

Longitudinal Short-Distance Constraints on Hadronic Light-by-Light Scattering and Tensor-Meson Contributions to the Muon $g - 2$

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(Received 21 March 2025; revised 23 May 2025; accepted 21 July 2025; published 28 August 2025)

Short-distance constraints from the operator product expansion in QCD play an important role in the evaluation of the hadronic light-by-light scattering contribution to the anomalous magnetic moment of the muon. Holographic QCD has been shown to naturally incorporate the Melnikov-Vainshtein constraint on the longitudinal amplitude following from the triangle anomaly in the asymmetric limit, where one photon virtuality remains small compared to the others. This is saturated by an infinite tower of axial vector mesons, and their numerical contribution to the muon $g - 2$ in AdS/QCD models agrees rather well with a recent dispersive analysis. However, in AdS/QCD, the longitudinal short-distance constraint where all virtualities are large turns out to be matched only at the level of 81%. In this Letter we show that tensor mesons, whose contribution to the muon $g - 2$ has recently been found to be underestimated, can fill this gap because in holographic QCD their infinite tower of excited mesons only contributes to the symmetric longitudinal short-distance constraint. Numerically, they give rise to a sizeable positive contribution from the low-energy region below 1.5 GeV, a small one from the mixed region, and a negligible one from the high-energy region, which could explain the remaining gap between the most recent dispersive and lattice results for the complete hadronic light-by-light contribution.

DOI: [10.1103/dxwr-gpsl](https://doi.org/10.1103/dxwr-gpsl)

Introduction—In its upcoming final result for the anomalous magnetic moment of the muon $a_\mu = (g - 2)_\mu/2$, the Fermilab experiment [1,2] is expected to reduce the current experimental error from 22×10^{-11} to about half of this size, challenging the standard model prediction [3], which needs further scrutiny and improvements regarding its main source of uncertainties to match this precision. At present, the main uncertainty arises from discrepancies in the contribution from hadronic vacuum polarization [4], for which new lattice QCD results [5–12] hold the promise of a more precise evaluation while also removing most of the deviation between the experimental result and the standard model value of the Muon $g - 2$ Theory Initiative from 2020 [3]. The latter involves as the second most important source of uncertainty the effects of hadronic light-by-light scattering (HLbL), at the time determined as $a_\mu^{\text{HLbL}} = 92(19) \times 10^{-11}$ [3,13–27], where the error needs to be reduced substantially as well in view of the expected experimental improvements for a_μ .

A new data-driven dispersive analysis [28,29] has achieved a significant reduction of previous uncertainties related to the contribution of axial vector mesons and short-distance constraints (SDCs), agreeing quite well with results obtained with holographic models of QCD that have indicated larger contributions than previously assumed [30–36], as also found with resonance models [37] and Dyson-Schwinger approximations [38].

Previous results [24] for the contribution of tensor mesons amounted to the almost negligibly small value of $+0.64(13) \times 10^{-11}$ for the ground state tensor multiplet with a quark model ansatz for the tensor transition form factors (TFFs), but Refs. [28,29] found that this needs to be revised to a 4 times larger and negative value when a formalism free of kinematical singularities introduced recently in [39] is employed. However, as a full dispersive treatment of tensor meson contributions is not yet feasible, the same simple quark model, though with a different scale parameter, has been employed.

In this Letter we discuss tensor meson contributions to HLbL in holographic QCD (hQCD) models whose predictions for the axial-vector contributions have been validated numerically to a remarkable degree by Refs. [28,29], and we show that the infinite tower of tensor mesons plays an important role in the longitudinal short-distance constraint (LSDC) together with axial-vector

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mesons. In the holographic approach, the infinite towers of the latter account fully for the Melnikov-Vainshtein (MV) SDC (MV-SDC) [13] on the longitudinal amplitude, but universally contribute only 81% of the symmetric LSDC where all photon virtualities are large.

