potentials: Steady-state, relaxation, and first-passage properties, *Phys. Rev. E* **99**, 032132 (2019).
- [27] K. Malakar, V. Jemseena, A. Kundu, K. Vijay Kumar, S. Sabhapandit, S. N. Majumdar, S. Redner, and A. Dhar, Steady state, relaxation and first-passage properties of a

- run-and-tumble particle in one-dimension, *J. Stat. Mech. Theory Exp.* **2018**, 043215 (2018).
- [28] M. Guéneau, S. N. Majumdar, and G. Schehr, Optimal mean first-passage time of a run-and-tumble particle in a class of one-dimensional confining potentials, *Europhys. Lett.* **145**, 61002 (2024).
- [29] M. Guéneau, S. N. Majumdar, and G. Schehr, Run-and-tumble particle in one-dimensional potentials: Mean first-passage time and applications, *Phys. Rev. E* **111**, 014144 (2025).
- [30] B. De Bruyne, S. N. Majumdar, and G. Schehr, Survival probability of a run-and-tumble particle in the presence of a drift, *J. Stat. Mech. Theory Exp.* **2021**, 043211 (2021).
- [31] F. Mori, P. Le Doussal, S. N. Majumdar, and G. Schehr, Universal survival probability for a d -dimensional run-and-tumble particle, *Phys. Rev. Lett.* **124**, 090603 (2020).
- [32] J. Elgeti and G. Gompper, Self-propelled rods near surfaces, *Europhys. Lett.* **85**, 38002 (2009).
- [33] J. Elgeti and G. Gompper, Wall accumulation of self-propelled spheres, *Europhys. Lett.* **101**, 48003 (2013).
- [34] Y. Zhao, Active particles in a tube: A generalized entropy potential approach, *Phys. Rev. Res.* **7**, 013015 (2025).
- [35] A. P. Solon, Y. Fily, A. Baskaran, M. E. Cates, Y. Kafri, M. Kardar, and J. Tailleur, Pressure is not a state function for generic active fluids, *Nat. Phys.* **11**, 673 (2015).
- [36] F. Smalenburg and H. Löwen, Swim pressure on walls with curves and corners, *Phys. Rev. E* **92**, 032304 (2015).
- [37] A. Duzgun, Active Brownian particles near straight or curved walls: Pressure and boundary layers, *Phys. Rev. E* **97**, 032606 (2018).
- [38] D. Siegmund, The equivalence of absorbing and reflecting barrier problems for stochastically monotone Markov processes, *Ann. Probab.* **4**, 914 (1976).
- [39] M. Guéneau and L. Touzo, Siegmund duality for physicists: A bridge between spatial and first-passage properties of continuous- and discrete-time stochastic processes, *J. Stat. Mech. Theory Exp.* **2024**, 083208 (2024).
- [40] P. Lévy, *Processus Stochastiques et Mouvement Brownien*, 2nd ed., Les grands classiques Gauthier-Villars (Ed. J. Gabay, Sceaux, 1992).
- [41] D. V. Lindley, The theory of queues with a single server, *Math. Proc. Cambridge Philos. Soc.* **48**, 277 (1952).
- [42] J. Cox and U. Rösler, A duality relation for entrance and exit laws for Markov processes, *Stochastic Processes Appl.* **16**, 141 (1984).
- [43] S. Jansen and N. Kurt, On the notion(s) of duality for Markov processes, *Probab. Surv.* **11**, 59 (2014).
- [44] C. Monthus, Markov dualities via the spectral decompositions of the two Markov generators in their bi-orthogonal basis of right and left eigenvectors, *J. Stat. Mech. Theory Exp.* **2025**, 103201 (2025).
- [45] M. Guéneau and L. Touzo, Relating absorbing and hard wall boundary conditions for a one-dimensional run-and-tumble particle, *J. Phys. A Math. Theor.* **57**, 225005 (2024).
- [46] M. Guéneau, Non-equilibrium dynamics and first-passage properties of stochastic processes: From Brownian motion to active particles, Ph. D. thesis, Sorbonne Université, 2025, 10.70675/a1944f89z8395z495bzb3a3z612db0595343.
