

Black Hole Airy Tail

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In Jackiw-Teitelboim (JT) gravity, which is dual to a random matrix ensemble, the annealed entropy differs from the quenched entropy at low temperatures and goes negative. However, computing the quenched entropy in JT gravity requires a replica limit that is poorly understood. To circumvent this, we define an intermediate quantity called the semiquenched entropy, which has the positivity properties of the quenched entropy, while requiring a much simpler replica trick. We compute this in JT gravity in different regimes using (i) a bulk calculation involving wormholes corresponding to the Airy limit of the dual matrix integral and (ii) a boundary calculation involving one-eigenvalue instanton saddles proposed by Hernández-Cuenca, demonstrating consistency between these two calculations in their common regime of validity. We also clarify why similar one-eigenvalue instanton saddles cannot be used to compute the quenched entropy due to a breakdown of the saddle-point approximation for the one-eigenvalue instanton in the replica limit. Our results show how to use the gravitational path integral to prove that black holes in JT gravity have isolated ground states and to study their properties.

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Introduction—Do black holes have isolated ground states? In this Letter, we shall answer this question using a gravitational calculation in a two-dimensional toy model, JT gravity, which has served in recent years as a tool for understanding quantum gravity effects in realistic black holes. As a two-dimensional toy model of quantum gravity, it is computationally tractable while still exhibiting interesting nonperturbative effects. The action of this theory is given by [1]

$$S_{\text{JT}} = S_0 \chi_{g,n} - \frac{1}{2} \int_M \Phi(R+2) - \int_{\partial M} \Phi_b(K-1). \quad (1)$$

The first term is a topological term with $\chi_{g,n} = 2 - 2g - n$ the Euler characteristic of the spacetime M with genus g and n boundaries ∂M . The boundary term encodes the dynamics of the theory, whose coupling is controlled by the boundary value of the dilaton Φ_b .

In [2], the Euclidean gravitational path integral of JT gravity was computed exactly to all perturbative orders in e^{-S_0} , in the $S_0 \rightarrow \infty$ limit. The partition function at finite temperature $Z(\beta)$ is computed by fixing the length of the

boundary and summing over all hyperbolic geometries with different topologies satisfying the boundary conditions. The resulting asymptotic genus expansion agrees with the double-scaling limit of a random matrix theory (RMT):

$$Z = \int \mathcal{D}H e^{-N \text{Tr} V(H)}, \quad (2)$$

where H is a Hermitian matrix and the potential $V(H)$ is specified by the leading-order density of states:

$$\rho_0(E) = \frac{\gamma e^{S_0}}{2\pi^2} \sinh\left(2\pi\sqrt{2\gamma E}\right), \quad (3)$$

where e^{S_0} controls the local eigenvalue density of the spectrum in the double-scaling limit, in which the size of matrices in Eq. (2) is taken to infinity, while focusing on the region near the edge of the spectrum [3]. Thus, the ensemble of random Hamiltonians provides a nonperturbative completion of the gravitational theory at finite S_0 .

In the absence of matter fields, the only observable in pure JT gravity is the energy spectrum. The obvious quantity of interest that probes our initial question is the thermal entropy, obtained from the thermal partition function $Z(\beta)$. If the thermal entropy vanishes at small temperatures (for which we probe the ground state), then black holes would indeed have a single isolated ground state, separated from the rest of the spectrum by a gap. However, in an ensemble of theories where one only computes averaged quantities, one has to be careful about which entropy is being computed, since the average can be taken in different ways. Conventionally, the quantities discussed in the literature are the annealed entropy and the quenched

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TABLE I. A summary of the types of entropies and their properties. The difficulty of computation in gravity decreases as we go down the table.

