

Kinematic Flow and the Emergence of Time

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Perhaps the most basic question we can ask about cosmological correlations is how their strength changes as we smoothly vary kinematic parameters. The answer is encoded in differential equations that govern this evolution in kinematic space. In this Letter, we introduce a new perspective on these differential equations. We show that, in the simplified setting of conformally coupled scalars in power-law Friedmann-Robertson-Walker spacetimes, the equations for arbitrary tree-level processes can be obtained from a small number of simple combinatorial rules. While this “kinematic flow” is defined purely in terms of boundary data, it reflects the physics of bulk time evolution. The unexpected regularity of the equations suggests the existence of an autonomously defined mathematical structure from which cosmological correlations and the time evolution of the associated spacetime emerge.

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Cosmology gives us the strongest motivations to replace the seemingly foundational notion of spacetime with deeper, and likely more abstract, principles. From the breakdown of Einstein gravity coupled to matter near the big bang singularity, to the profound mysteries associated with the accelerated expansion of our Universe, it is cosmology that most powerfully calls for an overhaul of our description of the laws of nature, jettisoning the phrase “time evolution” from the vocabulary of fundamental physics at the deepest level and, instead, seeing the concept of time emerge as an extremely useful approximation when this becomes possible.

It is also suggestive that our actual observations of the Universe are static, correlating measurements at different positions in the late Universe. We introduce a cosmological history to give a rational accounting of these spatial patterns, but ultimately the notion of cosmological time is an auxiliary one, not present in the observables themselves. There should therefore be a description of the same

physics that focuses only on the observed static correlations and in which time evolution becomes a derived concept.

Another motivation for reformulating cosmological perturbation theory in a way that dispenses with explicit evolution in time is that the standard time-centered computations are very complex. This complexity can be traced to the time integrals over interactions in the quantum-mechanical sum over histories, and it likely conceals fascinating new physical and mathematical structures underlying cosmological correlators.

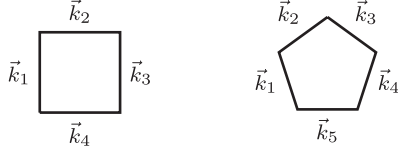
For several years, these developments have been calling for the formulation of a radically new, timeless approach to cosmological correlators. While some partial results in this direction have been found [1–19]—notably the cosmological polytopes associated with a new combinatorial and geometric understanding of the flat-space wave function [20–22]—we have not yet seen a single example of a completely autonomous set of ideas for determining the cosmological wave function, purely in terms of the kinematic data specifying the observable itself, capturing the dynamics of bulk time evolution in purely boundary terms.

Our goal in this Letter is to change this state of affairs. We will study an evolution in the space of kinematic configurations—called kinematic flow—that replaces the ordinary notion of cosmological time evolution. Remarkably, this flow can be defined autonomously in the space of boundary kinematics, and a few simple rules

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allow us to predict the differential equations for arbitrary tree graphs (at least in a toy model of cosmological evolution).

Boundary kinematics—We describe cosmological fluctuations in Fourier space, where the kinematic data are a set of wave vectors (momenta) $\vec{k}_a$, which form a closed polygon as a consequence of momentum conservation,



Correlators are then functions of the lengths of the sides and diagonals of these kinematic polygons.

Certain kinematic configurations are special. First, we can analytically continue the side lengths, so that the perimeters of some subpolygons (including the full polygon) vanish. As we will see, cosmological correlators have singularities at these locations [2,20,23]. It will be illuminating to represent these energy singularities graphically by shading the relevant subpolygons, such as

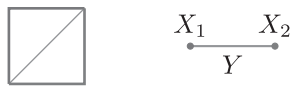
$$\square = k_{1234}, \quad \square_{\text{diag}} = k_{12} + s, \quad (1)$$

where $k_{n_1 \dots n_j} \equiv |\vec{k}_{n_1}| + \dots + |\vec{k}_{n_j}|$ and $s \equiv |\vec{k}_1 + \vec{k}_2|$. Correlators can, in principle, also have singularities when the sum of some external edges become collinear with a diagonal. We represent this by the following shading:

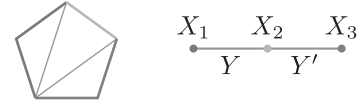
$$\square_{\text{diag}}^{\text{dashed}} = k_{12} - s, \quad (2)$$

where the dashed line indicates that the sign of the internal line is flipped when computing the perimeter. The absence of these singularities will be part of the boundary conditions for the differential equations we derive, along with the correct factorization of the physical object.

Graphs and tubings—It is useful to introduce a bit of extra structure in kinematic space. Polygons beg to be triangulated, and so we will pick a particular triangulation of the kinematic polygon. Any triangulation of a polygon has an associated dual graph that encodes the adjacency of triangles. For example, triangulations of the square correspond to graphs with two vertices



where $X_1 \equiv k_{12}$, $X_2 \equiv k_{34}$, and $Y \equiv |\vec{k}_1 + \vec{k}_2|$. Higher-point triangulations have similar representations, such as



where each vertex is associated with a triangle.

Above, we described how the singularities of correlators are related to the perimeters of subpolygons. This also has a useful representation on the dual graph. We first decorate the graph with markings (crosses on internal lines), and then draw connected tubes (circulings of the vertices and crosses) of these marked graphs. To each tubing, we then assign the sum of vertex energies enclosed by the tube and the energies of the internal lines piercing the tube. For tubes that intersect an internal line and enclose the corresponding cross, we flip the sign of the internal energy. For example, the singularities shown in (1) and (2) correspond to

$$\text{tube} \equiv \square, \quad \text{tube} \equiv \square_{\text{diag}}, \quad \text{tube} \equiv \square_{\text{diag}}^{\text{dashed}}.$$

These marked graphs and tubings will play an important role in the construction of differential equations.

Differential equations—The kinematic space of correlations can be expressed in terms of either a polygon or a marked graph. However, these are completely static objects, so how do they capture actual cosmological correlations? The most basic question we can ask is how the strength of correlations changes as we vary the external energies (i.e., the shape of the polygon) infinitesimally. This is a differential question, and so we should expect that the answer is captured by a differential equation. Remarkably, these differential equations can also be understood from a sort of flow through the space of graph tubings.

We denote the correlation function associated with a kinematic graph by ψ . (More properly, we will find that ψ can be simply related to “wave function coefficients”—see Supplemental Material [24]—but physically measurable correlation functions satisfy the same differential equations.) We then take derivatives of ψ with respect to the energies Z_I (which include both the external energies X_i and the exchange energies Y_j). Generically, this will generate new functions (and possibly the function ψ itself). Differentiating these functions produces additional new functions, and so on. *A priori* there is no reason that this procedure should ever terminate, but in our case [conformally coupled scalars in power-law Friedmann-Robertson-Walker (FRW) spacetime] it will. We therefore have a finite basis of functions S_a and can assemble them into a vector $\vec{I} \equiv (\psi, S_1, \dots, S_{n-1})^T$. Since the system is finite, it must satisfy a differential equation of the form

$$d\vec{I} = A\vec{I}, \quad (3)$$

where A is a matrix-valued 1-form and we have defined the total differential $d \equiv \sum_I \partial_{Z_I} dZ_I$.

The challenge now is to generate the connection matrix A . In the following, we will describe a combinatorial procedure that accomplishes this task in terms of a kinematic flow through the space of (marked) graph tubings. The output of this procedure will be a connection of the form

$$A = \sum_i \alpha_i d \log \Phi_i(Z), \quad (4)$$

where α_i are constant matrices and $\Phi_i(Z)$ are the letters of the differential equation. Notably, this equation is a sum of $d \log$ forms and thus is Abelian flat ($dA = 0$).