- [47] L. Touzo, Exact results for active particle models: From long-range interactions to first-passage properties, [arXiv:2507.17504](https://arxiv.org/abs/2507.17504)
- [48] Rothschild, Non-random distribution of bull spermatozoa in a drop of sperm suspension, *Nature (London)* **198**, 1221 (1963).
- [49] H. Winet, G. S. Bernstein, and J. Head, Observations on the response of human spermatozoa to gravity, boundaries and fluid shear, *Reproduction* **70**, 511 (1984).
- [50] A. P. Berke, L. Turner, H. C. Berg, and E. Lauga, Hydrodynamic attraction of swimming microorganisms by surfaces, *Phys. Rev. Lett.* **101**, 038102 (2008).
- [51] L. Caprini and U. Marini Bettolo Marconi, Active particles under confinement and effective force generation among surfaces, *Soft Matter* **14**, 9044 (2018).
- [52] L. Caprini and U. Marini Bettolo Marconi, Active chiral particles under confinement: Surface currents and bulk accumulation phenomena, *Soft Matter* **15**, 2627 (2019).
- [53] L. Caprini, F. Cecconi, and U. Marini Bettolo Marconi, Transport of active particles in an open-wedge channel, *J. Chem. Phys.* **150**, 144903 (2019).
- [54] N. Levernier, M. Dolgushev, O. Bénichou, R. Voituriez, and T. Guérin, Survival probability of stochastic processes beyond persistence exponents, *Nat. Commun.* **10**, 2990 (2019).
- [55] S. Redner, *A Guide to First-Passage Processes* (Cambridge University Press, Cambridge, UK; New York, 2001).
- [56] J. Klinger, R. Voituriez, and O. Bénichou, Splitting probabilities of symmetric jump processes, *Phys. Rev. Lett.* **129**, 140603 (2022).
- [57] S. N. Majumdar, A. Rosso, and A. Zoia, Hitting probability for anomalous diffusion processes, *Phys. Rev. Lett.* **104**, 020602 (2010).
- [58] S. A. Iyaniwura and Z. Peng, Splitting probabilities of confined chiral active Brownian particles, *New J. Phys.* **28**, 054401 (2026).
- [59] M. E. Cates and J. Tailleur, When are active Brownian particles and run-and-tumble particles equivalent? Consequences for motility-induced phase separation, *Europhys. Lett.* **101**, 20010 (2013).
- [60] C. Kurzthaler, Y. Zhao, N. Zhou, J. Schwarz-Linek, C. Devailly, J. Arlt, J.-D. Huang, W. C. K. Poon, T. Franosch, J. Tailleur, and V. A. Martinez, Characterization and control of the run-and-tumble dynamics of *Escherichia coli*, *Phys. Rev. Lett.* **132**, 038302 (2024).
- [61] M. Guéneau, S. N. Majumdar, and G. Schehr, Exact stationary state of a d -dimensional run-and-tumble particle in a harmonic potential, *Phys. Rev. E* **114**, 014144 (2026).
- [62] D. Frydel, Run-and-tumble particles in slit geometry as a splitting probability problem, *Phys. Fluids* **36**, 111901 (2024).
- [63] M. R. Evans and S. N. Majumdar, Diffusion with stochastic resetting, *Phys. Rev. Lett.* **106**, 160601 (2011).
- [64] M. R. Evans, S. N. Majumdar, and G. Schehr, Stochastic resetting and applications, *J. Phys. A Math. Theor.* **53**, 193001 (2020).
- [65] Y. Baouche and C. Kurzthaler, Optimal first-passage times of active Brownian particles under stochastic resetting, *Soft Matter* **21**, 5998 (2025).
- [66] V. Kumar, O. Sadekar, and U. Basu, Active Brownian motion in two dimensions under stochastic resetting, *Phys. Rev. E* **102**, 052129 (2020).
- [67] T. Arnoulx De Pirey, From an exact solution of dynamics in the vicinity of hard walls to extreme-value statistics of non-Markovian processes, *Phys. Rev. E* **110**, L062105 (2024).