Types	Rényi entropies	No. of replica limits for Rényi	Positivity requirement	Entropies ($n \rightarrow 1$)
Quenched	$S_Q^{(n)} \equiv (1/1-n) \langle \log(Z_n/Z_1^n) \rangle$	2	✓	$S_Q = S_{QQ} \equiv \langle \log \text{tr} \rho \rangle - \langle [\text{tr}(\rho \log \rho)] / \text{tr} \rho \rangle$
Quasiquenched	$S_{QQ}^{(n)} \equiv (1/1-n) \log \langle (Z_n/Z_1^n) \rangle$	1	✓	
Semiquenched	$S_{SQ}^{(n)} \equiv (1/1-n) \log(\langle Z_n \rangle / \langle Z_1^n \rangle)$	0	✓	$S_{SQ} \equiv [\langle \text{tr} \rho \log \text{tr} \rho \rangle - \langle \text{tr}(\rho \log \rho) \rangle] / \langle \text{tr} \rho \rangle$
Annealed	$S_A^{(n)} \equiv (1/1-n) \log(\langle Z_n \rangle / \langle Z_1^n \rangle)$	0	✗	$S_A \equiv \log \langle \text{tr} \rho \rangle - [\langle \text{tr}(\rho \log \rho) \rangle] / \langle \text{tr} \rho \rangle$

entropy. The annealed entropy $S_A(\beta) \equiv (1 - \beta \partial_\beta) \log \langle Z(\beta) \rangle$ differs from the quenched entropy, $S_Q(\beta) \equiv (1 - \beta \partial_\beta) \langle \log Z(\beta) \rangle$, which is truly the average entropy of the ensemble. An important distinction between these entropies is that the quenched entropy is positive, while the annealed entropy can be negative.

In gravitational theories, the annealed entropy is much easier to compute. When the entropy is self-averaging, $S_A = S_Q$. However, in JT gravity, S_A fails to be self-averaging at sufficiently low temperatures. The consequent mismatch between the two entropies was analyzed in [4], where they found $S_A < 0$ at $\beta = O(e^{2S_0/3})$ [6]. The negativity of the annealed entropy is not unique to the JT toy model; rather, it persists in higher-dimensional theories for all near-extremal black holes that are not supersymmetric at extremality [7–10]. This low temperature regime is thus among the most promising to probe the physics of only a small number of black hole microstates near the ground state. To address our initial question, we need to instead understand how the quenched entropy behaves in this regime.

However, calculating the quenched entropy at low temperatures is not easy. To compute the quenched entropy, one uses the so-called no-replica trick: $\langle \log Z(\beta) \rangle = \partial_m \langle Z(\beta)^m \rangle|_{m=0}$. At integer $m > 0$, this is an m -boundary calculation which can be done using the gravitational path integral and involves wormholes connecting different boundaries. This calculation was attempted [4,11], but the analytic continuation in m is complicated, and so far, there is no general method to compute S_Q in gravitational theories. Recently, [5], among other interesting results, proposed a calculation of S_Q using the RMT dual to JT gravity. The interesting proposal was that a new saddle, a one-eigenvalue instanton, dominates in the RMT due to the insertion of the operator $\langle Z(\beta)^m \rangle$ in Eq. (2) for large β [12]. The one-eigenvalue instanton indeed correctly computes $\langle Z(\beta)^m \rangle$ and, consequently, the annealed entropy. However, we will point out issues with the replica limit when calculating the quenched entropy using this technique.

To ameliorate the situation, we are motivated to study other entropic quantities in this regime. We consider various ensemble-averaged Rényi entropies for the thermal

density matrix $\rho = e^{-\beta H}$, summarized in Table I. As with conventional entropies, the quenched Rényi entropy is positive while the annealed is not sign definite; as before, evaluating the annealed entropy is simple, whereas the quenched entropy requires taking a delicate replica limit. Because of the subtleties in this replica limit, we propose a new intermediate quantity: the semiquenched Rényi entropy $S_{SQ}^{(n)}$, which is positive like the quenched Rényi entropy and, nevertheless, does not require a replica limit. While the semiquenched entropy might be easy to compute, it is as powerful as the quenched entropy in answering our starting question. In fact, we show that if the semiquenched entropy remains positive for all temperatures, then the ground state of the spectrum is indeed isolated and the quenched entropy, even if not computable analytically, is also provably positive.

