

Carrollian Holographic Duals Are Nonlocal

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Mapping the S matrix of a generic theory of flat-space gravity coupled to matter to correlation functions of a putative Carrollian dual, we show that bulk interactions imply boundary nonlocality.

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Introduction—The basic idea of the Carrollian holography program is rooted in symmetry. In an asymptotically flat spacetime, Poincaré transformations act on null infinity as a conformal extension of Carroll symmetry [1–4], motivating the possibility that theories of flat-space gravity are dual to conformal Carrollian theories [5,6]. (There is a related proposal of a celestial dual living at spatial infinity [7–9].) The pursuit of this idea has met some successes and challenges (see Ref. [10] for a recent review with an exhaustive list of references). One success is a reinterpretation of the flat-space limit of AdS/CFT [11–16] as a double-scaling limit, in which one simultaneously takes the large N parameter of a dual CFT to infinity while also taking the boundary speed of light to zero [17–20]. However, there are also challenges, in that the quantum mechanical behavior of Carrollian field theories is markedly different from that inferred from flat-space gravity [21–39].

The standard approach in Carrollian holography so far has been to use an analog [18,40–44] of the AdS/CFT dictionary to map the S matrix to functions. On account of the bulk Poincaré symmetry, these functions obey conformal Carrollian Ward identities and are interpreted as correlation functions of operators in a dual description. Much work has been done in this vein, inferring properties of putative duals from the S matrix of bulk theories. So far, there has been rather little work done concerning the operator interpretation of these functions. Besides studies of the quantization of Carrollian field theories, we note the work of [45,46], which studied aspects of an operator product expansion.

In this Letter, we are interested in the operator interpretation of putative Carrollian duals, in particular, the rather basic question of whether these duals are local or not. At this point, it is useful to note that nonlocality of the dual theory is a generic feature of non-AdS holography [47–49]. In relativistic field theory, the definition of locality is that commutators of local operators vanish outside the light cone [50], which, in turn, implies causality. In Carrollian field theories, the situation is more fragile, since light cones collapse and different spatial locations are causally disconnected. Locality then mandates that all commutators of local operators vanish at nonzero spatial separation. We would like to see if this stringent condition is met.

To answer this question, we must first tackle the problem of boundary operator ordering. The standard dictionary relating the S matrix to Carrollian correlators [18,40–44] borrows from the Hilbert space and operators of the flat-space theory, implicitly building a Hilbert space by acting with creation and annihilation operators for massless fields on the bulk vacuum. The basic boundary operators are the integral transform (in null boundary time) of those creation and annihilation operators. In this dictionary, operators fall into one of two types, future and past, respectively, superpositions of positive (negative)-frequency modes built from annihilation (creation) operators on future (past) null infinity. Future operators are, of course, related to past ones by the S matrix. Bulk causality implies that future operators commute with each other, and past operators commute with each other. In free theories, future operators and past operators commute unless they have the same spatial momentum, which in a “dual” means they commute at nonzero spatial separation. However, unsurprisingly, we find that this is no longer true in the presence of bulk interactions, implying a violation of boundary locality. This is the main result of our Letter.

One might wonder if the existing dictionary mapping the S matrix to Carrollian correlators is the culprit and that this work implies the need for an improved version. While in our opinion the current dictionary is not satisfactory, since operator insertions are built from superpositions of only positive- or negative-frequency modes, the central physical

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fact underlying our analysis—that bulk interactions imply in operators no longer commute with out operators—will appear in any improved dictionary and we expect will lead to the same result. Indeed, we attempt to construct a modified dictionary that treats past and future equally, and, yet under it, bulk interactions still imply boundary nonlocality.

Our result can also be understood intuitively using the flat-space limit of AdS/CFT [13,14,16]. There, past operators are really CFT operators acting in a small time band concentrated around a fixed boundary time, while future operators live in a small time band a light-crossing time $\Delta\tau = L\pi$ later, with L the AdS radius. From this point of view, future operators live in the light cone of past operators, and so, of course, they do not commute.

