

Entropy Inequalities Constrain Holographic Erasure Correction

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We interpret holographic entropy inequalities in terms of erasure correction. The nonsaturation of an inequality is a necessary condition for certain schemes of holographic erasure correction, manifested in the bulk as nonempty overlaps of corresponding entanglement wedges.

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Introduction—The AdS/CFT correspondence [1]—probably the best understood UV-complete theory of gravity—is a duality between gravity with asymptotically anti-de Sitter (AdS) boundary conditions and a conformal field theory (CFT) living on its asymptotic boundary. One key element of the duality is the Ryu-Takayanagi proposal [2–4], which under the simplest circumstances posits that the entanglement entropy of a boundary (CFT) subsystem A equals the minimal area of a bulk (AdS) surface homologous to A . Another key fact about the duality is that the physics in the bulk region between A and its homologous minimal surface, called the entanglement wedge $E(A)$, can be recovered from A alone—a statement known as “subregion duality” [5,6]. These assertions impose a degree of locality on the holographic (bulk-to-boundary) map.

The holographic map has interesting properties with respect to set inclusion. Select a number of disjoint subsystems A_s on the boundary. The entanglement wedge nesting theorem [7] says that the wedges of two non-overlapping subsystems are disjoint,

$$A_s \cap A_t = \emptyset \Rightarrow E(A_s) \cap E(A_t) = \emptyset. \quad (1)$$

However, a similar statement where three subsystems have no common overlap does not hold—for example [8],

$$E(A_1 A_2) \cap E(A_2 A_3) \cap E(A_3 A_1) \neq \emptyset \text{ is possible} \quad (2)$$

even though $A_1 A_2 \cap A_2 A_3 \cap A_3 A_1 = \emptyset$. (Here and in the following $A_1 A_2 \equiv A_1 \cup A_2$.) Reference [8] traced the origin of phenomenon (2) to erasure-correcting properties of the holographic map. In this view, bulk physics comprises logical degrees of freedom, which are redundantly encoded

in the CFT. If a limited portion of the CFT data is erased (e.g., erasing A_3 to retain $A_1 A_2$), the retained data may suffice to recover the bulk information. The erasure-correcting aspects of the bulk-to-boundary map have been extensively studied [9], including explicit tensor network models [10,11].

The Ryu-Takayanagi proposal also has a noteworthy consequence: it imposes conditions on the entanglement entropies of the boundary quantum state. Suppose the boundary is divided into $2k + 1$ subregions A_s , with the indices interpreted modulo $2k + 1$ ($A_{2k+2} \equiv A_1$ etc.). If the quantum state is holographically dual to a semiclassical geometry and the Ryu-Takayanagi proposal holds, then the entanglement entropies satisfy [12,13]

$$\sum_{s=1}^{2k+1} (S_{A_s A_{s+1} \dots A_{s+k}} - S_{A_s \dots A_{s+k-1}}) \geq S_{A_1 A_2 \dots A_{2k+1}}. \quad (3)$$

These conditions eliminate certain entanglement structures, such as the GHZ-type entanglement on four or more constituents [14], from representing semiclassical bulk geometries. Beyond (3), we currently know two infinite families of holographic entropy inequalities [15–17] [see Eq. (20) below and Supplemental Material (SM) [18]] and 1868 isolated inequalities [21,25], which do not seem to conform to any discernible pattern.

The multitude of the inequalities, and the lack of a known principle to organize them, suggest a question: Can we interpret the holographic entropy inequalities in physical, ideally operational terms? The inequalities express inviolable properties of Ryu-Takayanagi surfaces, which in turn (by subregion duality) determine which bulk regions are protected against which boundary erasures. It is therefore natural to suspect that a physical interpretation of the inequalities should concern erasure correction. This Letter offers such an interpretation.