There are three ingredients to Eq. (3). First, we have to enumerate the possible letters that can appear in the connection matrix (4). Next, we have to determine how many functions appear in the vector $\vec{I}$ and organize them in some way. Finally, we need a way to derive the differential of $\vec{I}$ and hence obtain the coefficient matrices α_i . We will address each of these needs in turn.

Letters—We start by describing the possible singularities of the differential equations (i.e., its letters). They are naturally encoded by the same graph tubings that we introduced to label kinematic limits. For example, the letters (represented as $d \log$ forms) of the two-site chain are the following tubings of the marked graph:

$$\begin{aligned} \textcircled{\bullet} \text{---} \bullet &\equiv d \log(X_1 + Y), & \textcircled{\bullet} \text{---} \bullet &\equiv d \log(X_1 - Y), \\ \bullet \text{---} \textcircled{\bullet} &\equiv d \log(X_2 + Y), & \bullet \text{---} \textcircled{\bullet} &\equiv d \log(X_2 - Y), \\ \textcircled{\bullet} \text{---} \textcircled{\bullet} &\equiv d \log(X_1 + X_2). \end{aligned}$$

Starting at three sites, vertices can connect to multiple lines, leading to letters with multiple sign flips, such as

$$\begin{aligned} \textcircled{\bullet} \text{---} \bullet \text{---} \bullet &\equiv d \log(X_2 - Y + Y'), \\ \bullet \text{---} \textcircled{\bullet} \text{---} \bullet &\equiv d \log(X_2 - Y - Y'). \end{aligned}$$

In total, there are 13 letters for the three-site graph.

Functions—Next, we define the set of basis functions of the differential equation. For this purpose, we introduce a second type of graph tubings. This time, we draw all “complete tubings” of the graph (non-nested and nonintersecting tubings that enclose all vertices). Each tubing corresponds to a unique basis function. Tubings that contain at least one cross are associated with source functions, while the wave function itself has no enclosed crosses. For the two-site chain, the set of functions is

$$\begin{array}{ccc} \psi \textcircled{\bullet} \text{---} \bullet & F \textcircled{\bullet} \text{---} \bullet & Z \textcircled{\bullet} \text{---} \bullet \\ & \tilde{F} \textcircled{\bullet} \text{---} \bullet & \end{array}$$

while, for the three-site chain, we have

$$\begin{array}{ccc} \psi \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & f \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & g \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \\ q_1 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & q_2 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \tilde{g} \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \\ F \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & q_3 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & Z \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \\ \tilde{F} \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \tilde{q}_1 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \\ Q_1 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \tilde{q}_2 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \\ Q_2 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \tilde{q}_3 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & \\ Q_3 \textcircled{\bullet} \text{---} \bullet \text{---} \bullet & & \end{array}$$

Here, we have ordered the graphs by the number of vertices enclosed by tubes that also contain a cross.

Kinematic flow—The final ingredients are the coefficient matrices α_i in (4). In the following, we introduce a set of rules that dictate how the graph tubings associated with the basis functions evolve when we take derivatives. This will allow us to write down—by hand—the differential equations for all tree graphs.

We start with the tubing associated with a parent function of interest and then generate a family tree of its descendants according to a set of rules, from which we can then read off the differential of the parent function. The procedure is best illustrated by an example, and a particularly instructive case is the function Q_1 of the three-site chain:

(1) **Activation**: We first move through the graph and “activate” each tube enclosing a vertex. Each activation forms a branch of the tree.

For the function Q_1 , we generate

$$\textcircled{\bullet} \text{---} \bullet \text{---} \bullet \rightarrow \begin{array}{l} \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \\ \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \\ \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \end{array} \quad (5)$$

where the colored tubes are activated in each branch.

