

Universal Time Evolution of Holographic and Quantum Complexity

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Holographic complexity, as the bulk dual of quantum complexity, encodes the geometric structure of black hole interiors. Motivated by the complexity = anything proposal, we introduce the spectral representation for generating functions associated with codimension-one and codimension-zero holographic complexity measures. These generating functions exhibit a universal slope-ramp-plateau structure analogous to the spectral form factor in chaotic quantum systems. In such systems, quantum complexity evolves universally, displaying long-time linear growth followed by saturation at late times. By employing the generating function formalism, we show that this universal behavior has two origins: a particular pole structure of the matrix elements of the generating functions in the energy eigenbasis and random matrix universality in spectral statistics. Using the residue theorem, we prove that the existence of this pole structure is a necessary and sufficient condition for the linear growth of holographic complexity measures. Furthermore, we show that the late-time saturation plateau arises directly from the spectral level repulsion, a hallmark of quantum chaos.

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Introduction—Quantum gravity is confronted by profound tensions between the smooth geometry of classical spacetime and the discrete and finite nature of quantum Hilbert space. A prominent illustration of this tension appears in the spectral statistics of black holes. One notable example is Maldacena’s black hole information paradox [1], which can be characterized by the time evolution of holographic correlators, such as two-sided correlation functions $\langle \text{TFD}_\beta | \mathcal{O}(t_L) \mathcal{O}(t_R) | \text{TFD}_\beta \rangle$ (TFD, thermofield double). Its spectral representation is given by

$$\frac{1}{Z} \sum_{i,j} e^{-\frac{\beta}{2}(E_i + E_j)} e^{-iT(E_i - E_j)} |\langle \mathbf{E}_j | \mathcal{O} | \mathbf{E}_i \rangle|^2, \quad (1)$$

with the time shift defined as $T = t_L + t_R$ [2] and $Z = \text{Tr} e^{-\beta H}$ representing the partition function. While semiclassical gravity predicts an indefinite decay of these correlation functions, a unitary and finite quantum theory requires erratic fluctuations and a late-time plateau. This apparent contradiction is succinctly captured by the spectral

form factor, which exhibits a characteristic slope-ramp-plateau structure [3] in chaotic quantum systems, e.g., the Sachdev-Ye-Kitaev (SYK) model [9–11] and Jackiw-Teitelboim (JT) gravity [12–19].

Another striking manifestation of this tension is Susskind’s wormhole size paradox. Specifically, the wormhole’s size in classical spacetime, e.g., the volume of the black hole interior or Einstein-Rosen bridge (see Fig. 2), grows linearly without bound at late times. According to the complexity = volume conjecture [20–28] or the related proposal identifying volume with the information metric [29–31], this infinite growth appears to contradict the finiteness of the Hilbert space dimension in quantum gravity. Brown and Susskind conjectured [32] that quantum complexity [33] in chaotic quantum systems exhibits a long-time linear growth [34] and eventually transitions to a plateau after a timescale e^{S_0} , as depicted in Fig. 1. This transition was first holographically realized in JT gravity by incorporating nonperturbative quantum corrections to geodesic lengths [35]; see also Refs. [36–44]. For more recent progress on complexity measures, see Ref. [45] for a review.

Remarkably, these two paradoxes are fundamentally connected via the holographic dictionary. In particular, the geodesic approximation of holographic correlation functions [46,47] implies that late-time saturation in both wormhole size and quantum correlators arises from the same underlying mechanism, governed by quantum chaos and random matrix universality. This connection was explicitly established in [41] via generating functions for geodesics in JT gravity, as summarized in Fig. 1.

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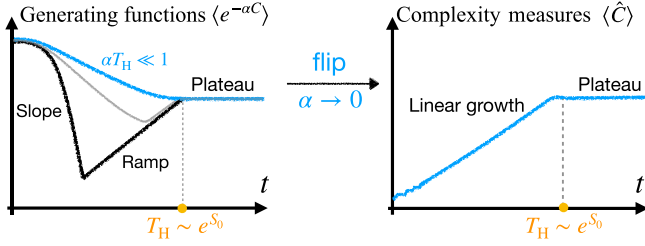


FIG. 1. Left: typical time evolution of generating functions of complexity $\langle e^{-\alpha \mathcal{C}} \rangle$ presents the well-known slope-ramp-plateau structure. Right: the time evolution of complexity measures could be derived from its generating function in the limit $\alpha \rightarrow 0$.