We explore these ideas using the simplest possible bulk model, scalar ϕ^3 theory in four dimensions, but, as we have stressed above, the underlying physics is general.

While known Carrollian quantum theories behave dramatically differently from what we infer from the flat-space S matrix, one might hope that there are yet unknown Carrollian duals to flat-space gravity that exist as solutions to a suitable bootstrap. However, the absence of locality in a Carrollian dual description implies the lack of an operator product expansion, an important ingredient in known bootstraps.

Our focus here is exclusively with the possibility of a Carrollian dual to the flat-space S matrix, but it is worth noting that putative celestial duals are also nonlocal, although for different reasons [51–54].

The remainder of this Letter is organized as follows. In the next section, we review the existing dictionary between the S matrix of massless fields and conformal Carrollian correlators. We then demonstrate our central result, that bulk interactions mandate boundary nonlocality. We go on to construct an improved dictionary, but this improvement does not restore locality. In Supplemental Material [55], we discuss other regulatory schemes for local Carrollian theories and why old computations of “entanglement entropy” in a putative Carrollian dual to 3D gravity coupled to matter cannot be interpreted as a measure of entanglement.

Review—We briefly recall the map between S matrices in asymptotically flat spacetimes and Carrollian correlation functions [40,41,60]. There is a similar dictionary relevant for “celestial holography.” For definiteness, consider a real scalar ϕ in $3+1$ -dimensional Minkowski space. In the free theory, one has the standard mode expansion

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a(\mathbf{p}) e^{-ip \cdot x} + a^\dagger(\mathbf{p}) e^{ip \cdot x} \right). \quad (1)$$

With local interactions, the asymptotic field is expressed in terms of $a_{\text{in/out}}$ operators. Since our interest is massless

scattering, we use the standard null-momentum parametrization [61]

$$p^\mu = \omega(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}), \quad (2)$$

where $\omega > 0$ is the energy and $(z, \bar{z})$ are stereographic coordinates on the celestial sphere.

In the conformal compactification, $\mathcal{S}^+$ is coordinatized by $(u, z, \bar{z})$ and $\mathcal{S}^-$ by $(v, z, \bar{z})$. The map from asymptotic operators to Carrollian ones is implemented by a Mellin-like transform in null time [40,41,60,62,63]:

$$\begin{aligned} \Phi_{\text{out}}^\Delta(u, z, \bar{z}) &= \int_0^\infty d\omega \omega^{\Delta-1} \left(a_{\text{out}}(\omega, z, \bar{z}) e^{-i\omega u} + (\text{H.c.}) \right), \\ \Phi_{\text{in}}^\Delta(v, z, \bar{z}) &= \int_0^\infty d\omega \omega^{\Delta-1} \left(a_{\text{in}}(\omega, z, \bar{z}) e^{-i\omega v} + (\text{H.c.}) \right), \end{aligned} \quad (3)$$

where we keep the conformal dimension Δ arbitrary but, for clarity, we have not included the arbitrary normalization constant. This flexibility captures multiparticle states [64] and simultaneously avoids the IR issues that afflict correlators where we assign the naive dimension $\Delta = 1$ to the dual of massless fields [42]. Indeed, [64] emphasizes that multitrace primaries in the parent CFT generically lead to Δ differing from the extrapolate dictionary of [41].