Fitting the tensor coupling such that the symmetric LSDC is saturated in combination with the axial-vector mesons leads to a two-photon coupling of the ground-state tensor modes in remarkable agreement with available data from the BELLE experiment [40] for the singly virtual TFF. However, in contrast to the simple quark model employed in Refs. [28,29], it predicts a positive contribution to a_μ . Moreover, the combined contribution of the full tensor tower leads to a surprisingly large result, $a_\mu^T \sim +11 \times 10^{-11}$, which could explain the remaining difference between the new dispersive result of [28,29] and current lattice results for the complete HLbL contribution of Refs. [41–44].

HLbL tensor and $(g-2)_\mu$ —The central object in the calculation of the HLbL contribution to $(g-2)_\mu$ is the HLbL tensor, i.e., the hadronic four-point function of electromagnetic currents ($q_4 = q_1 + q_2 + q_3$),

$$\begin{aligned} \Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = & -i \int d^4x d^4y d^4z e^{-i(q_1 \cdot x + q_2 \cdot y + q_3 \cdot z)} \\ & \times \langle 0 | T \{ j_{\text{em}}^\mu(x) j_{\text{em}}^\nu(y) j_{\text{em}}^\lambda(z) j_{\text{em}}^\sigma(0) \} | 0 \rangle, \end{aligned} \quad (1)$$

where

$$j_{\text{em}}^\mu = \bar{q} Q \gamma^\mu q, \quad Q = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right), \quad (2)$$

with $N_f = 3$ quark fields $q = (u, d, s)^T$.

A very useful representation of the HLbL tensor can be obtained using the Bardeen, Tung [45], and Tarrach [46] decomposition

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i, \quad (3)$$

with scalar functions Π_i depending on the virtualities q_i^2 , and Lorentz structures $T_i^{\mu\nu\lambda\sigma}$ defined in [47]. This decomposition is manifestly gauge-invariant and satisfies crossing symmetries.

The contribution to $(g-2)_\mu$ is then given by the master equation [16]

$$\begin{aligned} a_\mu^{\text{HLbL}} = & \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \\ & \times \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, Q_3), \end{aligned} \quad (4)$$

where 12 hadronic scalar functions $\bar{\Pi}_i$ built from the Π_i in (3) are evaluated for the reduced kinematics

$$(q_1^2, q_2^2, q_3^2, q_4^2) = (-Q_1^2, -Q_2^2, -Q_1^2 - 2Q_1 Q_2 \tau - Q_2^2, 0). \quad (5)$$

Isolating single resonance exchange contributions to the $\bar{\Pi}_i$ in terms of their TFFs is a nontrivial task, which has recently been extended to spin-1 and spin-2 resonances using a new optimized basis [39]. In the case of tensor meson exchanges, the absence of kinematic singularities is however given only when the tensor TFFs have certain simplifications. (Nevertheless, in triangle kinematics, the problem this poses for the dispersive approach can be avoided in the strategy developed in [48,49].)

A particular problem for hadronic models is the fact that TFFs of meson resonances have a power-law decay at large virtualities [50] that makes it impossible to satisfy the SDCs for the HLbL tensor following from the operator product expansion (OPE) of perturbative QCD by a finite number of resonances. Especially important is the SDC derived for the longitudinal amplitude $\bar{\Pi}_1$ by Melnikov and Vainshtein [13] from the triangle anomaly in combination with the OPE when two virtualities are much larger than the third,

$$C_{\text{MV}} = \lim_{Q_3 \rightarrow \infty} \lim_{Q \rightarrow \infty} Q^2 Q_3^2 \bar{\Pi}_1(Q, Q, Q_3) = -\frac{2}{3\pi^2}, \quad (6)$$

while the symmetric limit gives

$$C_{\text{sym}} = \lim_{Q \rightarrow \infty} Q^4 \bar{\Pi}_1(Q, Q, Q) = -\frac{4}{9\pi^2}, \quad (7)$$

as proved (and extended) in the subsequent analyses of Refs. [21,51,52].