(2) **Growth**: Activated tubes with no crosses can “grow” to enclose adjacent crosses in all possible ways. For example, in the bottom branch of (5), we have

$$\textcircled{\bullet} \text{---} \bullet \text{---} \bullet \rightarrow \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \quad (6)$$

(3) **Merger**: If a grown tube intersects another tube, they “merge” (i.e., their union becomes activated). Such a merger occurs in the top branch of (5),

$$\textcircled{\bullet} \text{---} \bullet \text{---} \bullet \rightarrow \textcircled{\bullet} \text{---} \bullet \text{---} \bullet \quad (7)$$

where $\textcircled{\bullet} \text{---} \bullet \text{---} \bullet \Rightarrow \textcircled{\bullet} \text{---} \bullet \text{---} \bullet$. Tubes that grow and merge lead to additional descendant layers in the tree.

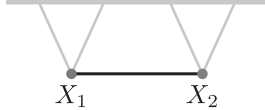
(4) **Absorption**: If an activated tube is adjacent to another tube with a cross, it further merges with this tube (“absorbs” it). This absorption is directional and only occurs if the cross in the activated tube points in the direction of the other tube. Absorption again generates an additional level of the tree.

$$\square\psi = g(X_1 + X_2)^{-(1-2\varepsilon)}, \quad (14)$$

and the relevant differential operator involves *only* X_1 ,

$$\square \equiv (X_1^2 - Y^2)\partial_{X_1}^2 + 2(1 - \varepsilon)X_1\partial_{X_1} - \varepsilon(1 - \varepsilon) \quad (15)$$

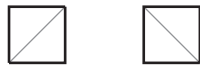
(and similarly for X_2). The nontrivial feature that this equation encodes is that there is a *local* differential operator that simplifies the singularity structure of the wave function ψ . This reflects the fact that the wave function can equally be thought of as arising from a bulk time integral associated with the Feynman diagram



where the existence of such a differential operator is a consequence of the Green's function equation $\square G(x) = i\delta(x)$ [6], where $\square$ is now the d'Alembertian in the bulk spacetime. It is remarkable that, by asking a completely static question in the space of kinematic variables, we are led to a function that can equally well be interpreted as coming from cosmological time evolution.

Sum over graphs—So far, we have discussed the differential equations for individual graphs (or specific triangulations of the kinematic polygon). For scattering amplitudes, however, the most stunning simplifications arise in the sum over Feynman graphs [26–30], while the individual graphs are often not even physically meaningful objects. It remains an important open problem to find similar irreducible structures for the cosmological wave function that emerge when summing over graphs. We will see some hints of deeper structure by removing the scaffolding of a choice of triangulation of the kinematic polygon and, instead, summing over triangulations. The output will again have a physical interpretation, but now as the (flavor-ordered) wave function in $\text{tr}\phi^3$ theory, which involves a sum over planar graphs [31,32].

A four-point function has two different triangulations



corresponding to s - and t -channel exchanges. Similarly, a five-point function is associated with a pentagon, which has five different triangulations

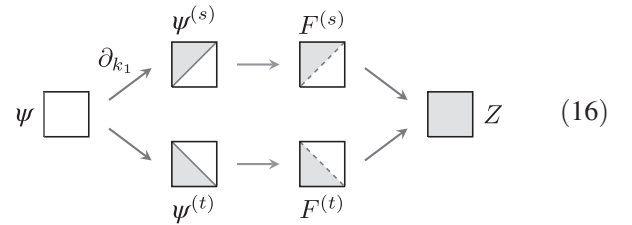


In general, the triangulations of an n -gon are counted by the Catalan numbers C_{n-2} .

The full wave function is again a member of a finite space of functions, which satisfy differential equations.