Here, we compute $S_{SQ}^{(n)}$ in JT gravity in the interesting regimes where $S_A < 0$. This low-temperature regime is generally difficult to understand since the genus expansion in JT gravity breaks down. We sum over all wormhole contributions to demonstrate positivity of $S_{SQ}^{(n)}$ in the so-called Airy limit when $\beta = O(e^{2S_0/3})$. We show that, when $\beta \gg O(e^{2S_0/3})$, the matrix integral dual to JT gravity is dominated by a new saddle, the one-eigenvalue instanton proposed in [5], in which the lowest eigenvalue is well-separated from the rest of the eigenvalues. This saddle gives the correct positive result for $S_{SQ}^{(n)}$. Moreover, we show that these two calculations agree in their overlapping regime of validity. The positivity of the semiquenched entropy for all values of β leads us to the conclusion that we have an isolated ground state in each member of the ensemble. This is our first important result.

Our second goal is, for completeness, to clarify the behavior of the quenched entropy in the low-temperature regime. We show that while the one-eigenvalue instanton provides the correct answer for $\langle Z(\beta)^m \rangle$ for $m \neq 0$, the analytic continuation to $m = 0$ required for the quenched entropy is subtle: in the limit $m \rightarrow 0$, the one-eigenvalue instanton reunites with the rest of the spectrum and the saddle-point approximation in its derivation breaks down due to a noncommuting order of limits between $S_0 \rightarrow \infty$

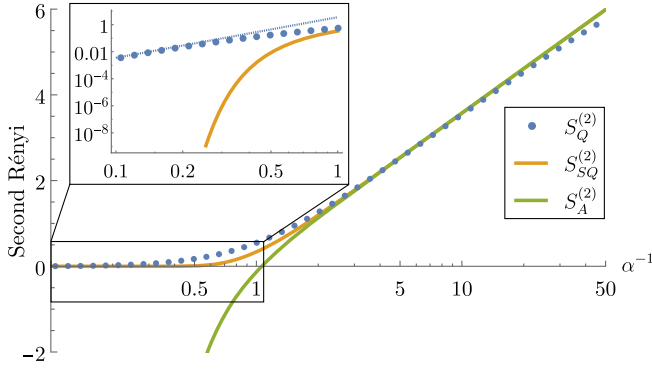


FIG. 1. Comparing semiquenched, annealed, and quenched second Rényi entropies, focusing on the region near the Airy edge $\beta = \alpha e^{2S_0/3}$ (with $\gamma = 1$). Semiquenched and annealed Rényis are obtained analytically, the quenched Rényi is obtained numerically. Notice the different low temperature behavior (inset): the quenched entropy has a α^{-3} power-law decay which can be obtained analytically (dotted blue line), whereas the semi-quenched entropy decays exponentially. Unlike the annealed entropy, both these entropies remain positive.

and $m \rightarrow 0$. We demonstrate that the one-eigenvalue instanton answer disagrees with previous matrix model calculations of the quenched entropy in the low temperature limit [13,14]. Thus, the one-eigenvalue instanton provides the correct result for computationally simpler quantities like the annealed and semiquenched entropy, but not for the quenched entropy.

Our results are summarized in Fig. 1 where we plot all the different Rényi entropies as a function of β , some obtained analytically and others numerically.

In Supplemental Material [15], we report technical details to support the results of this Letter.

A zoo of entropies—In studying entanglement and entropy in ensembles of quantum theories, such as disordered systems, gravitational theories, or random tensor networks, it is essential to distinguish among several natural generalizations of entropy to ensemble-averaged settings. The n th Rényi entropy for a given un-normalized density matrix ρ is

$$S^{(n)}(\rho) = \frac{1}{1-n} \log \left(\frac{\text{tr}(\rho^n)}{\text{tr}(\rho)} \right) \equiv \frac{1}{1-n} \log \left(\frac{Z_n}{Z_1} \right). \quad (4)$$

The different generalizations of the Rényi entropy differ in their ordering of operations: taking logarithms, powers, and ensemble averages. Different choices lead to different quantities, with distinct physical interpretations and computational challenges. Conventionally, the two quantities discussed in the literature are the quenched and annealed entropies. Here, we discuss a larger class of quantities, summarized in Table I.

Annealed entropy: We begin with the simplest quantity, the *annealed Rényi entropy*, defined by

$$S_A^{(n)} \equiv \frac{1}{1-n} \log \frac{\langle Z_n \rangle}{\langle Z_1 \rangle^n}, \quad (5)$$

which treats the partition functions in a fully averaged manner. This quantity is the simplest to compute using the gravitational path integral since, at integer n , it does not require any analytic continuation.