- [68] A. V. Straube and F. Höfling, Depinning transition of self-propelled particles, *Phys. Rev. E* **110**, L062601 (2024).
- [69] P. Maggaretti and H. Stark, Model microswimmers in channels with varying cross section, *J. Chem. Phys.* **146**, 174901 (2017).
- [70] O. Granek, Y. Kafri, M. Kardar, S. Ro, J. Tailleur, and A. Solon, Colloquium: Inclusions, boundaries, and disorder in scalar active matter, *Rev. Mod. Phys.* **96**, 031003 (2024).
- [71] R. D. Schumm and P. C. Bressloff, Search processes with stochastic resetting and partially absorbing targets, *J. Phys. A Math. Theor.* **54**, 404004 (2021).
- [72] P. C. Bressloff, Trapping of an active Brownian particle at a partially absorbing wall, *Proc. R. Soc. A* **479**, 20230086 (2023).
- [73] P. C. Bressloff, Run-and-tumble particle with diffusion: Boundary local times and the zero-diffusion limit, *J. Stat. Mech. Theory Exp.* **2025**, 113201 (2025).
- [74] E. Woillez, Y. Zhao, Y. Kafri, V. Lecomte, and J. Tailleur, Activated escape of a self-propelled particle from a metastable state, *Phys. Rev. Lett.* **122**, 258001 (2019).
- [75] The file is available on Zenodo, [10.5281/zenodo.18982482](https://zenodo.org/record/18982482).

End Matter

Perturbation expansion—The probability density of an ABP between absorbing walls $P_A(z, \vartheta, t|z_0, \vartheta_0)$ satisfies the FPE: $\partial_t P_A = \mathcal{L}_{z, \vartheta} P_A = [\mathcal{H}_0 + \text{Pe}\mathcal{V}]P_A$, where $\mathcal{H}_0 = D\partial_z^2 + D_{\text{rot}}\partial_\vartheta^2$ and $\mathcal{V} = -(D/L)\cos\vartheta\partial_z$. Transforming to Laplace space and inserting $\hat{P}_A = \hat{P}_A^{(0)} + \text{Pe}\hat{P}_A^{(1)} + \text{Pe}^2\hat{P}_A^{(2)} + \dots$ yields the coupled equations:

$$(s - \mathcal{H}_0)\hat{P}_A^{(0)} = \delta(z - z_0)\delta(\vartheta - \vartheta_0), \quad (\text{A1a})$$

$$(s - \mathcal{H}_0)\hat{P}_A^{(i+1)} = \mathcal{V}\hat{P}_A^{(i)}, \quad i \geq 0, \quad (\text{A1b})$$

where the zeroth order $P_A^{(0)}$ corresponds to the propagator of a passive particle with rotational diffusion between two

absorbing walls. It is possible to perform an eigenfunction expansion of the unperturbed FPE and to obtain the Green's function of the unbounded problem:

$$G^u(s, z, \vartheta, z', \vartheta') = \frac{1}{4\pi} \sum_{\ell=-\infty}^{\infty} \frac{1}{Dp_\ell} e^{-p_\ell|z-z'|} e^{i\ell(\vartheta'-\vartheta)}, \quad (\text{A2})$$

where $p_\ell^2 = (s + D_{\text{rot}}\ell^2)/D$. The absorbing boundaries at $z = 0$ and $z = L$ are then enforced by the method of reflections, which gives

$$\hat{P}_A^{(0)}(z, \vartheta, s|z', \vartheta') = \frac{1}{2\pi} \sum_{\ell=-\infty}^{\infty} e^{i\ell(\vartheta'-\vartheta)} \frac{1}{Dp_\ell \sinh(p_\ell L)} \begin{cases} \sinh(p_\ell z) \sinh[p_\ell(L - z')], & z < z', \\ \sinh(p_\ell z') \sinh[p_\ell(L - z)], & z > z'. \end{cases} \quad (\text{A3})$$