In this Letter, we provide a universal resolution to the wormhole size paradox by introducing generating functions $\langle e^{-\alpha \mathcal{C}} \rangle$ for holographic complexity measures $\mathcal{C}$. In light of the complexity = anything proposal [27,28,48], which suggests that there exist infinitely many analogous holographic complexity measures (see Fig. 2), our framework applies universally to both codimension-zero and codimension-one gravitational observables within a generic holographic gravity setting. By employing spectral representations of generating functions, we explicitly identify their universal slope-ramp-plateau structure [49]. Moreover, we prove that the universal time evolution of quantum or holographic complexity is governed by spectral correlations dictated by random matrix universality, as well as a specific pole structure in the matrix elements of these generating functions expressed in the energy eigenbasis.

In holographic quantum gravity, we generally assume that the Hilbert space of black hole microstates $\mathcal{H}_{\text{BH}}$ is finite and discrete with a small but finite G_N . However, as a powerful approximation, we expand the Hamiltonian eigenbasis using a continuous spectrum E_i and generalized eigenstates $|E_i\rangle$ [51], which are rigorously defined in the associated *rigged Hilbert space*. In this distributional sense, they allow for a resolution of the identity operator,

$$\hat{1} = e^{S_0} \int_{E_0 - \Delta E/2}^{E_0 + \Delta E/2} dE D(E) |E\rangle \langle E|, \quad (2)$$

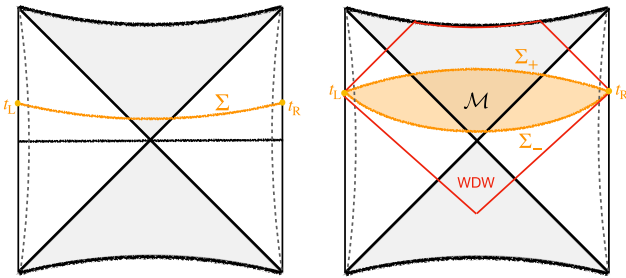


FIG. 2. Complexity = anything proposal. Left: codimension-one observables are supported on extremal hypersurface Σ . Right: codimension-zero observables are defined by extremal subregion $\mathcal{M}$ with boundaries $\Sigma_{\pm}$.

where we focus on a microcanonical ensemble characterized by a finite energy window centered around E_0 , i.e., $E_i \in [E_0 - (\Delta E/2), E_0 + (\Delta E/2)]$. Here, $D(E)$ denotes the rescaled density of states, and S_0 is the coarse-grained entropy characterizing the total dimension of the Hilbert space. Of particular interest to holography is the microcanonical thermofield double (TFD) state,

$$|\text{TFD}(t)\rangle = \frac{e^{S_0}}{\sqrt{Z}} \int_{E_0 - \Delta E/2}^{E_0 + \Delta E/2} dE D(E) e^{-iEt} |E\rangle. \quad (3)$$

which is dual to a two-sided anti-de Sitter (AdS) black hole [52]. The normalization factor $Z \approx e^{S_0} \bar{D}(E_0) \Delta E$ represents the total number of microcanonical states, with $\bar{D}(E_0)$ being the *ensemble-averaged* density of states. A fundamental timescale for finite quantum systems is the Heisenberg time T_H , defined as the inverse of the mean energy level spacing, namely

$$T_H := \frac{2\pi\hbar}{\text{average energy spacing}} = 2\pi e^{S_0} \bar{D}(E_0). \quad (4)$$

We will find that this timescale marks the transition in the time evolution of complexity from a linear growth regime to a late-time plateau, as illustrated in Fig. 1.

Codimension-one observables and spectral decomposition—To explicitly illustrate the time evolution of holographic complexity, we first consider codimension-one gravitational observables, denoted by $\mathcal{C}$ [27], can be interpreted as generalized volume functionals [53] defined on the extremal hypersurface $\Sigma(t_L, t_R)$ (see Fig. 2). Analogous to the geodesic-length operator studied in [41], we formally define the *operator of gravitational observables* corresponding to holographic complexity measures by

$$\hat{\mathcal{C}} := \sum_{\mathcal{C}\text{-basis}} \mathcal{C} |\mathcal{C}\rangle \langle \mathcal{C}| = \int \mathcal{C} |\mathcal{C}\rangle \langle \mathcal{C}| d\mathcal{C}, \quad (5)$$

where $|\mathcal{C}\rangle$ represents the quantum state with a fixed generalized volume $\mathcal{C}$. However, the holographic complexity operator $\hat{\mathcal{C}}$ is not always well-defined, because the set of states $|\mathcal{C}\rangle$ can become overcomplete due to quantum corrections from Euclidean wormholes in the gravitational path integral. In such cases, the complexity operator develops divergences and can deviate drastically from classical expectations [41]. A natural resolution is to select a complete orthonormal basis (denoted by $|\mathcal{C}_O\rangle$), which can be obtained systematically via the Gram-Schmidt orthogonalization procedure [39,43]. Correspondingly, one can define a complexity operator in terms of the resulting discrete basis,

$$\hat{\mathcal{C}}_O := \sum_{\text{orthonormal basis}} \mathcal{C} |\mathcal{C}_O\rangle \langle \mathcal{C}_O|. \quad (6)$$

This construction generalizes the notion of Krylov complexity, which has been extensively studied in recent literature

[40,45,56–71]. Details of this approach are presented in the End Matter.