However, $S_A^{(n)}$ is not necessarily positive like an entropy. This is because it computes average moments of the matrix $\tilde{\rho} = (\rho / \langle \text{tr} \rho \rangle)$, as seen by rewriting Eq. (5) as

$$S_A^{(n)} = \frac{1}{1-n} \log \langle \text{tr}(\tilde{\rho}^n) \rangle. \quad (6)$$

Since $\tilde{\rho}$ is only normalized on average, the Rényi entropy can be negative (e.g., $\tilde{\rho}$ can have support on eigenvalues larger than 1).

Taking the $n \rightarrow 1$ limit, one obtains the annealed entropy

$$S_A = \log \langle \text{tr} \rho \rangle - \frac{\langle \text{tr}(\rho \log \rho) \rangle}{\langle \text{tr} \rho \rangle} \quad (7a)$$

$$= (1 - n \partial_n) \log \langle Z_n \rangle \Big|_{n=1}, \quad (7b)$$

where the last line explains the connection to the usual thermal annealed entropy. In the thermal case, n and β appear together in $Z_n = \text{tr} e^{-n\beta H}$, and we have $n \partial_n f(n\beta) = \beta \partial_\beta f(n\beta)$.

Quenched entropy: The most physically well-motivated quantity is the *quenched Rényi entropy* defined as

$$S_Q^{(n)} \equiv \frac{1}{1-n} \left\langle \log \frac{Z_n}{Z_1} \right\rangle = \frac{\langle \log Z_n \rangle - \langle \log Z_1 \rangle}{1-n}, \quad (8)$$

This definition corresponds to computing the Rényi entropy in each member of the ensemble and then averaging. Thus, it retains the positivity of the Rényi entropy.

However, it is much harder to compute since it requires the following replica trick:

$$S_Q^{(n)} = \frac{1}{1-n} \lim_{m \rightarrow 0} \left[\frac{\langle (Z_n)^m \rangle - \langle (Z_1)^{nm} \rangle}{m} \right], \quad (9)$$

which can be computed at positive integer m, n, k using the gravitational path integral and requires analytic continuation for the two replica limits shown above. In the von Neumann limit ($n \rightarrow 1$), this gives

$$S_Q = \langle \log \text{tr} \rho \rangle - \left\langle \frac{\text{tr}(\rho \log \rho)}{\text{tr} \rho} \right\rangle \quad (10a)$$

$$= (1 - n \partial_n) \langle \log Z_n \rangle \Big|_{n=1}, \quad (10b)$$

which again agrees with the standard definition of the thermal quenched entropy.

Semiquenched entropy: Since the annealed entropy goes negative and the quenched entropy is hard to compute,

we now construct an intermediate quantity that is positive and, nevertheless, easier to compute since it does not require a replica limit. We define the *semiquenched Rényi entropy* as

$$S_{\text{SQ}}^{(n)} \equiv \frac{1}{1-n} \log \frac{\langle Z_n \rangle}{\langle Z_1^n \rangle}. \quad (11)$$

This quantity does not require any analytic continuation to be computed at integer $n > 1$. We will exploit this fact in our calculation using the gravitational path integral. The level of difficulty in analytically continuing this quantity in n is precisely the same as that of computing entanglement entropy in a quantum field theory given the integer Rényi entropies.

In the $n \rightarrow 1$ limit, we find a new quantity:

$$S_{\text{SQ}} = \frac{\langle \text{tr} \rho \log \text{tr} \rho \rangle - \langle \text{tr}(\rho \log \rho) \rangle}{\langle \text{tr} \rho \rangle}, \quad (12)$$

which we call the semiquenched entropy [17].