Holographic QCD—Hadronic models in hQCD [53–57] are inspired by the original AdS/CFT duality between a four-dimensional (4D) conformal large- N_c gauge theory at strong coupling and a classical five-dimensional (5D) field theory in a curved gravitational background with anti-de Sitter (AdS) metric [58–60]: for every quantum operator $\mathcal{O}(x)$ of the 4D (strongly coupled) gauge theory, there exists a corresponding 5D field $\phi(x, z)$ living in an asymptotically AdS space, whose value on the conformal boundary (taken at $z = \epsilon \rightarrow 0$), $\phi(x, 0) \equiv \phi_0(x)$, is identified (modulo some specific powers of ϵ) with the four-dimensional source of $\mathcal{O}(x)$. In this way correlation functions of the 4D operators can be obtained by functional derivatives of the 5D Lagrangian with respect to the values of the 5D fields on the conformal boundary.

Correlation functions of (conserved) $\text{SU}(3)_L \times \text{SU}(3)_R$ [or $\text{U}(3)_L \times \text{U}(3)_R$] flavor chiral currents of QCD, in the large- N_c limit, are obtained from a 5D Yang-Mills action with coupling constant g_5 plus a Chern-Simons term in a curved 5D AdS_5 space. In the simplest case that we shall

consider here, in so-called hard-wall (HW) models, the extra fifth (holographic) dimension extends over the finite interval $z \in (0, z_0]$,

$$S_{\text{YM}} = -\frac{1}{4g_5^2} \int d^4x \int_0^{z_0} dz \sqrt{-g} g^{PR} g^{QS} \times \text{tr}(\mathcal{F}_{PQ}^L \mathcal{F}_{RS}^L + \mathcal{F}_{PQ}^R \mathcal{F}_{RS}^R), \quad (8)$$

where $P, Q, R, S = 0, \dots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$, $\mathcal{B}_N = L_N, R_N$ being 5D gauge fields transforming under $U(3)_{L,R}$ respectively; vector and axial-vector fields are given by $V_\mu = \frac{1}{2}(L_\mu + R_\mu)$ and $A_\mu = \frac{1}{2}(L_\mu - R_\mu)$.

Interactions with photons involve the vector bulk-to-boundary propagator $\mathcal{J}$, which is a solution of the vector 5D equation of motion for generic value of the 4D momentum q^2 satisfying the boundary conditions $\mathcal{J}(q, 0) = 1$ and $\partial_z \mathcal{J}(q, z_0) = 0$. For Euclidean momentum $q^2 = -Q^2$, it is given in HW models by

$$\mathcal{J}(Q, z) = Qz \left[K_1(Qz) + \frac{K_0(Qz_0)}{I_0(Qz_0)} I_1(Qz) \right], \quad (9)$$

where I and K denote modified Bessel functions.

According to the holographic recipe, the 4D vector current two-point function follows from the asymptotic behavior of (9), and matching it to the leading-order perturbative QCD result for $Q^2 \rightarrow \infty$ [54],

$$\Pi_V(Q^2) = -\frac{1}{g_5^2 Q^2} \left(\frac{1}{z} \partial_z \mathcal{J}(Q, z) \right) \Big|_{z \rightarrow 0} \sim -\frac{N_c}{24\pi^2} \ln Q^2, \quad (10)$$

determines $g_5^2 = 12\pi^2/N_c = (2\pi)^2$ for $N_c = 3$.

Single resonances appear as the coefficients of the expansion of the 5D fields in terms of a series of normalizable eigenfunctions. In the case of the vector field the masses are given by $m_n = \gamma_{0,n}/z_0$, where $\gamma_{0,n}$ is the n th zero of the Bessel function $J_0(\gamma_{0,n}) = 0$. Identifying the mass of the lowest vector resonance with the mass of the ρ meson $m_1 = \gamma_{0,1}/z_0 = M_\rho = 775.26 \text{ MeV}$ fixes $z_0 = \gamma_{0,1}/m_1 = 3.102 \text{ GeV}^{-1}$.