To obtain physically meaningful expectation values of holographic complexity measures $\mathcal{C}$, a complementary approach was introduced in [35,41]. Instead of defining the complexity operator directly, one may focus on the regulated operator,

$$\widehat{e^{-\alpha\mathcal{C}}} := \int e^{-\alpha\mathcal{C}} |\mathcal{C}\rangle \langle \mathcal{C}| d\mathcal{C}, \quad (7)$$

which is well-defined even when $|\mathcal{C}\rangle$ is overcomplete, since the parameter α effectively acts as a regulator. The corresponding expectation values, namely,

$$\langle e^{-\alpha\mathcal{C}} \rangle := \langle \text{TFD}(t) | \widehat{e^{-\alpha\mathcal{C}}} | \text{TFD}(t) \rangle, \quad (8)$$

serve as generating functions for holographic complexity measures $\mathcal{C}$. We then subtract the unphysical divergence by imposing that the generating operator reduces to the identity in the limit $\alpha \rightarrow 0$, i.e.,

$$\lim_{\alpha \rightarrow 0} \widehat{e^{-\alpha\mathcal{C}}} = \hat{1} = \sum_{\text{complete basis}} |\mathcal{C}\rangle \langle \mathcal{C}|, \quad (9)$$

where the sum is taken over a complete $\mathcal{C}$ basis.

The main result of this Letter is that holographic complexity measures $\mathcal{C}$ and their generating functions exhibit a universal time evolution that is fixed by a small set of holographic inputs, without requiring detailed knowledge of the microscopic structure of the holographic system or of the precise definition of $\mathcal{C}$. The central observation is that the generating function admits a spectral decomposition,

$$\frac{e^{2S_0}}{\mathcal{Z}} \int d\bar{E} dE_{ij} e^{-iE_{ij}t} \overline{\langle D(E_i) D(E_j) \rangle} \langle E_i | \widehat{e^{-\alpha\mathcal{C}}} | E_j \rangle, \quad (10)$$

with $\bar{E} = (E_i + E_j)/2$, $E_{ij} = E_i - E_j$. This decomposition makes manifest that the time evolution of the generating function is governed by two independent ingredients: the matrix elements $\langle E_i | \widehat{e^{-\alpha\mathcal{C}}} | E_j \rangle$ and the spectral correlator $\overline{\langle D(E_i) D(E_j) \rangle}$. The universal time evolution follows from the fact that, in holographic systems, these two ingredients are constrained by the following two holographic inputs: (i) holographic complexity grows linearly in time in the semiclassical regime; (ii) black holes are dual to quantum chaotic systems. The first requirement imposes a strong constraint on the analytic structure of the matrix elements $\langle E_i | \widehat{e^{-\alpha\mathcal{C}}} | E_j \rangle$ of the generating function. As discussed in later sections, we find that linear growth occurs if and only if the matrix elements after analytic continuation develop a particular pole structure, i.e.,

$$\langle E_i | \widehat{e^{-\alpha\mathcal{C}}} | E_j \rangle \sim \left(\frac{\tilde{\alpha}}{e^{S_0} \pi \bar{D}(\bar{E})} \right) \frac{1}{(\tilde{\alpha} + iE_{ij})(\tilde{\alpha} - iE_{ij})} + \dots \quad (11)$$

Here, we focus on the regime with $\alpha \sim E_{ij} \ll 1$ and define $\tilde{\alpha} = M\alpha + \mathcal{O}(\alpha^2)$. The essential feature is the presence of a simple pole located precisely on the imaginary axis at $E_{ij} = \pm i\tilde{\alpha}$. The overall coefficient is fixed by the completeness of the $\mathcal{C}$ basis [cf. Eq. (9)] together with the normalization of the energy eigenstates.

The second input instead constrains the ensemble-averaged spectral correlator. One key universal feature of quantum chaotic systems is *level repulsion*, which expresses the fact that nearby energy levels avoid one another. Mathematically it implies that the off-diagonal part of the n -point spectral correlator vanishes as two levels coincide, e.g.,

$$\lim_{E_{ij} \rightarrow 0} \overline{\langle D(E_i) D(E_j) \rangle}_{\text{off-diag}} = 0. \quad (12)$$

As we will show, this property ensures that the linear growth dictated by the pole structure does not persist indefinitely, but instead crosses over to a late-time plateau.