The Bondi-Metzner-Sachs (BMS₄) algebra [65,66] takes the form

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m}, & [\bar{L}_n, \bar{L}_m] &= (n-m)\bar{L}_{n+m}, \\ [L_n, M_{r,s}] &= \left(\frac{n+1}{2} - r \right) M_{r+n,s}, \\ [\bar{L}_n, M_{r,s}] &= \left(\frac{n+1}{2} - s \right) M_{r,s+n}, & [M_{r,s}, M_{p,q}] &= 0. \end{aligned} \quad (4)$$

Here, $\{L_n, \bar{L}_n\}$ generate superrotations on the celestial sphere, and $M_{r,s}$ generate supertranslations. The inspiration behind the Carrollian approach to flat-space holography rests on the isomorphism between (4) and the infinite-dimensional lift of the 3D conformal Carrollian algebra [3,4]. Using the transformation properties of the creation and annihilation operators, one finds that the boundary fields in (3) transform as BMS₄ primaries [62]. Thus, (3) effects a change of basis from plane waves to conformal primaries [40,41,60,62,67].

Inserting (3) into S -matrix elements, specifically matrix elements of products of in and out operators, gives a Mellin integral representation of celestial correlators. For a process with p outgoing and $n-p$ incoming particles, we have

$$\begin{aligned} & \langle \Phi_{\text{out}}^{\Delta_1}(u_1, z_1, \bar{z}_1) \cdots \Phi_{\text{out}}^{\Delta_p}(u_p, z_p, \bar{z}_p) \Phi_{\text{in}}^{\Delta_{p+1}}(v_{p+1}, z_{p+1}, \bar{z}_{p+1}) \cdots \Phi_{\text{in}}^{\Delta_n}(v_n, z_n, \bar{z}_n) \rangle \\ &= \prod_{i=1}^p \int_0^\infty d\omega_i \omega_i^{\Delta_i-1} e^{-i\omega_i(u_i-i\epsilon)} \prod_{j=p+1}^n \int_0^\infty d\omega_j \omega_j^{\Delta_j-1} e^{i\omega_j(v_j+i\epsilon)} S(\omega_1, z_1, \bar{z}_1; \dots, \omega_n, z_n, \bar{z}_n), \end{aligned} \quad (5)$$

with $\epsilon > 0$ implementing the usual $i\epsilon$ prescription to render the Mellin integrals convergent [68].

Using the (antipodal) identification of the celestial spheres at $\mathcal{S}^+$ and $\mathcal{S}^-$ [41], one may trade v for u and work entirely at $\mathcal{S}^+$ [69]. The dynamical content of scattering is then encoded in the “out” sector and, correspondingly, in the resulting Carrollian correlators.

Boundary nonlocality—Consider massless scalar ϕ^3 theory. In perturbation theory, the interacting theory has an interacting vacuum $|\Omega\rangle$ evolved from the free field vacuum $|0\rangle$ using the interaction picture Hamiltonian. Using standard quantum field theory techniques to express $a_{\text{out}}(\mathbf{p})$ in terms of $a_{\text{in}}(\mathbf{p})$, we find [70]

$$\begin{aligned} & \langle \Omega | a_{\text{out}}(\mathbf{p}_1) [a_{\text{out}}(\mathbf{p}_2), a_{\text{in}}^\dagger(\mathbf{p}_3)] a_{\text{in}}^\dagger(\mathbf{p}_4) | \Omega \rangle \\ &= \mathcal{A}_{\text{free}} + \mathcal{A}_{\text{exchange}} + \mathcal{A}_{\text{collinear}} + \mathcal{O}(\lambda^3), \end{aligned} \quad (6)$$

where

$$\begin{aligned} \mathcal{A}_{\text{free}} &= (2\pi)^6 (2E_{\mathbf{p}_1}) (2E_{\mathbf{p}_2}) \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_4) \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_3), \\ \mathcal{A}_{\text{exchange}} &= i(2\pi)^4 \lambda^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\quad \times \left[\frac{1}{s + i\epsilon} + \frac{1}{t + i\epsilon} + \frac{1}{u + i\epsilon} \right], \end{aligned} \quad (7)$$

where $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$, and there is a $\mathcal{O}(\lambda^2)$ collinear contribution $\mathcal{A}_{\text{collinear}}$ delta function supported on $p_2 \propto p_3$ or $p_2 \propto p_4$ whose details depend on the renormalization scheme.