These entropies can be easily seen to be positive since $Z_n \leq Z_1^n$ for $n \geq 1$ for any member of an ensemble of theories with discrete eigenenergies. Taking an ensemble average of this inequality results in the positivity of $S_{\text{SQ}}^{(n)}$. For $n < 1$, the argument is similar except that the inequalities are reversed. An important fact is that $Z_n > Z_1^n$ for $n \geq 1$ as $T \rightarrow 0$ for any member of the ensemble for which the ground state is not isolated from the rest of the spectrum, proven in Supplemental Material, Sec. I [15]; for all other members $Z_n - Z_1^n$ decreases with T and vanishes as $T \rightarrow 0$. Therefore, if there is even one member with a nonisolated ground state, this would be reflected in the semiquenched entropy eventually becoming negative. Consequently, positivity of the semiquenched entropy implies an isolated ground state for all members of the ensemble. Positivity of the quenched entropy also follows, due to its positivity in each ensemble member.

Semiquenched entropy—

Airy wormholes at $\beta = O(e^{2S_0/3})$: Consider JT gravity in the limit $\beta \rightarrow \infty$, $S_0 \rightarrow \infty$ with

$$\alpha \equiv \beta e^{-2S_0/3} \quad (13)$$

fixed. In this limit, $Z(\beta)$ is dominated by eigenvalues near the edge of the spectrum. From the perspective of RMT, it is well known that the square-root edge of the spectrum, Eq. (3), is controlled by the Airy model [18], see Ref. [5] for a nice review.

From the bulk perspective, the standard genus expansion of JT gravity is

$$\langle Z(\beta)^m \rangle = \sum_{g=0}^{\infty} e^{(2-2g-m)S_0} Z_{g,m}(\beta), \quad (14)$$

with $Z_{g,m}(\beta) = O((\beta^{3/2})^{2g+m-2})$, implying that the above expansion breaks down in our regime of interest. To rectify

this, we can consider a different low-temperature expansion, where we expand each $Z_{g,m}(\beta)$ in powers of β . Swapping the order of summation, we then have

$$\langle Z(\beta)^m \rangle = \sum_{l=0}^{\infty} \beta^{-l} \sum_{g=0}^{\infty} \alpha^{3g+3m/2-3} Z_{g,m,l}. \quad (15)$$

The $l=0$ term dominates in our limit of interest. Remarkably, for $l=0$, the sum over genus is convergent and can be explicitly performed; see Refs. [4,19] for a review. The geometries dominating this sum are well understood [20], and the resulting partition function matches the Airy matrix model [21,22].

We will now use the above idea to compute the semiquenched Rényi entropy, focusing on the $n=2$ case where we can give explicit formulas. In Sec. II of [15], we show that for all integer n , $S_{\text{SQ}}^{(n)} \rightarrow 0$ as $\alpha \rightarrow \infty$. We have

$$S_{\text{SQ}}^{(2)} = -\log \frac{\langle Z(2\beta) \rangle}{\langle Z(\beta)^2 \rangle} = \log \frac{\text{disk}_{\text{res}}(\beta)^2 + \text{cyl}_{\text{res}}(\beta, \beta)}{\text{disk}_{\text{res}}(2\beta)}, \quad (16)$$

where $\text{disk}_{\text{res}}(x)$ is the one-boundary partition function with boundary length x and $\text{cyl}_{\text{res}}(x, y)$ is the connected two-boundary partition function with boundary lengths x, y . In this limit, both $\text{disk}_{\text{res}}(x)$ and $\text{cyl}_{\text{res}}(x, y)$ receive important contributions from higher-genus geometries. These quantities can be computed exactly by resumming the genus expansion and the relevant formulas are [4,21,25,26]

$$\text{disk}_{\text{res}}(\alpha e^{2S_0/3}) = \frac{\gamma^{3/2} e^{\frac{\alpha^3}{24\gamma^3}}}{\sqrt{2\pi\alpha^{3/2}}}, \quad (17a)$$

$$\text{cyl}_{\text{res}}(\alpha e^{2S_0/3}, \alpha e^{2S_0/3}) = \frac{\gamma^{3/2} e^{\frac{\alpha^3}{3\gamma^3}}}{4\sqrt{\pi\alpha^{3/2}}} \text{erf}\left(\frac{1}{2} \sqrt{\frac{\alpha^3}{\gamma^3}}\right), \quad (17b)$$

where $\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x dt e^{-t^2}$. Using these formulas, we obtain