Explicit example from Gaussian unitary ensemble—To make our results more concrete, we now illustrate the universal time evolution by using an explicit example. It is known that spectral correlations exhibit random-matrix universality when the energy difference is less than a characteristic value called the “Thouless energy” [72]. More precisely, for $|E_i - E_j| \ll E_{\text{Th}}$, the ensemble-averaged spectral correlator coincides with that of the appropriate Wigner-Dyson ensemble. As a representative example, we consider the Gaussian unitary ensemble, whose two-point spectral correlator is governed by the following well-known sine kernel [73]:

$$\overline{\langle D(E_i) D(E_j) \rangle}_{E_i \neq E_j} \approx \bar{D}(E_i) \bar{D}(E_j) - \frac{\sin^2[e^{S_0} \pi E_{ij} \bar{D}(\bar{E})]}{(e^{S_0} \pi E_{ij})^2}. \quad (13)$$

As $E_{ij} \rightarrow 0$, the off-diagonal part of this correlator vanishes, explicitly realizing the universal phenomenon of level repulsion.

Combining this universal spectral correlation (13) with the particular pole structure (11) of the matrix elements, we can explicitly derive the time evolution of the generating function by substituting these two ingredients into the spectral representation (10). The resulting integral can be evaluated over the complex E_{ij} plane. For example, the integral contour corresponding to the classical contributions is illustrated in Fig. 3. For a generic value of the regulator α satisfying $\alpha T_{\text{H}} \gg 1$, we find that the time evolution of the generating function is dominated by the following contribution (see Supplemental Material [74] for a detailed derivation),

$$\langle \widehat{e^{-\alpha\mathcal{C}}} \rangle \approx \begin{cases} e^{-\tilde{\alpha}t} + \mathcal{O}\left(\frac{\tilde{\alpha}}{t\Delta E}\right), & \text{slope } \tilde{\alpha}t \sim 1, \\ \frac{2t}{\tilde{\alpha}T_{\text{H}}^2} + \mathcal{O}(e^{-\tilde{\alpha}t}), & \text{ramp } t < T_{\text{H}}, \\ \frac{2}{\tilde{\alpha}T_{\text{H}}} + \mathcal{O}(e^{-\tilde{\alpha}(t-T_{\text{H}})}), & \text{plateau } t > T_{\text{H}}. \end{cases} \quad (14)$$

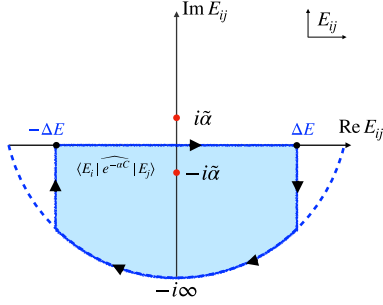


FIG. 3. The integration contour corresponding to both the classical contributions and quantum corrections at late times.

A similar behavior has been explicitly obtained for geodesic lengths and time shifts in JT gravity [41]. Interestingly, as illustrated in [41], the linear ramp region would gradually disappear as the control parameter α decreases. The nonperturbative expectation value of quantum (holographic) complexity $\mathcal{C}$ is derived from its regularized generating function by

$$\mathcal{C} := -\lim_{\alpha \rightarrow 0} \partial_{\alpha} \left(\langle e^{-\alpha \mathcal{C}} \rangle_{\text{reg}} \right), \quad (15)$$

where we perform the regularization according to Eq. (9). By explicitly using the pole structure given in Eq. (11) and the sine kernel (13), we obtain a simpler time evolution,

$$\mathcal{C} \approx \text{const} + \begin{cases} Mt \left(1 - \frac{t}{T_H} + \frac{t^2}{3T_H^2} \right), & (t < T_H) \\ \frac{1}{3} MT_H, & (t \geq T_H) \end{cases}. \quad (16)$$

This result clearly exhibits a long-time linear growth and saturates on the plateau after the Heisenberg time T_H . We further note that the classical linear growth rate M is also determined by the location of the pole, i.e., $E_{ij} = \pm iM\alpha$. The linear growth rate M can be interpreted as the *momentum* of the associated complexity measures; see e.g., Refs. [27,28,77,78]. From the viewpoint of holographic complexity, the coefficient M is typically related to the Arnowitt-Deser-Misner (ADM) mass of AdS black holes.

Codimension-zero observables—The infinitely many codimension-one holographic complexity measures discussed above capture only a limited amount of spectral information, as they exclusively involve two-point spectral correlation functions. In the spirit of the complexity = anything proposal, the most general class of holographic complexity measures is associated with extremal codimension-zero bulk subregions $\mathcal{M}$. Such regions are bounded by future and past spacelike hypersurfaces $\Sigma_{\pm}$, as shown in Fig. 2. As the second part of this Letter, we propose an extension of the previous analysis to codimension-zero holographic complexity $\mathcal{C}_{\text{Any}}$, based on the specific pole structure in Eq. (11).