Here, $\mathcal{A}_{\text{exchange}}$ is the standard sum of exchange diagrams contributing to the S matrix and is the one responsible for boundary nonlocality. It comes solely from the time-ordered part of (6). There is no such contribution in the out-of-time-ordered term $\langle \Omega | a_{\text{out}}(\mathbf{p}_1) a_{\text{in}}^\dagger(\mathbf{p}_3) a_{\text{out}}(\mathbf{p}_2) a_{\text{in}}^\dagger(\mathbf{p}_4) | \Omega \rangle$. To see this, note that, at the level of asymptotic states, we have $\hat{S} a_{\text{in}}^\dagger(p) | \Omega \rangle = a_{\text{out}}^\dagger(p) | \Omega \rangle + [\text{multiparticle states}]$ [71]. In massless $\lambda\phi^3$ theory, the first subleading terms at $\mathcal{O}(\lambda)$ come from $1 \rightarrow 2$ splittings which are kinematically forced to be collinear when all legs are on shell. In the time-ordered matrix element $\langle \Omega | a_{\text{out}}(\mathbf{p}_1) a_{\text{out}}(\mathbf{p}_2) a_{\text{in}}^\dagger(\mathbf{p}_3) a_{\text{in}}^\dagger(\mathbf{p}_4) | \Omega \rangle$, each $a_{\text{in}}^\dagger$ is converted by $\hat{S}$ into an $a_{\text{out}}^\dagger$ plus its collinear multiparticle tail, and the $\mathcal{O}(\lambda^2)$ piece in which the two resulting one-particle out states scatter through the usual $2 \rightarrow 2$ kernel produces $\mathcal{A}_{\text{exchange}}$. By contrast, in the mixed ordering $\langle \Omega | a_{\text{out}}(\mathbf{p}_1) a_{\text{in}}^\dagger(\mathbf{p}_3) a_{\text{out}}(\mathbf{p}_2) a_{\text{in}}^\dagger(\mathbf{p}_4) | \Omega \rangle$, each $a_{\text{out}}(\mathbf{p}_i)$ is already adjacent to an $a_{\text{in}}^\dagger(\mathbf{p}_j)$, so at generic kinematics the only contribution at $\mathcal{O}(\lambda^2)$ comes from the diagonal $1 \rightarrow 1$ part $\hat{S} a_{\text{in}}^\dagger(p) | \Omega \rangle \sim a_{\text{out}}^\dagger(p) | \Omega \rangle$, which enforces $\mathbf{p}_1 = \mathbf{p}_3$ and $\mathbf{p}_2 = \mathbf{p}_4$. The multiparticle components in $\hat{S} a_{\text{in}}^\dagger(p) | \Omega \rangle$ can contribute only when the extra on-shell quanta are exactly collinear with one of the external legs; these degenerate configurations are precisely isolated by the delta functions in $\mathcal{A}_{\text{collinear}}$ and so do not build up an additional exchange kernel. From this discussion, it is clear that $\mathcal{A}_{\text{collinear}}$ arises solely from the mixed-ordering term. Since $\hat{S} a_{\text{in}}^\dagger(p) | \Omega \rangle \sim a_{\text{out}}^\dagger(p) | \Omega \rangle$, we need $a_{\text{out}}(\mathbf{p}_i)$ adjacent to $a_{\text{in}}(\mathbf{p}_i)$ in order to pick out the collinear contributions.

It is useful to record the commutator

$$\begin{aligned} [a_{\text{out}}(\mathbf{p}_1), a_{\text{in}}^\dagger(\mathbf{p}_2)] &= (2\pi)^3 2E_{\mathbf{p}_1} \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2) + i\lambda \int d^4x e^{i(p_1-p_2)\cdot x} \phi_I(x) \\ &\quad + \frac{\lambda^2}{12} \int \int d^4x d^4x' e^{ip_1\cdot x} \theta(x^0 - x'^0) \left(-2e^{-ip_2\cdot x} [\phi_I(x), \phi_I^3(x')] + 6e^{-ip_2\cdot x'} [\phi_I^2(x), \phi_I^2(x')] \right) + \mathcal{O}(\lambda^3), \end{aligned} \quad (8)$$

where ϕ_I is the interaction picture field. It is these ϕ_I -dependent terms that imply boundary nonlocality.