A rich dynamics is contained in the sector of axial gauge fields, which yields a TFF for axial-vector mesons [30] in agreement with the Landau-Yang theorem [61,62] and with the (subsequently derived) doubly virtual asymptotic behavior deduced from QCD light-cone expansions [50]. Summing over the infinite tower of axial-vector mesons one finds that the MV-SDC is realized in the form [30–32]

$$C_{\text{MV}}^A = -\left(\frac{g_5}{2\pi}\right)^2 \frac{1}{\pi^2} \int_0^\infty d\xi \xi [\xi K_1(\xi)]^2 = -\frac{2}{3\pi^2} \left(\frac{g_5}{2\pi}\right)^2. \quad (11)$$

The symmetric LSDC, on the other hand, is reproduced as far as the power-law behavior is concerned, but the coefficient therein is 19% too small,

$$C_{\text{sym}}^A = -\left(\frac{g_5}{2\pi}\right)^2 \frac{1}{\pi^2} \int_0^\infty d\xi \xi [\xi K_1(\xi)]^3 = -0.8122 \times \frac{4}{9\pi^2} \left(\frac{g_5}{2\pi}\right)^2. \quad (12)$$

Tensor mesons in hQCD—Following [35,63,64], the tensor meson is introduced in the model as a deformation of the 4D part of the AdS metric

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu - \frac{1}{z^2} dz^2, \quad (13)$$

whose dynamics is governed by the 5D Einstein-Hilbert action

$$S_{\text{EH}} = -2k_T \int d^5x \sqrt{g} (\mathcal{R} + 2\Lambda), \quad (14)$$

with k_T determining the strength of the coupling to other fields.

Tensor mesons arise as solutions for traceless-transverse metric fluctuation $h_{\mu\nu} = \epsilon_{\mu\nu}^T(q) h_n(z)$ with $h_n(0) = 0 = h'_n(z_0)$. Requiring canonical normalization $k_T \int dz z^{-3} (\partial_z h_n)^2 = (m_n^T)^2$, they are given explicitly by

$$h_n(z) = \frac{\sqrt{2/k_T}}{z_0 J_2(\gamma_{1,n})} z^2 J_2(\gamma_{1,n} z / z_0), \quad (15)$$

yielding an infinite tower of (flavor singlet) tensor meson modes with masses fixed by the zeros of the Bessel function J_1 ,

$$m_n^T / M_\rho = \gamma_{1,n} / \gamma_{0,1} = 1.593, 2.917, 4.230, \dots \quad (16)$$

The lowest tensor resonance has thus a mass $m_1^T = 1.235 \text{ GeV}$, which is only 3% below the physical value $M_{f_2} = 1.2754 \text{ GeV}$ of $f_2(1270)$. (To distinguish physical and model parameters, we use uppercase M for experimental values of masses and lower-case m for corresponding mass parameters given by the hQCD model.)

A single tensor mode gives rise to the $T \rightarrow \gamma^* \gamma^*$ amplitude

$$\mathcal{M}^{\mu\nu\alpha\beta} = T_1^{\mu\nu\alpha\beta} \frac{1}{m_T} \mathcal{F}_1 + T_3^{\mu\nu\alpha\beta} \frac{1}{m_T^3} \mathcal{F}_3, \quad (17)$$

with structure functions (following the notation of [50])