$$S_{\text{SQ}}^{(2)} = \log \left(2\sqrt{\frac{\gamma^3}{\pi\alpha^3}} e^{-\frac{\alpha^3}{4\gamma^3}} + \text{erf}\left(\frac{1}{2} \sqrt{\frac{\alpha^3}{\gamma^3}}\right) \right), \quad (18)$$

which we plot in Fig. 1 to emphasize positivity. At large α , Eq. (18) becomes exponentially small:

$$S_{\text{SQ}}^{(2)} \approx \frac{4\gamma^{9/2}}{\sqrt{\pi\alpha^{9/2}}} e^{-\frac{\alpha^3}{4\gamma^3}}. \quad (19)$$

From Eq. (17), we see that at large α , the cylinder contribution dominates over the disk in $\langle Z(\beta)^2 \rangle$ [27]. It is these wormholes that differentiate $S_{\text{SQ}}^{(2)}$ from $S_A^{(2)}$ —which is simply given by the first term in Eq. (18)—rescuing positivity. The fact that $\text{cyl}_{\text{res}}(\beta, \beta)$ approaches $\text{disk}_{\text{res}}(2\beta)$ at large β is a highly nontrivial consistency condition on

wormhole contributions to the gravitational path integral, satisfied by JT gravity.

Eigenvalue instantons at $\beta \gg O(e^{2S_0/3})$: At even lower temperatures, the above calculation breaks down [28]. In this regime, we appeal to RMT to understand what happens. In the RMT, we are computing

$$\langle Z(\beta)^m \rangle = \int \mathcal{D}H e^{-N \text{Tr} V(H)} (\text{tr}(e^{-\beta H})^m). \quad (20)$$

Following [5], we can exponentiate the insertion to obtain an effective potential $\text{tr} V(H) - (m/N) \log \text{tr}(e^{-\beta H})$ and we expect a new saddle, since the insertion enhances the contribution from unlikely instances of H where the minimum eigenvalue λ_0 is very negative. This new saddle, called a one-eigenvalue instanton, was obtained in [5].

We discuss aspects of the general solution in Sec. III of Supplemental Material [15]. To compare the instanton answer with the Airy answer, we focus on the regime $\beta = \alpha e^{2S_0/3}$ with $\alpha = O(1)$ but large. Since the remaining eigenvalues remain in their original saddle, we will focus on the effective action for the lowest eigenvalue given by

$$I[\lambda_0] = e^{S_0} V_{\text{eff}}(\lambda_0) + m\beta\lambda_0, \quad (21)$$

where $V_{\text{eff}}(\lambda_0) = (4\sqrt{2}/3)|\gamma\lambda_0|^{3/2}$ is the effective potential near the edge. Solving for the saddle point, we obtain $\lambda_0^* = -(m^2\beta^2/8\gamma^3 e^{2S_0})$. Expanding the action around the saddle point, we find

$$I[\lambda_0^* + \delta\lambda_0] = -\frac{m^3\beta^3}{24\gamma^3 e^{2S_0}} \left(1 + \sum_{k \geq 2} c_k \left(\frac{\gamma e^{2S_0/3}}{m\beta} \right)^{3k/2} \frac{\delta\lambda_0^k}{\delta\lambda_0^k} \right), \quad (22)$$

where $\overline{\delta\lambda_0} = \sqrt{(\gamma^3 e^{2S_0}/m\beta)} \delta\lambda_0$ has $O(1)$ quadratic fluctuations. Thus, the expansion is controlled if $(\gamma e^{2S_0/3}/m\beta)$ is small, in which case higher loop terms are suppressed. Thus, the saddle-point analysis of the one-eigenvalue instanton is trustworthy for our cases of interest, $m = 1, 2$, as long as $\alpha \gg 1$. The analysis also remains valid for $\beta = \omega(e^{2S_0/3})$.

Equation (22) implies that $\exp(-I[\lambda_0^*])$ agrees with the leading behaviour of Eq. (17). To compute $S_{\text{SQ}}^{(2)}$, we need to find the first subleading correction to the instanton action. In Sec. IV of Supplemental Material [15], we argue that this comes from a two-eigenvalue instanton that gives us

$$S_{\text{SQ}}^{(2)} \approx e^{-\frac{\alpha^3}{4\gamma^3}}, \quad (23)$$

where we only estimate the right exponent (without the subleading multiplicative power-law factors), which smoothly interpolates with Eq. (19) [29].