From the standard dictionary (5), an expression of the form (6) corresponds to

$$\begin{aligned} & \langle \Phi_{\text{out}}^{\Delta_1}(u_1, z_1, \bar{z}_1) [\Phi_{\text{out}}^{\Delta_2}(u_2, z_2, \bar{z}_2), \Phi_{\text{in}}^{\Delta_3}(u_3, z_3, \bar{z}_3)] \Phi_{\text{in}}^{\Delta_4}(u_4, z_4, \bar{z}_4) \rangle \\ &= \int \left(\prod_{i=1}^4 d\omega_i \omega_i^{\Delta_i-1} e^{i\epsilon_i \omega_i u_i} \right) \langle \Omega | a_{\text{out}}(\mathbf{p}_1) [a_{\text{out}}(\mathbf{p}_2), a_{\text{in}}^\dagger(\mathbf{p}_3)] a_{\text{in}}^\dagger(\mathbf{p}_4) | \Omega \rangle. \end{aligned} \quad (9)$$

From the boundary perspective, one would naïvely expect this expression to result in a linear combination of a product of two point structures weighted by appropriate space-time-dependent factors. Indeed according to the standard Heemskerk-Penedones-Polchinski-Sully argument [72], bulk locality in AdS arises from the solutions to the crossing conditions on the CFT four-point correlator. One would obtain those crossing conditions via an operator product expansion (OPE) and equating the resulting OPE blocks. If one is drawing analogy from AdS/CFT, one would expect

that (9) would result in a linear combination of a product of Carrollian OPEs [45,46,73] weighted by appropriate OPE blocks. This expectation is simply wrong, and (9) is inconsistent with an OPE on account of the exchange term. To our knowledge, an expression of the form (9) has not been computed before in the literature and we emphasize that (9) is fairly generic, as the only elements that go into (5) are symmetries and the existence of the map (5).

The dictionary (5) is known to give the complicated result [18,42,63,74]

$$\int \left(\prod_{i=1}^4 d\omega_i \omega_i^{\Delta_i-1} e^{ie_i \omega_i u_i} \right) \mathcal{A}_{\text{exchange}} = \frac{\lambda^2}{4} \left[\frac{\Gamma(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 - 6)}{z_{13}^2 \bar{z}_{13}^2 z_{24} \bar{z}_{24}} \frac{(\sigma_1^*)^{\Delta_1-2} (\sigma_2^*)^{\Delta_2-1} (\sigma_3^*)^{\Delta_3-2} (\sigma_4^*)^{\Delta_4-1}}{[i(\sigma_1^* u_1 + \sigma_3^* u_3 - \sigma_2^* u_2 - \sigma_4^* u_4)]^{\Delta_1+\Delta_2+\Delta_3+\Delta_4-6}} \right. \\ \left. \times \prod_{i=1}^4 \mathbb{I}_{[0,1]}(\sigma_i^*) + (s\text{-channel}) + (u\text{-channel}) \right] \delta(|z - \bar{z}|), \quad (10)$$

where

$$\sigma_1^* = \frac{1}{D} \frac{z_{24} \bar{z}_{34}}{z_{12} \bar{z}_{13}}, \quad \sigma_2^* = \frac{1}{D} \frac{z_{34} \bar{z}_{14}}{z_{23} \bar{z}_{12}}, \quad \sigma_3^* = \frac{1}{D} \frac{z_{24} \bar{z}_{14}}{z_{23} \bar{z}_{13}}, \\ \sigma_4^* = \frac{1}{D}, \quad D = 2 \frac{z_{24} \bar{z}_{34}}{z_{12} \bar{z}_{13}} + 2 \frac{z_{24} \bar{z}_{14}}{z_{23} \bar{z}_{13}}, \quad (11)$$

and z and $\bar{z}$ are the conformal cross ratios

$$z = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \bar{z} = \frac{\bar{z}_{12} \bar{z}_{34}}{\bar{z}_{13} \bar{z}_{24}}. \quad (12)$$