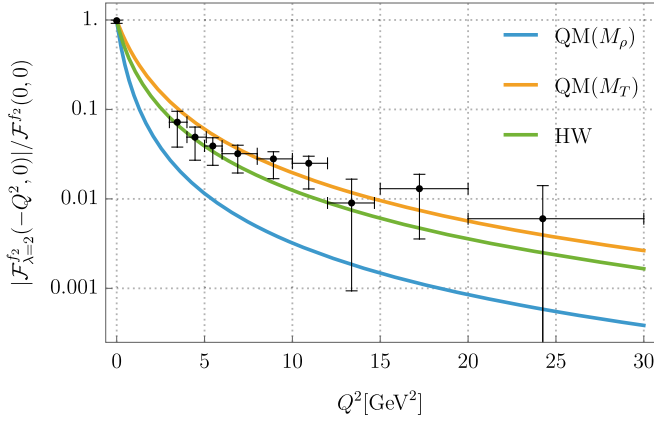


FIG. 1. Comparison of singly virtual tensor TFFs for helicity $\lambda = 2$ with Belle data [40] for the $f_2(1270)$, normalized by $\mathcal{F}^{f_2}(0, 0) \equiv \sqrt{\mathcal{F}_1^T(0, 0)^2 + \mathcal{F}_2^T(0, 0)^2/24} = \sqrt{5\Gamma_{\gamma\gamma}/(\pi\alpha^2 M_T)}$. HW is the hard-wall hQCD result, $\text{QM}(\Lambda_T)$ refers to the quark model of Ref. [65], with scale parameter Λ_T set to either M_ρ [28] or $M_T = M_{f_2}$ [50,65]. (Figure taken from [66], which also shows the comparison for the remaining helicities.).

$$\begin{aligned} \mathcal{F}_1^T(-Q_1^2, -Q_2^2)/m_T \\ = -\frac{1}{g_5^2} \text{tr} Q^2 \int \frac{dz}{z} h_n(z) \mathcal{J}(z, Q_1) \mathcal{J}(z, Q_2), \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{F}_3^T(-Q_1^2, -Q_2^2)/m_T^3 \\ = -\frac{1}{g_5^2} \text{tr} Q^2 \int \frac{dz}{z} h_n(z) \frac{\partial_z \mathcal{J}(z, Q_1)}{Q_1^2} \frac{\partial_z \mathcal{J}(z, Q_2)}{Q_2^2}. \end{aligned} \quad (19)$$

Evaluating this for the lowest tensor mode in the singly virtual case where experimental data are available [40] for the $f_2(1270)$ tensor meson, one obtains a remarkably good agreement as shown in Fig. 1, where it is compared with the quark model (QM) ansatz [65] with dipole scale Λ_T set to either M_ρ as in [28] or M_T as in [50,65].

Summing over the infinite tower of tensor modes corresponds to a Witten diagram with bulk-to-bulk tensor propagator involving

$$G(z, z'; q^2) = \sum_0^\infty \frac{h_n(z) h_n(z')}{q^2 - (m_n^T)^2}, \quad (20)$$