Equations (A1) and (A2) lead to the iterative integral relation in Eq. (15), which we use to compute each order of the perturbative expansion. Note that this method is adapted from Ref. [16], where the case of a single absorbing wall was studied. The explicit corrections to the propagator are lengthy and not reported here (see the *Mathematica* Script [75]). Instead, we only quote the first corrections to the main observables.

For the mean first-passage time, we obtain

$$\frac{\langle T(z_0, \vartheta_0) \rangle}{\tau} = \frac{z_0(1 - z_0/L)}{2L} + \cos\vartheta_0 \frac{\sinh\left[\sqrt{\gamma}\left(z_0/L - \frac{1}{2}\right)\right] - 2\left[z_0/L - \frac{1}{2}\right] \sinh\left(\frac{1}{2}\sqrt{\gamma}\right)}{2\gamma \sinh\left(\frac{1}{2}\sqrt{\gamma}\right)} \text{Pe} + O(\text{Pe}^2). \quad (\text{A4})$$

The splitting probability with fixed initial orientation reads

$$\pi_L(z_0, \vartheta_0) = \frac{z_0}{L} + \cos\vartheta_0 \frac{\cosh\left(\frac{1}{2}\sqrt{\gamma}\right) - \cosh\left[\sqrt{\gamma}\left(z_0/L - \frac{1}{2}\right)\right]}{\gamma \cosh\left(\frac{1}{2}\sqrt{\gamma}\right)} \text{Pe} + O(\text{Pe}^2). \quad (\text{A5})$$

Finally, averaging over the initial orientation, we find (in agreement with Ref. [58])

$$\pi_L(z_0) = \frac{1}{2\pi} \int_0^{2\pi} d\vartheta_0 \pi_L(z_0, \vartheta_0) = \frac{z_0}{L} + \frac{\sinh\left[\sqrt{\gamma}\left(z_0/L - \frac{1}{2}\right)\right] - 2\left(z_0/L - \frac{1}{2}\right) \sinh\left(\frac{1}{2}\sqrt{\gamma}\right)}{2\gamma^{3/2} \cosh\left(\frac{1}{2}\sqrt{\gamma}\right)} \text{Pe}^2 + O(\text{Pe}^4). \quad (\text{A6})$$