$\mathbb{I}_{[0,1]}(x)$ is the indicator function

$$\mathbb{I}_{[0,1]}(x) = \begin{cases} 1, & \text{if } x \in [0, 1], \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

The role of the indicator function is simply to constrain the magnitude of the conformal cross ratio depending on the particular channel under consideration [75]. By choosing $\Delta_i \neq 1$, we avoided IR singularities in (10). A necessary and sufficient condition for avoiding the IR singularities of the Γ function is $\sum_{i=1}^4 \Delta_i > 6$. For the t -channel process, the indicator functions imply $0 < z < 1$. To avoid clutter, we have indicated only the t -channel process. The others have analogous structures but, most importantly, are proportional to $\delta(|z - \bar{z}|)$.

The delta function $\delta(|z - \bar{z}|)$ cannot arise from an ultralocal Carrollian OPE like that appearing in the free field result. Rather, it enforces the constraint that the conformal cross ratio is real. We can understand this from the following fact. Using the $SL(2; \mathbb{C})$ invariance to move three of the insertions to the equator of the celestial sphere, momentum conservation enforces that the fourth point lies

on the equator as well. This is a nonlocal structure that depends on all four insertions in the boundary field theory. We reiterate that the novel point of our Letter is not merely the appearance of distributional terms in Carrollian correlators, but the persistence of this nonlocality in the commutator diagnostic. It has no counterpart in the standard way AdS bulk locality arises from the solutions to the crossing equation of the boundary CFT four-point function [72]. Instead, in flat space, local bulk interactions generate nonlocal structures in a Carrollian dual.

Let us ask if, as in ordinary field theory, analytic continuation in boundary time relates different orderings to each other. Consider the free theory. Since $\langle 0 | a_{\text{in}}^\dagger(\mathbf{p}_2) a_{\text{out}}(\mathbf{p}_1) | 0 \rangle = 0$, we infer

$$\langle [\Phi_{\text{out}}^\Delta(u_1, z_1, \bar{z}_1), \Phi_{\text{in}}^\Delta(u_2, z_2, \bar{z}_2)] \rangle \\ = \langle \Phi_{\text{out}}^\Delta(u_1, z_1, \bar{z}_1) \Phi_{\text{in}}^\Delta(u_2, z_2, \bar{z}_2) \rangle, \quad (14)$$

which is *not* what one infers from analytic continuation of the rhs in boundary time. This is because analytic continuation results in a Lorentzian time-ordered correlator with an appropriate $i\epsilon$ prescription. Thus, generically, lhs $\neq$ rhs in (14) because the commutator is obtained from a difference between two Lorentzian orderings.

For this reason, we obtain out-of-time-ordered correlators not by analytically continuing in boundary time but by directly considering the matrix elements of out-of-time-ordered strings of a_{out} 's and $a_{\text{in}}^\dagger$'s.

Nonlocality from the flat-space limit of AdS/CFT—The source of the nonlocality in the four-point function (10) can be traced to the interaction terms in the second line of the commutator of boundary operators in (8). We will now see how this is consistent with the flat-space limit of AdS/CFT

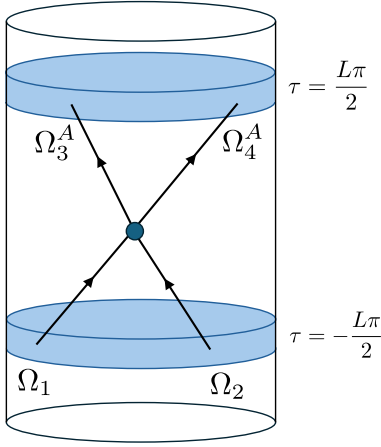


FIG. 1. A cartoon of how a flat-space scattering amplitude is obtained from the large radius limit of AdS/CFT.