which at $q^2 = 0$ has the simple form

$$G(z, z'; 0) = -\frac{1}{4} \min(z^4, z'^4). \quad (21)$$

For the longitudinal amplitude this yields

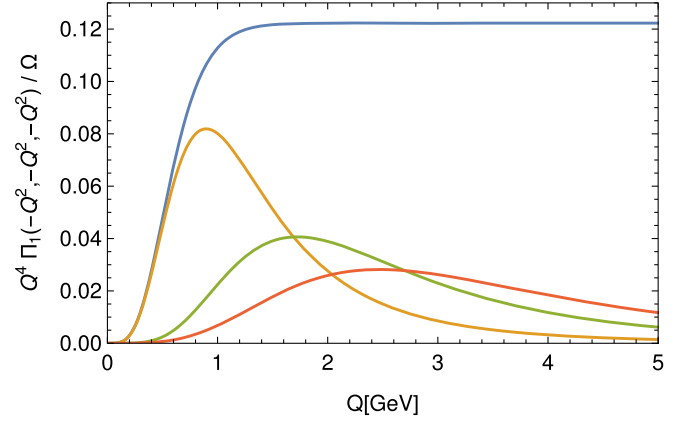


FIG. 2. Contribution of tensor mesons to the symmetric longitudinal SDC: $Q^4 \hat{\Pi}_1(-Q^2, -Q^2, -Q^2)$ with full tensor bulk-to-bulk propagator (blue) and the contributions from the first three modes (orange, green, red), normalized to the full symmetric LSDC value $\Omega = -4/(9\pi^2)$ with k_T fixed by (24). By contrast, adding tensor contributions to Fig. 4 in [30], which shows how the tower of axial-vector mesons saturates the MV-SDC in the asymmetric limit $Q_3 \ll Q_1 \sim Q_2 \rightarrow \infty$, those contributions would not be discernible as different from zero (cf. Ref. [66].).

$$\begin{aligned} \bar{\Pi}_1(Q, Q, Q_3) = & -\frac{4}{k_T} \left(\frac{\text{tr} Q^2}{g_5^2} \right)^2 \int_0^{z_0} \frac{dz dz'}{z z'} \\ & \times \mathcal{J}(z, Q) \mathcal{J}(z', Q) \\ & \times \frac{\partial_{z'} \mathcal{J}(z', Q_3)}{Q_3^2} \partial_{z'} G(z, z'; 0). \end{aligned} \quad (22)$$

In [66] we have given the results for the individual tensor contributions to $\bar{\Pi}_i$, evaluated alternatively with only the pole contribution as defined by the dispersive approach [39] and without this restriction, which differ strongly. Summing over the whole tower of tensor modes leads to exactly the same result (22), whereas in the case of axial-vector mesons there is equivalence between the two possibilities already at the level of individual modes.

In the limit $Q_3 \ll Q \rightarrow \infty$ the result (22) decays faster than $1/Q^2$, giving no contribution to the MV-SDC. But in the symmetric limit $Q_3 = Q \rightarrow \infty$ one obtains (cf. Fig. 2)

$$\begin{aligned} \bar{\Pi}_1(Q, Q, Q) \rightarrow & \frac{4}{Q^4} \frac{1}{k_T} \left(\frac{\text{tr} Q^2}{g_5^2} \right)^2 \int_0^\infty d\xi K_1(\xi) \int_0^\xi d\xi' \xi'^3 \\ & \times K_1(\xi') \partial_{\xi'} [\xi' K_1(\xi')], \\ = & \frac{4}{Q^4} \frac{1}{k_T} \left(\frac{\text{tr} Q^2}{g_5^2} \right)^2 \times (-0.15285). \end{aligned} \quad (23)$$

This is 12.23% of the OPE result for $N_c = N_f = 3$ when k_T is matched through the energy-momentum tensor two-point function as done originally in [63], i.e.,

$$k_T \equiv \frac{1}{g_f^2} = \left(\frac{N_c N_f}{80\pi^2} + \frac{N_c^2 - 1}{40\pi^2} \right) = \frac{5}{16\pi^2}. \quad (24)$$

Together with the contribution from the axial-vector mesons, the symmetric SDC is matched at the level of $81.22 + 12.23 = 93.45\%$, and this could be increased to 98.2% if (24) is reduced to the term proportional to N_c^2 that would be selected in a large- N_c expansion.