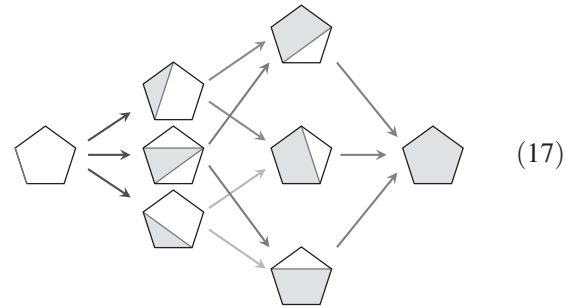
This time, we take derivatives with respect to the external energies k_i (i.e., the side lengths of the kinematic polygon). The letters in the differential equation are now associated with shaded subpolygons, where we allow internal lines to be either solid or dashed (see above). Similarly, functions are associated with (possibly disconnected) shadings of polyangulations, which are the analog of the disjoint tubings in the single-graph case.

The kinematic flow now determines the evolution of shaded polyangulations. The precise rules are given in [25]. Here, we will simplify the discussion by restricting ourselves to taking successive partial derivatives with respect to a single edge of the polygon (say the edge corresponding to the energy k_1). The k_1 derivative of the wave function ψ then generates shadings of all triangles that contain the edge associated with k_1 . Internal lines can be solid or dashed. Taking further k_1 derivatives, the shadings grow by attaching further triangles. For example, in the case of the four-point function, the evolution of shadings is



There are two different paths for reaching the shaded square, which introduces two different functions $\psi^{(s)}$ and $\psi^{(t)}$, which ultimately correspond to different exchange channels that contribute to the full ψ from the bulk perspective. The function Z associated with the full shading is then “shared” between the s - and t -channel equations.

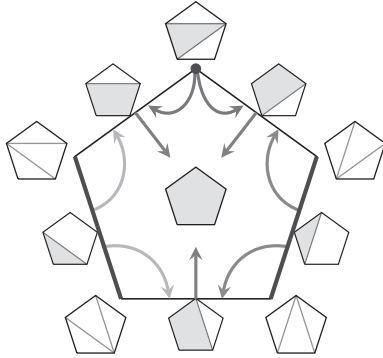
For the four-point function, only one function is shared between channels, but the structure becomes more intricate at higher points [25]. For example, the five-point function has the following sequence of shadings:



To reduce the size of the figure, we have suppressed the solid and dashed distinction of the internal lines. This case has multiple shared functions between the channels.

A convenient way to visualize the relationship between the functions is to assign them to the geometry of an associahedron. To each vertex of the associahedron, we attach a complete triangulation of the kinematic polygon

(and hence functions that are unique to the different channels), while the facets of the geometry correspond to partial triangulations (and hence shared functions). For the case of the five-point function, we have



Shown here are the shadings appearing in (17). We have also indicated how the shadings evolve along the associahedron under the kinematic flow. Other shadings would appear if we were to take derivatives with respect to the other energies (and not just k_1). In general, we can think of the associahedron as being dressed with the basis functions and letters corresponding to the (partial) triangulations on its facets and vertices. For the five-point function, there are seven functions associated with each vertex, four shared functions at each edge, and one shared function on the face of the pentagon [25].

Outlook—In this Letter, we revealed a simple and universal structure underlying the apparent complexity of the differential equations for the FRW wave function of conformally coupled scalars. These rules govern a flow in the boundary kinematic space and make no explicit reference to time evolution in the bulk spacetime. It is intriguing that the components of the construction are defined naturally in the kinematic space of cosmological correlators and are related to interesting combinatorial objects. At its core, this flow in kinematic space is a reflection of the local and causal nature of bulk time evolution. Beyond the conformally coupled case, there are other situations of cosmological and observational interest (for example, general mass fields in exact de Sitter space) with a similar bulk time integral structure, it is natural to suspect that analogous rules will exist in these settings, and it will be interesting to uncover them.

We motivated this endeavor as the search for a viewpoint where time is a derived concept. To view this as a true emergence of time, however, requires that this kinematic flow takes on a life of its own, by being derived from a different set of physical and mathematical principles. The mere existence of this structure is already a hint that a more elemental formulation exists, and we have seen glimpses of such hidden magical features. We hope that this Letter serves as inspiration for finding the deeper mathematical structure underlying cosmological correlations.

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