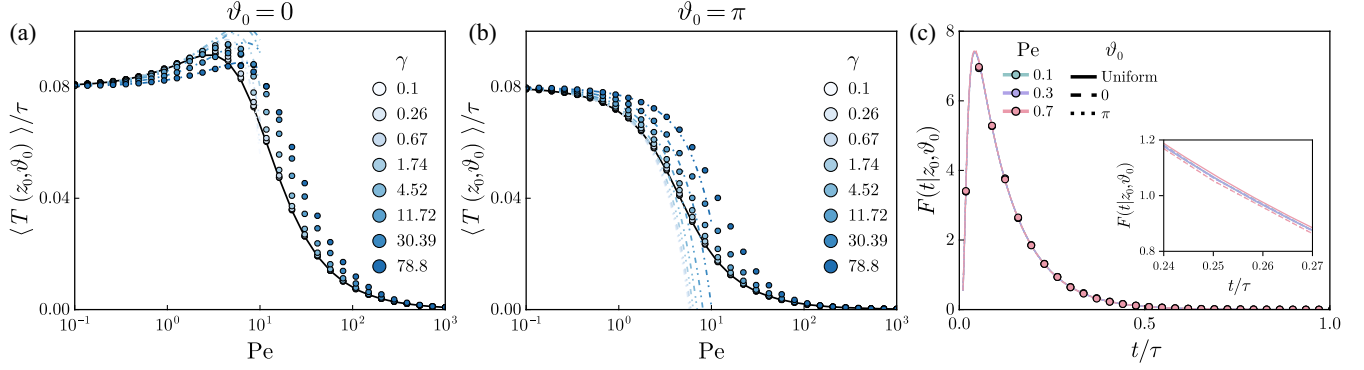


FIG. 4. (a),(b) Mean-first-passage time $\langle T \rangle$ as a function of the Péclet number for different γ and for $z_0/L = 0.2$. Dash-dotted and solid lines denote the low-Pe expansion up to $O(Pe^2)$ and the high-activity result, respectively. (c) FPT probability density $F(t|z_0, \vartheta_0)$ as a function of time for different Péclet numbers and initial orientations, with initial position $z_0/L = 0.5$ and $\gamma = 3$. Inset shows an enlargement of $F(t|z_0, \vartheta_0)$ at short times.

Consequences of Siegmund duality—From the Siegmund identity [Eq. (10)], one can relate first-passage observables of the dynamics with absorbing walls to spatial properties of the dual dynamics with hard walls. Although illustrated here for ABPs, these relations hold generally for any pair of Siegmund-dual processes.

First, the survival probability of the ABP between two absorbing walls at $z = 0$ and $z = L$ is defined by

$$S(z_0, t) = \mathbb{P}_A[0 < z(t) < L | z_0], \quad (B1a)$$

$$= 1 - \mathbb{P}_A[z(t) = 0 | z_0] - \mathbb{P}_A[z(t) = L | z_0], \quad (B1b)$$

$$= \mathbb{P}_A[z(t) \geq 0^+ | z_0] - \mathbb{P}_A[z(t) \geq L | z_0], \quad (B1c)$$

where, in the last equality, we used the presence of an absorbing wall at L and that $1 - \mathbb{P}_A[z(t) = 0 | z_0] = \mathbb{P}_A[z(t) \geq 0^+ | z_0]$. Using the Siegmund duality [Eq. (10)], it follows that

$$S(z_0, t) = \mathbb{P}_H[z(t) \leq z_0 | 0^+] - \mathbb{P}_H[z(t) \leq z_0 | L], \quad (B2)$$

yielding Eq. (11) of the main text. In particular, in the presence of one wall (i.e., when $L \rightarrow \infty$), we find

$$S(z_0, t) = \int_0^{z_0} dz p_H(z, t | 0). \quad (B3)$$

Hence, the survival probability of a particle initially at z_0 with an absorbing boundary at the origin can be interpreted as the probability that a dual particle, starting at the origin and evolving in the presence of a hard wall at the origin, is located in the interval $[0, z_0]$ at time t .

Second, the MFPT follows directly from the survival probability, $\langle T \rangle = \int_0^\infty dt S(z_0, t)$. Using Eq. (11) leads straightforwardly to Eq. (12). The MFPT to either absorbing wall for a particle starting at z_0 can thus be interpreted as the difference between the expected local times

accumulated in the interval $[0, z_0]$ by the dual hard-wall dynamics when initialized at 0 and L .

Finally, setting $\tilde{z}_0 = L$ in Eq. (10) relates the exit probability at wall L , defined in Eq. (14) for a particle starting at z_0 , to the probability that the dual hard-wall dynamics lies in $[0, z_0]$ [39],

$$E_L(z_0, t) = \int_0^{z_0} dz p_H(z, t | L). \quad (B4)$$

In the long-time limit $t \rightarrow \infty$, the initial condition is forgotten. Taking a derivative with respect to z_0 then yields a direct relation between the splitting probability and the stationary distribution of the hard-wall dynamics [39],

$$p_H(z) = \frac{\partial \pi_L(z)}{\partial z}, \quad (B5)$$

where $\pi_L(z) = \lim_{t \rightarrow \infty} E_L(z, t)$.