However, this nearly perfect match is somewhat coincidental, since it does not reproduce the correct large- N_c scaling. The OPE result is proportional to $N_c \text{tr} Q^4$, whereas the above result involves $N_c^0 (\text{tr} Q^2)^2$ because $g_5^2 = 12\pi^2/N_c$. The correct behavior could be obtained by a multiplet of tensor contributions with $k_T \propto N_c$ as provided by the quark part of the energy-momentum tensor, contributing proportional to the sum $\sum_{a=0,3,8} (\text{tr} Q^2 T^a)^2 = \frac{1}{2} \text{tr} Q^4$. Instead of fixing k_T by (24), one could then determine it by matching the combined contribution of axial-vector and tensor meson contributions to the symmetric LSDC. With degenerate tensor mesons as given by the AdS/QCD model, the result is simply an extra overall factor compared to the above result. For $g_5^2 = (2\pi)^2$ as given by the OPE of the vector correlator, the tensor contribution to the HLbL amplitude and thus to a_μ needed to be enhanced by a factor 1.536; if one chooses a reduced $g_5^2 = 0.894(2\pi)^2$ by fitting F_ρ as proposed in [34], the contribution from tensor mesons is enhanced by a factor $1/0.894^2$ relative to the axial-vector contributions so that another factor of 1.097 suffices to have the correct ratio between symmetric and MV-SDC. In these two scenarios, the contributions from the tensor mesons to a_μ would have to be enhanced simply by a factor 1.536 and 1.373, respectively, compared to the initial choice (24) together with $g_5^2 = (2\pi)^2$. (This also mitigates somewhat the numerical deficit that was obtained in [63] for the ratio $\Gamma(f_2 \rightarrow \pi\pi)/\Gamma(f_2 \rightarrow \gamma\gamma)$ [66].)

In fact, matching the lowest tensor meson pole contribution in the AdS/QCD model with (24) to mass and two-photon decay rates of the experimentally observed $f_2(1270), a_2(1320), f_2'(1525)$ one obtains also a result that is about a factor 1.5 higher. The two options for fitting g_5^2 , OPE fit and F_ρ fit, and then fixing k_T through the OPE ratio of the symmetric and asymmetric LSDC give a result that tightly brackets the one obtained by matching the experimental data for f_2, a_2, f_2' ; see Table I. Here, more than 90% of the tensor contributions to a_μ are from the low-energy region $Q_i \leq Q_0 = 1.5$ GeV, while contributions from all $Q_i > Q_0$ are below 10^{-12} . (For more details, see [66].)

Summing over the infinite tower of excited tensor modes by using the full bulk-to-bulk propagator still gives tiny (a few 10^{-12}) contributions from the high-energy region $Q_i > Q_0$, but rather large contributions in the IR region

$$\begin{aligned} a_\mu^{T,\text{total}} \times 10^{11} &= \begin{cases} 11.1 & (F_\rho\text{-fit}) \\ 12.4 & (\text{OPE-fit}) \end{cases}, \\ a_\mu^{T,\text{total}}|_{\text{Mixed}} \times 10^{11} &= \begin{cases} 1.9 & (F_\rho\text{-fit}) \\ 2.1 & (\text{OPE-fit}) \end{cases}, \\ a_\mu^{T,\text{total}}|_{\text{IR}} \times 10^{11} &= \begin{cases} 8.5 & (F_\rho\text{-fit}) \\ 9.5 & (\text{OPE-fit}) \end{cases}. \end{aligned} \quad (25)$$

In line with the “best-guess” model for the hQCD axial-sector contributions of [34] we would adopt the F_ρ -fit case as central value, the OPE-fit case as upper, and the original parameter choice of [63] as lower values for a theoretical error estimate within hQCD,

$$\begin{aligned} a_\mu^{T,\text{total}} &= 11.1_{-3.0}^{+1.3} \times 10^{-11}, \\ a_\mu^{T,\text{total}}|_{\text{IR}} &= 8.5_{-2.3}^{+1.0} \times 10^{-11}. \end{aligned} \quad (26)$$

A contribution in the IR region of this size from tensor mesons would move the central value for $a_\mu^{\text{HLbL},\text{total}}$