We have thus proven positivity of the semiquenched entropy for any value of β , and therefore that all members of the ensemble have an isolated ground state. In turn, this implies that the gravitational quenched entropy also stays positive for any value of β .

Comments on quenched entropy—At $\beta \gtrsim e^{2S_0/3}$, the answer for the quenched entropy is also controlled by the Airy limit [13,14]. Because of the replica limits involved in this calculation, it is much harder to obtain directly but a matrix model analysis was done in [13] where they found that the low temperature entropy at leading order term at large α is $S_Q^{(n)} \approx (7\pi^4\gamma^3/360\alpha^3)[(1+n)(1+n^2)/n^3]$ [30]. The quenched entropy, obtained numerically by sampling 1000 GUE 1000×1000 matrices, is plotted in Fig. 1, matching the aforementioned analytic result at large α .

In contrast, the proposal of using the one-eigenvalue instanton for the quenched entropy [5] does not reproduce this, leading to $S_Q \approx T^{3/2}$ at low temperatures, independent of the symmetry class. The location of the instanton is a function of m and approaches the continuum edge as we take $m \rightarrow 0$. In this limit, the saddle point approximation breaks down since higher-loop contributions in Eq. (22) become important. This is also problematic because the leading answer for quenched in [5] comes from the continuum spectrum, whereas the assumption used in obtaining the instanton was that the leading eigenvalue dominates $Z(\beta)$. This is a consequence of a wrong order of limits between $\beta \rightarrow \infty$ and $m \rightarrow 0$ [31].

The key difference for the semiquenched and annealed entropies is that they are dominated at large β by configurations with large gaps between the two leading eigenvalues, whereas the quenched entropy gets its leading (power-law) contribution from configurations with a small gap.

Discussion—We showed how wormholes in JT gravity restore positivity of $S_{\text{SQ}}^{(n)}$, which is sufficient to prove that the ground state is isolated and the quenched entropy also remains positive. This required a resummation of the genus expansion, and no single geometry was sufficient. Computing other observables sensitive to the isolated ground state is now also possible: they will also include the same genus resummation. Our results suggest that requiring positivity of entropies can put constraints on wormhole contributions in a putative gravitational theory: a wormhole bootstrap.

We probed a regime where all the entropies are non-self-averaging and thus are not necessarily representative of a typical member of the ensemble. Both $S_{\text{SQ}}^{(n)}$ and $S_Q^{(n)}$ in this regime are dominated by atypical members of the ensemble. This is most obvious for $S_{\text{SQ}}^{(n)}$ from the one-eigenvalue instanton analysis, which is a highly atypical member of the ensemble with a large gap between the two smallest eigenvalues. Thus, our results for the entropies are not as valuable in higher dimensions if gravity is not dual to an ensemble of theories. Nonetheless, the positivity of the semiquenched entropy is a consequence of the presence of an isolated ground state in every member of the ensemble. This is a self-averaging property that is meaningful in higher dimensions.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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$$\begin{aligned} S_{QQ}^{(n)} &= \frac{1}{1-n} \log \left\langle \frac{Z_n}{Z_1^n} \right\rangle \\ &= \frac{1}{1-n} \log \lim_{k \rightarrow -n} \left\langle Z_n Z_1^k \right\rangle. \end{aligned}$$

This still requires a replica limit to compute, and is positive by a similar argument used above. In the $n \rightarrow 1$ limit, it yields the quenched entropy. Since it is as difficult to compute as the quenched entropy (which also requires one replica limit), we will not focus on it in this Letter.

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- [29] Notice that both the Airy analysis for $\alpha \sim O(1)$ (excluding the cyl_{res} contribution) and the one-eigenvalue instanton analysis for $\alpha \gg 1$ also apply to the annealed entropy.

The corrections, however, do not rescue positivity for the annealed entropy.

- [30] This behavior holds at arbitrarily low temperatures, up to doubly nonperturbative corrections [\[13\]](#).
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