[11,13,16,76–79]. Consider a bulk theory that arises from the large radius limit of a theory in AdS, and work in coordinates where the global AdS line element is

$$ds^2 = \frac{L^2}{\cos^2(\rho)} \left(-\frac{d\tau^2}{L^2} + d\rho^2 + \sin^2(\rho)d\Omega^2 \right), \quad (15)$$

with L the AdS radius, so that boundary time is τ . The operators encoding the flat-space S matrix in the deep interior are smeared over time bands of $\mathcal{O}(1)$ width located at $\tau = \pm(L\pi/2)$. The operators encoding out states are inserted in the band at $\tau = +(L\pi/2)$, whereas the operators encoding in states are in the band at $\tau = -(L\pi/2)$. Moreover, there is a relative antipodal flip between past and future, in the sense that in states of definite spatial momentum are inserted at an angle Ω on the boundary sphere, and out states of the same spatial momentum are inserted at the antipode Ω_A . See Fig. 1. One can see this antipodal flip at the level of symmetry algebra when you restrict the CFT isometries to the time bands around $\tau = \pm(L\pi/2)$ [80], and this antipodal flip is crucial to obtain Carrollian correlators in the limit [17].

If one considers a free theory in the bulk AdS, then, of course, one has the standard mode decomposition [81,82]. One can take the boundary limit and obtain the CFT operators in terms of those modes. If the bulk Hamiltonian is free, bulk time evolution evolves the modes at $\tau = -(L\pi/2)$ to antipodally flipped ones at $\tau = (L\pi/2)$, so that the commutator of past and future modes is the standard equal time commutation relation. However, bulk interactions lead to corrections. The resulting evolution operator can still be perturbatively corrected as long as the bulk interactions obey microcausality [83]. Such bulk interactions result in anomalous dimensions for the boundary operators $O(\tau, \Omega)$ dual to the bulk field, but nevertheless

$$\left[O\left(\tau = +\frac{L\pi}{2}, \Omega_{1A}\right), O\left(\tau = -\frac{L\pi}{2}, \Omega_2\right) \right] \quad (16)$$

will be proportional to (8). From this perspective, the nonlocality of Carrollian correlators arises simply because future operators are related to the past ones by evolution by the Arnowitt-Deser-Misner Hamiltonian of global AdS.

A modified dictionary—The dictionary suggested by (5) generically implies that the codimension-one boundary description of S matrices is nonlocal. Intuitively, this is expected from the fact that the degrees of freedom that encode scattering are, respectively, supported in $\mathcal{I}^-$ and $\mathcal{I}^+$ with antipodal matching conditions across i^0 [84]. Equation (10) embodies this simple fact. Under the current map (5), the out and in operators become Carrollian fields at $\mathcal{I}^+$ and $\mathcal{I}^-$, respectively. Here, we postulate another dictionary that maps out and in operators to a single Carrollian field [85]. However, this is not enough to cure the nonlocality uncovered above in (10).

Consider the boundary field built from both out and in oscillators,

$$\Psi(u, z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} [a_{\text{out}}(\omega, z, \bar{z}) e^{-i\omega u} - a_{\text{in}}^\dagger(\omega, z, \bar{z}) e^{i\omega u}]. \quad (17)$$