TABLE I. a_μ results for the tensor meson pole contributions obtained by inserting the hQCD results for $\mathcal{F}_{1,3}^T$ in the formulas obtained in [39] within the dispersive approach, when experimental values of masses and two-photon decay rates are matched (upper four lines), compared to the results in the AdS/QCD model when k_T is fixed by the symmetric SDC with g_5 alternatively determined by a fit of F_ρ or the OPE of the vector correlator. The columns IR and Mixed give the contributions from the low-energy (IR) region defined by $Q_i \leq Q_0 = 1.5$ GeV and from the region where one or two virtualities are above Q_0 . (Contributions from the region where $Q_i > Q_0$ for all Q_i are below 10^{-13} .) In the last two lines $\Gamma_{\gamma\gamma}$ for the (degenerate) AdS/QCD result with $m_1^T = 1.235$ GeV has been split as $(69 + 25 + 6)\%$ corresponding to ideal mixing for $f_2 + a_2 + f_2'$.

k_T	M_T [GeV]	$\Gamma_{\gamma\gamma}$ [keV]	IR	Mixed	$a_\mu [10^{-11}]$
By $\Gamma_{\gamma\gamma}$	1.2754(8)	2.65(45)	2.28	0.16	2.4(4)
	1.3182(6)	1.01(9)	0.85	0.05	0.9(1)
	1.5173(24)	0.08(2)	0.06	0.003	0.06(2)
	$f_2 + a_2 + f_2'$		3.19	0.21	3.4(4)
F_ρ fit	1.235	$2.3 + 0.8 + 0.2$	2.93	0.23	3.17
OPE fit	1.235	$2.6 + 0.9 + 0.2$	3.28	0.25	3.55

obtained therein in a dispersive approach from [67] $101.9(7.9)$ to about 113×10^{-11} . This would then agree perfectly with the recent lattice results of $109.6(15.9)$, $124.7(14.9)$, and $125.5(11.6) \times 10^{-11}$ of the Mainz group [41,42], the RBC-UKQCD group [43], and the BMWc lattice group [44], respectively.

As shown in [66], the sizable *positive* contribution from tensor mesons is a consequence of the presence of the structure function $\mathcal{F}_3^T$, which is not constrained by existing data on singly virtual TFFs for tensor mesons. Dropping it would give even larger but negative results for the tensor contributions, but within the holographic approach this cannot be done without violating the symmetries of AdS; moreover, because of chiral symmetry, dropping the corresponding term in the vector part of the action would require one to drop also the same kind of term for axial vectors, completely losing the coupling between tensor mesons and pions in the Hirn-Sanz [56] model.

In [50], employing a light-cone expansion generalizing the Brodsky-Lepage approach [68,69], it was found that $\mathcal{F}_{2,4,5}^T$ are asymptotically of comparable magnitude as $\mathcal{F}_3^T$, while they do not arise in the present hQCD model. Moreover, the asymptotic asymmetry functions obtained for $\mathcal{F}_{1,3}^T$ differ [66], while there is agreement between the holographic results and the results of [50] in the case of axial-vector mesons [30]. This calls for further studies, in particular regarding the remaining SDCs [51,52] on the HLbL amplitude.

However, the significant contributions obtained for the tensor-meson contributions to a_μ^{HLbL} in hQCD arise primarily from the low-energy region. As shown in Appendix A of [66], the two structure functions $\mathcal{F}_{1,3}^T$ are directly related to the two structures of kinetic and mass terms of vector meson fields interacting with tensor fields, making it physically plausible that both contribute importantly at low energies. In the case of $\mathcal{F}_3^T$, this prediction of hQCD can be tested in the doubly virtual $T \rightarrow \gamma^* \gamma^*$ processes, for which experimental data will hopefully become available in the future. Already now it is clear from our hQCD results that there is room for further sizeable contributions in the dispersive approach of [28,29] coming from the tensor meson sector, which will require additional experimental input for a conclusive treatment.

Acknowledgments—We would like to thank Martin Hoferichter for very useful discussions. This work has been supported by the Austrian Science Fund (FWF) [grant DOI: 10.55776/PAT7221623]. L.C. acknowledges the support of the INFN research project ENP (Exploring New Physics).

Data availability—The data that support the findings of this Letter are not publicly available upon publication because it is not technically feasible and/or the cost of

preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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