Boundary layer analysis for the MFPT—The MFPT obeys the backward FPE: $\mathcal{L}_{z_0, \vartheta_0}^\dagger \langle T \rangle = -1$. Away from the boundaries, the frozen-orientation approximation is justified when the rotational diffusion time is large compared to the persistent crossing time, $L/v \ll D_{\text{rot}}^{-1}$, or equivalently $\gamma \ll Pe$. The condition is more restrictive inside the boundary layer close to the walls. Near $z = L$, we introduce the stretched coordinate $y = (z_0 - L)/(L\epsilon)$ with $\epsilon = \gamma/Pe$, which resolves a layer of thickness $O(L\epsilon)$. The backward FPE then becomes

$$\left(\frac{D}{L^2 \epsilon^2} \partial_y^2 + \frac{v}{L \epsilon} \cos \vartheta_0 \partial_y + D_{\text{rot}} \partial_{\vartheta_0}^2 \right) \langle T \rangle = -1. \quad (C1)$$

Multiplying by $\epsilon^2 L^2 / D$, using the definitions of Pe and γ , and rewriting $Pe \epsilon = \gamma$, we arrive at

$$(\partial_y^2 + \gamma \cos \vartheta_0 \partial_y + \gamma \epsilon^2 \partial_{\vartheta_0}^2) \frac{\langle T \rangle}{\tau} = -\epsilon^2. \quad (\text{C2})$$

Thus rotational diffusion is negligible in the boundary-layer equation provided $\gamma \epsilon^2 \ll 1$, or $\gamma \ll \text{Pe}^{2/3}$.

Impact of γ on the MFPT—The MFPT depends on rotational diffusion via γ . For $\vartheta_0 = 0$, an increase in γ shifts the location of the peak towards larger Pe [Fig. 4(a)]. This can be understood by considering the particle's persistence length: $\ell_p = v/D_{\text{rot}} = L\text{Pe}/\gamma$. The largest MFPT is expected when the persistence length is comparable to (or shorter than) the distance to the wall at $z = L$: $\ell_p \sim L - z_0$, giving a critical Péclet number $\text{Pe}^* \sim \gamma(L - z_0)/L$ that increases with γ . We further find that at small Pe, the MFPT decreases for large γ , indicating that rotational diffusion reorients particles towards the closer wall and thus speeds up their arrival. For large Pe, the MFPT increases with γ , as rotational diffusion perturbs the persistent motion towards the other wall (at $z = L$). This is accompanied by nonmonotonic behavior of the maximal MFPT with respect to γ : Weak rotational diffusion (small γ) enhances the maximal MFPT compared to the

high-activity limit, while the MFPT decreases at large γ and the peak smears out.

This effect is only prominent for particles oriented towards the wall, which is further away ($\vartheta_0 = 0$), while for particles oriented towards the closer wall ($\vartheta_0 = \pi$) rotational noise always increases the MFPT [Fig. 4(b)]. For selected cases in the small-Pe regime, the corresponding FPT probability densities shown in Fig. 4(c) decay rapidly and show no distinct features for different ϑ_0 .

Computer simulations—To corroborate our analytical results, we simulate the equations of motion [Eq. (2)] employing the Euler-Maruyama scheme:

$$\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + v\mathbf{e}(t)\Delta t + \sqrt{2D\Delta t}\mathbf{N}_\eta, \quad (\text{E1a})$$

$$\vartheta(t + \Delta t) = \vartheta(t) + \sqrt{2D_{\text{rot}}\Delta t}N_\xi, \quad (\text{E1b})$$

where $\mathbf{N}_\eta(0, 1)$ and $N_\xi(0, 1)$ are independent standard normal random variables with zero mean and unit variance and the time step is set to $\Delta t = 10^{-8}\tau$ ($\Delta t = 10^{-4}\tau$) when studying absorbing (hard) walls. We simulate 2×10^5 trajectories to obtain reliable statistics.