Using the relation $a_{\text{out}}(\mathbf{p}) = \hat{S}^\dagger a_{\text{in}}(\mathbf{p}) \hat{S}$, where $\hat{S}$ is the S matrix built from the interacting Hamiltonian, (17) [(and (18)] reduces to (3) for a free theory. In an interacting theory, however, the boundary field $\Psi(u, z, \bar{z})$ encodes both the initial state (through a_{in}) and the full evolution (through $\hat{S}$). An analogous construction at $\mathcal{I}^-$ is

$$\tilde{\Psi}(v, z, \bar{z}) = \int_0^\infty d\omega \omega^{\Delta-1} [a_{\text{in}}(\omega, z, \bar{z}) e^{-i\omega v} - \hat{S}^\dagger a_{\text{in}}^\dagger(\omega, z, \bar{z}) \hat{S} e^{i\omega v}]. \quad (18)$$

Antipodal matching, $\Psi(u, z, \bar{z})|_{\mathcal{I}^+} = \tilde{\Psi}(v, z, \bar{z})|_{\mathcal{I}^-}$, would be a nontrivial requirement in the interacting theory, since Ψ depends on the full history via $\hat{S}$.

A transparent diagnostic of nonlocality is

$$\begin{aligned} & [\Psi(u_1, z_1, \bar{z}_1), \Psi(u_2, z_2, \bar{z}_2)] \\ &= \int d\omega_1 d\omega_2 \left(e^{-i\omega_1 u_1 + i\omega_2 u_2} [a_{\text{out}}(\omega_1, z_1, \bar{z}_1), a_{\text{in}}^\dagger(\omega_2, z_2, \bar{z}_2)] \right. \\ & \quad \left. - e^{i\omega_1 u_1 - i\omega_2 u_2} [a_{\text{out}}(\omega_2, z_2, \bar{z}_2), a_{\text{in}}^\dagger(\omega_1, z_1, \bar{z}_1)] \right). \end{aligned} \quad (19)$$

Using (8), we find

$$\begin{aligned}
 [\Psi(u_1, z_1, \bar{z}_1), \Psi(u_2, z_2, \bar{z}_2)] &= (2\pi)^3 (-i\pi \text{sgn}(u_{12})) \delta^2(z_{12}) \\
 &+ i\lambda \int d\omega_1 d\omega_2 d^4x \phi_I(x) (e^{i(p_1-p_2)\cdot x} e^{-i\omega_1 u_1 + i\omega_2 u_2} - e^{i(p_2-p_1)\cdot x} e^{i\omega_1 u_1 - i\omega_2 u_2}) \\
 &+ \frac{\lambda^2}{12} \int d\omega_1 d\omega_2 d^4x_1 d^4x_2 \theta(x_1^0 - x_2^0) \left(-2(e^{i(p_1-p_2)\cdot x_1} e^{-i\omega_1 u_1 + i\omega_2 u_2} - e^{i(p_2-p_1)\cdot x_1} e^{i\omega_1 u_1 - i\omega_2 u_2}) \right. \\
 &\times [\phi_I(x_1), \phi_I^3(x_2)] + 6(e^{ip_1\cdot x_1 - ip_2\cdot x_2} e^{-i\omega_1 u_1 + i\omega_2 u_2} - e^{ip_2\cdot x_1 - ip_1\cdot x_2} e^{i\omega_1 u_1 - i\omega_2 u_2}) [\phi_I^2(x_1), \phi_I^2(x_2)] \Big),
 \end{aligned} \tag{20}$$

where we have used $u_{12} \equiv u_1 - u_2$ and $z_{12} \equiv z_1 - z_2$ for brevity [86]. Although the $\mathcal{O}(\lambda^2)$ term involves time ordering via $\theta(x_1^0 - x_2^0)$, the spatial integrations extend over all space. The resulting operator insertions are, therefore, nonlocal, producing precisely the kind of correlations indicated by (10). In particular, the commutator does not vanish outside the Carrollian light cone.

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- [69] Appendix A in [41] contains the map needed to work entirely at $\mathcal{I}^+$.

- [70] We follow the standard notation where boldface symbols of the form $\mathbf{p}_i$ are used to denote vectors in 3D space, whereas four momenta are denoted by p_i .
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