The key point developed in [27,28] to demonstrate the linear growth of codimension-zero holographic complexity

is that the bulk quantity defined within the subregion $\mathcal{M}$ can be reduced to contributions on its future and past boundaries $\Sigma_{\pm}$ by applying Stokes' theorem. Consequently, within the complexity = anything framework, the codimension-zero holographic complexity can be decomposed as the sum of two codimension-one gravitational observables defined on $\Sigma_{\pm}$, namely,

$$\mathcal{C}_{\text{Any}} = \mathcal{C}_+(\Sigma_+) + \mathcal{C}_-(\Sigma_-). \quad (17)$$

Motivated by this decomposition, we propose the generating function for codimension-zero complexity as

$$\langle e^{-\alpha_+ \mathcal{C}_+ - \alpha_- \mathcal{C}_-} \rangle := \langle \text{TFD} | e^{-\alpha_+ \mathcal{C}_+} e^{-\alpha_- \mathcal{C}_-} | \text{TFD} \rangle. \quad (18)$$

Its corresponding spectral representation reads

$$\frac{e^{3S_0}}{\mathcal{Z}} \int dE_1 dE_2 dE_3 e^{-iE_{12}t} \overline{\langle D(E_1) D(E_2) D(E_3) \rangle} \times \langle E_1 | e^{-\alpha_+ \mathcal{C}_+} | E_3 \rangle \times \langle E_3 | e^{-\alpha_- \mathcal{C}_-} | E_2 \rangle, \quad (19)$$

where we insert an identity operator (2) expanded in energy eigenstates. A crucial distinction between generalized volume and codimension-zero observables is that the latter incorporates ensemble-averaged three-point spectral correlation functions, thereby encoding richer information about the spectrum. Because of random-matrix universality, the spectral correlation functions can be expressed in terms of the Christoffel-Darboux kernel. Remarkably, we can decompose the ensemble-averaged three-point spectral correlator in terms of

$$\begin{aligned} \overline{\langle D(E_1) D(E_2) D(E_3) \rangle} &= D(E_1) D(E_2) D(E_3) + \langle D(E_1) D(E_2) D(E_3) \rangle_c \\ &+ \langle D(E_1) \rangle \langle D(E_2) D(E_3) \rangle_c \\ &+ \langle D(E_2) \rangle \langle D(E_3) D(E_1) \rangle_c \\ &+ \langle D(E_3) \rangle \langle D(E_2) D(E_1) \rangle_c. \end{aligned} \quad (20)$$

Interestingly, the spectral decomposition in Eq. (20) naturally *suggests* corresponding Euclidean wormhole contributions in the gravitational path integral. These contributions are organized by different connectivity patterns among the three sectors, as schematically illustrated in Fig. 4. Terms involving connected two-point correlators correspond to configurations in which only two sectors are connected. In contrast, fully connected three-point correlators represent topologies linking all three sectors.

Noting that each matrix element appearing in Eq. (19) exhibits the same characteristic pole located at $E_{13} = \pm iM_+ \alpha_+$ and $E_{32} = \pm iM_- \alpha_-$, as in Eq. (11), we can extend our analysis directly to codimension-zero generating functions. In particular, holographic complexity $\mathcal{C}_{\text{Any}}$ derived in the *classical* AdS black hole spacetime corresponds to the contribution from disconnected correlation functions [i.e., taking $(1/G_N) \sim e^{S_0} \rightarrow \infty$]. Hence, it is

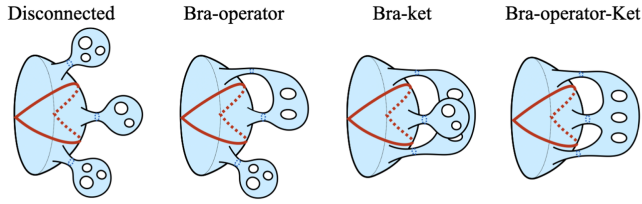


FIG. 4. Schematic illustration of Euclidean wormhole saddles suggested by the decomposition of the three-point spectral correlator in Eq. (20). Each diagram represents a distinct topology of connectivity among the three sectors (bra, operator insertion, and ket).

straightforward to get the time evolution of codimension-zero holographic complexity,

$$\mathcal{C}_{\text{Any}}|_{\text{classical}} \approx \text{const} + (M_+ + M_-)t, \quad (21)$$

which matches the linear growth derived previously within the complexity = anything proposal [27,28].

To incorporate quantum corrections associated with spacetime wormholes at finite S_0 , we apply the averaged three-point spectral correlation function (20). Similarly, we find that the generating function $\langle e^{-\alpha_+ \mathcal{C}_+ - \alpha_- \mathcal{C}_-} \rangle$ involved with the universal pole structure defined in Eq. (11) typically exhibits a slope-ramp-plateau structure analogous to the codimension-one case. After performing the same regularization in the $\alpha \rightarrow 0$ limit, we show that the time evolution of codimension-zero complexity measures is similar to Eq. (16) for the codimension-one case (see Supplemental Material [74]).

Although our motivation originates from holographic complexity within the complexity = anything framework, our construction extends naturally to generic quantum systems by defining quantum complexity in the same way. Explicitly, we define universal microscopic generating functions of quantum complexity measures in the micro-canonical ensemble [79] as follows:

$$G^{(0)}(\alpha_{\pm}, t) = \sum_{E_i, E_j, E_k} \frac{\tilde{\alpha}_+ \tilde{\alpha}_- e^{-i(E_{ik} + E_{kj})t}}{(E_{ik}^2 + \tilde{\alpha}_+^2)(E_{kj}^2 + \tilde{\alpha}_-^2)}. \quad (22)$$

The corresponding codimension-one counterpart has been defined in [41] and is associated with the spectral complexity introduced in [35]. These generating functions typically exhibit a universal slope-ramp-plateau structure for chaotic quantum systems governed by random-matrix universality. As a concrete illustration, numerical results in the SYK model [9,10] are shown in Fig. 5.

Toward universality in time evolution—The previous analysis illustrates that the characteristic linear growth and late-time plateau of infinite complexity measures in chaotic systems emerge directly from the specific pole structure described in Eq. (11) and the universal spectral statistics encoded by Eq. (13). Moreover, we can rigorously identify necessary and sufficient conditions for this universal time

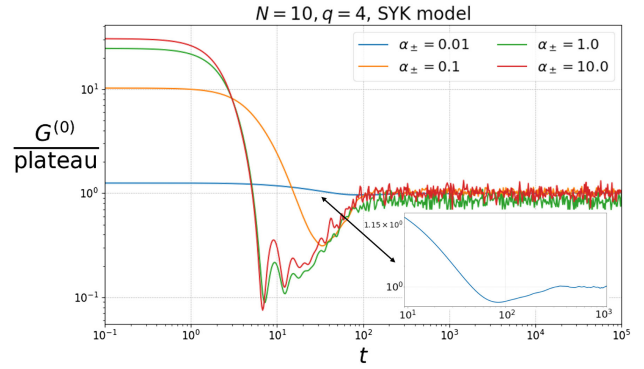


FIG. 5. The time evolution of generating functions $G^{(0)}(\alpha_{\pm}, t)$ defined in Eq. (22) (rescaled by its plateau value) for the SYK model with $N = 10$, $q = 4$. The characteristic linear ramp before the Heisenberg time ($T_H \sim 10^2$) would be suppressed with decreasing values of $\alpha_{\pm}$.

evolution using the generating function formalism introduced in Eqs. (10) and (19). A crucial observation is that, by construction, the time dependence in the spectral representation exclusively arises from the phase factor $e^{-iE_{ij}t}$, originating from the wave function of the TFD state. Consequently, upon deforming the integration contour in the complex E_{ij} plane (see Fig. 3) and applying the residue theorem, the time evolution is thus determined by the residues of poles enclosed by the contour. Schematically, we represent this connection as

Pole Structure $\Leftrightarrow$ Time Evolution.

In the classical limit ($e^{S_0} \rightarrow \infty$), all quantum corrections vanish, leaving only disconnected spectral correlators. Considering a generic pole located at $E_{ij} = E_R(\alpha) + iE_I(\alpha)$, the residue of the generating function yields a time dependence $e^{-iE_R t} e^{E_I t}$. To ensure linear growth of holographic complexity in the classical regime, it is necessary that the matrix element $\langle E_i | e^{-\alpha \mathcal{C}} | E_j \rangle$ possesses a *first-order pole* satisfying

$$\lim_{\alpha \rightarrow 0} E_R(\alpha) = 0, \text{ and } \lim_{\alpha \rightarrow 0} E_I(\alpha) \sim -M\alpha \rightarrow 0. \quad (23)$$

Conversely, we can explicitly verify that no other type of pole (including higher-order poles) can produce the desired linear growth of holographic complexity measures (see Supplemental Material [74] for a detailed proof).

It is intriguing to note that a corresponding geometric condition appears in the complexity = anything proposal, where the linear growth of holographic complexity follows from the extremization of hypersurfaces Σ or codimension-zero bulk subregions $\mathcal{M}$ [27,28]. This analogy motivates us to *conjecture* that the presence of this particular pole structure corresponds holographically to geometric extremization in the bulk spacetime, i.e.,

$$\text{Extremization} \xleftrightarrow{\text{conjecture}} \text{Pole at } E_{ij} = -i\tilde{\alpha},$$

thereby establishing a direct link between quantum complexity and its holographic dual description.

Furthermore, we introduce quantum corrections with a finite S_0 by incorporating the joint eigenvalue distributions $R_2(E_i, E_j)$ with a generic Christoffel-Darboux kernel [80]. They characterize spectral correlations without contact terms. We find that the necessary and sufficient condition for the late-time plateau of quantum complexity is explicitly given by (see Supplemental Material [74])

$$\lim_{E_{ij}=-i\tilde{\alpha}\rightarrow 0} (\bar{D}(E_i)\bar{D}(E_j) - \text{Kernel}(E_{ij}, \bar{E})) = 0. \quad (24)$$

Remarkably, this condition exactly coincides with spectral level repulsion as shown in Eq. (12). It is worth emphasizing that level repulsion is widely recognized as a hallmark of quantum chaos. Analogous conclusions straightforwardly apply to codimension-zero holographic complexity measures when incorporating three-point spectral correlation functions.

Conclusion—In this Letter, we have presented a universal framework for deriving the time evolution of quantum and holographic complexity based on spectral representations of generating functions. By identifying a characteristic pole structure in the associated matrix elements and incorporating universal spectral correlations from random-matrix theory, we have demonstrated that quantum complexity in chaotic quantum systems universally exhibits long-time linear growth followed by a saturation plateau at late times. In particular, we have rigorously established that the presence of this specific pole structure, the finiteness and discreteness of the Hilbert space, and quantum chaos characterized by spectral level repulsion constitute the necessary and sufficient conditions for this universal time evolution of complexity measures. Our results also open several promising directions for future investigation, such as clarifying the relation between holographic complexity measures and Krylov (spread) complexity [40,57,63,66,81–83], exploring the switchback effect [21], extending to quantum complexity measures of mixed states [84], and investigating more general complexity operators involving higher-point spectral correlations.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

Complete and orthonormal basis—In terms of the energy eigenbasis $|E_i\rangle$, fixed- C states can be formally defined as

$$|C\rangle = e^{S_0} \int dE D(E) \phi_E(C) |E\rangle. \quad (\text{A1})$$

All information about the state $|C\rangle$ is encoded in the wave function

$$\phi_E(C) \equiv \langle E|C\rangle. \quad (\text{A2})$$

Using this representation, the matrix element entering the generating function of complexity can be defined as

$$\langle E_i | e^{-\alpha C} | E_j \rangle \equiv \int dC \phi_{E_i}(C) \phi_{E_j}^*(C) e^{-\alpha C}. \quad (\text{A3})$$

As shown in the main text, the characteristic pole structure in Eq. (11) is both necessary and sufficient for

linear growth of holographic complexity in the semiclassical regime. This particular pole structure is realized by an infinite class of operators, including the geodesic length operator in JT gravity [39] and the chord-number operator in the double-scaled SYK model [43]. The latter is also equivalent to the Krylov spread complexity operator. From the perspective of the wave function $\phi_E(\mathcal{C})$, the origin of this pole can be traced to its characteristic oscillatory behavior. In particular, for $E_i \simeq E_j$ one finds

$$\phi_{E_i}(\mathcal{C})\phi_{E_j}^*(\mathcal{C}) \approx_{E_i \simeq E_j} \frac{1}{e^{S_0} \pi \bar{D}(\bar{E}) M} \cos(t_C E_{ij}), \quad (\text{A4})$$

with $t_C \approx \mathcal{C}/M$. Substituting this form into Eq. (A3) immediately reproduces the pole structure in Eq. (11).

An alternative approach to computing the expectation value of complexity, introduced in [41], is based on the squared overlap between a fixed- $\mathcal{C}$ state and the thermofield double state,

$$\begin{aligned} P_t(\mathcal{C}) &:= \langle \text{TFD}(t) | \mathcal{C} \rangle \langle \mathcal{C} | \text{TFD}(t) \rangle \\ &= \frac{e^{2S_0}}{Z} \int dE_i dE_j e^{-iE_{ij}t} \phi_{E_i}(\mathcal{C}) \phi_{E_j}^*(\mathcal{C}) \overline{\langle D(E_i) D(E_j) \rangle}. \end{aligned} \quad (\text{A5})$$

Using the characteristic wave function (A4), one finds that the dominant contribution depends only on $|t - t_C|$, since

$$e^{-iE_{ij}t} \phi_{E_i}(\mathcal{C}) \phi_{E_j}^*(\mathcal{C}) \sim e^{\pm iE_{ij}(t-t_C)}. \quad (\text{A6})$$

Crucially, we highlight that this squared overlap incorporates nonperturbative corrections because it is involved with the ensemble-averaged spectral correlator, i.e.,

$$e^{2S_0} \overline{\langle D(E_i) D(E_j) \rangle} = R_2(E_i, E_j) + \delta(E_{ij}) e^{S_0} D(E_i). \quad (\text{A7})$$

Equipped with this “probability distribution,” the expectation value of generating function and complexity measures can then be formally obtained as

$$\langle e^{-a\mathcal{C}} \rangle = \int d\mathcal{C} P_t(\mathcal{C}) \times e^{-a\mathcal{C}}, \quad (\text{A8})$$

and

$$\langle \hat{\mathcal{C}} \rangle = \int d\mathcal{C} P_t(\mathcal{C}) \times \mathcal{C}, \quad (\text{A9})$$

respectively. However, this integral (i.e., sum over all basis states) generically results in the divergence of $\langle \hat{\mathcal{C}} \rangle$ due to contributions from $t_C \gg T_H$. In particular, $P_t(\mathcal{C})$ develops a

plateau for $t_C \gg T_H$, with $P_t(\mathcal{C}) \simeq (2/MT_H)$. This divergence reflects the overcompleteness of the original $\mathcal{C}$ basis, which in turn originates from spacetime wormhole contributions.

In order to not only remove the divergence but also obtain a physically meaningful operator, one must perform a Gram-Schmidt orthogonalization [39,43], yielding an orthonormal basis $|\mathcal{C}_O\rangle$. The choice of orthogonalization is directly tied to the physical interpretation of the resulting operator $\hat{\mathcal{C}}_O$ in the classical regime, in close analogy with nonisometric codes [85–87]. The operator $\hat{\mathcal{C}}_O$ is defined in terms of Eq. (6), and its expectation value reads

$$\langle \hat{\mathcal{C}}_O \rangle = \sum_{\text{orthonormal basis}} P_t(\mathcal{C}_O) \times \mathcal{C}_O, \quad (\text{A10})$$

with $\sum_{\mathcal{C}_O} P_t(\mathcal{C}_O) = 1$. In the following, we still denote the total dimension of orthonormal $\mathcal{C}_O$ basis as the partition function Z . In the following, we aim to show that the same universal time evolution can be derived by evaluating $\langle \hat{\mathcal{C}}_O \rangle$, i.e., presenting a long-time linear growth and a transition to the plateau at late times.

In the semiclassical regime $t \ll T_H$, only the disconnected part of the spectral correlator contributes. Using Eq. (A4), we can easily find that the squared overlap is dominated by

$$P_t(\mathcal{C}) \approx \frac{2}{\pi M \Delta E} \frac{\sin^2\left((t_C - t) \frac{\Delta E}{2}\right)}{(t_C - t)^2}. \quad (\text{A11})$$

This expression reveals that the TFD state is sharply localized around $t_C \approx t$. Corrections due to orthogonalization are suppressed by powers of e^{-S_0} , so both the wave function structure (A4) and the pole structure (11) remain valid to leading order. As a result, the expectation value $\langle \hat{\mathcal{C}}_O \rangle$ reproduces the expected linear growth at early times, namely,

$$\langle \hat{\mathcal{C}}_O \rangle|_{t \ll T_H} \approx Mt. \quad (\text{A12})$$

As the evolution approaches the Heisenberg time $t \sim T_H$, the overlap becomes dominated by states with $\mathcal{C}_O < Mt$, causing the linear growth to slow down. In the late-time regime $t \gg T_H$, the probability distribution takes the form

$$P_t(\mathcal{C}_O) \approx \frac{1}{Z} + \frac{1}{Z} \int dE_i dE_j \phi_{E_i} \phi_{E_j}^* R_2(E_i, E_j) e^{iE_{ij}t}. \quad (\text{A13})$$

The crucial point is that off-diagonal contributions are suppressed at late times due to level repulsion, $R_2(E_i, E_j) \sim \mathcal{O}(|E_{ij}|^\beta)$, with $\beta = 2$ for Gaussian unitary ensemble.

As a consequence of this suppression, the probability distribution at late times becomes approximately uniform,

$$P_t(C_O) \approx \frac{1}{Z} + \mathcal{O}\left(\frac{1}{t}\right). \quad (\text{A14})$$

Here, the constant is fixed by the completeness of orthonormal C_O basis. Accordingly, the late-time expectation value saturates to a constant, i.e.,

$$\langle \hat{C}_O \rangle|_{t \gg T_H} \approx \frac{1}{Z} \sum_{C_O \text{ basis}} C_O. \quad (\text{A15})$$

Finally, we recover the same universal time evolution, characterized by linear growth and late-time saturation, in agreement with the result obtained from the regularization of generating functions in the $\alpha \rightarrow 0$ limit.