Example—Take inequality (3) with $k = 2$. Imagine designing a holographic erasure-correcting code where five parties A_s ($1 \leq s \leq 5$; $s \equiv s + 5$), possibly in the presence of a sixth party O , encode some logical

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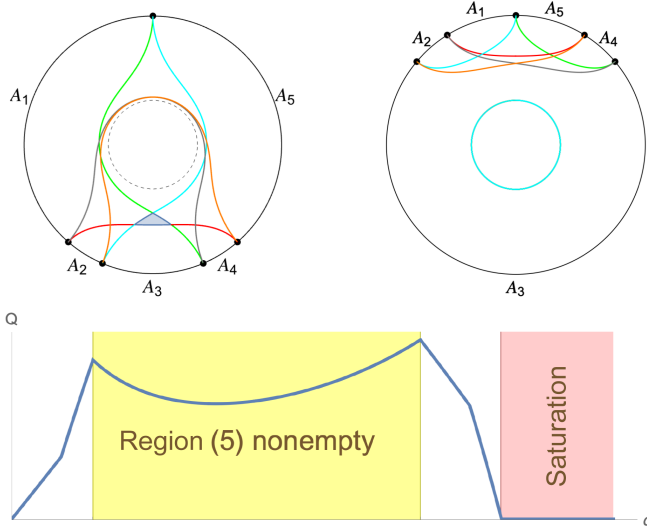


FIG. 1. Example (4), (5) applied to the static Bañados-Teitelboim-Zanelli black hole: we vary $A_3 = (-\alpha, \alpha)$ and keep A_2, A_4 of fixed small size. The highlighted region (5) is nonempty and quantity (4) is zero only when the Ryu-Takayanagi surfaces are in the phases shown. As claimed, the two circumstances never coincide.

information. The goal is to ensure that the logical information be recoverable from $A_1A_2A_3$ and from $A_3A_4A_5$ but *not* from $A_2A_3A_4$ nor $A_4A_5A_1$ nor $A_5A_1A_2$. As an illustration of our more general claim, we posit that if inequality (3) is saturated then this goal cannot be achieved.

Because bulk geometries encapsulate recoverable logical information in the form of entanglement wedges, our claim is really a statement about entanglement wedges; see Fig. 1. We can transcribe it explicitly as follows:

$$Q \equiv \sum_{1 \leq s \leq 5} S_{A_s^+} - \sum_{1 \leq s \leq 5} S_{A_s^-} - S_{A_1A_2A_3A_4A_5} = 0 \Rightarrow \quad (4)$$

$$E(A_1^+) \cap \overline{E(A_2^+)} \cap E(A_3^+) \cap \overline{E(A_4^+)} \cap \overline{E(A_5^+)} = \emptyset. \quad (5)$$

Here, we introduce the k -dependent definitions

$$A_s^+ = A_s A_{s+1} \dots A_{s+k-1} A_{s+k} \quad (6)$$

$$A_s^- = A_s A_{s+1} \dots A_{s+k-1} \quad (7)$$

for consecutively labeled combinations of subsystems A_s , which appear in inequalities (3). By comparing with statement (2), we see that ours is a *sufficient* condition for when a certain type of erasure correction does *not* work. By contrapositive, the *non* saturation of a holographic inequality is a *necessary* condition for certain schemes of erasure correction.

Figure 1 presents one instance of (4), (5). Five parties A_s control consecutive intervals on the circular boundary of a static Bañados-Teitelboim-Zanelli black hole [26]; the sixth

party O represents the purifying system, i.e., the second asymptotic region of the Kruskal-extended spacetime [27]. We vary $A_3 = (-\alpha, \alpha)$ while keeping the sizes of A_1, A_5 and A_2, A_4 (the latter fixed and small) pairwise equal. As claimed, the bulk region in question is nonempty only when the inequality is not saturated.

Preliminaries—To state our claim in full generality and to prove it, we need the concept of a “contraction map” [13]. Exhibiting a contraction that obeys appropriate boundary conditions is how all the presently known holographic entropy inequalities were proven. It is not known whether every holographic entropy inequality can be proven using a contraction (but see Refs. [28,29]) or whether this method of proof is valid when the bulk geometry and its dual quantum state are time-dependent (but see Refs. [30–32]). We sidestep both complications by considering only inequalities for which a contraction proof exists, and only working with holographic erasure-correcting codes that are time-reversal-symmetric.

Consider a schematic inequality,

$$\sum_{i=1}^l \alpha_i S_{X_i} \geq \sum_{j=1}^r \beta_j S_{Y_j}, \quad (8)$$

where all coefficients α_i and β_j are positive. Under the stated assumptions, the minimal surface that computes S_{X_i} divides the time-reversal-symmetric bulk slice into two halves: $\tilde{E}(X_i)$ that lives within $E(X_i)$, and its complement. The entanglement wedge $E(X_i)$ is the bulk domain of dependence of $\tilde{E}(X_i)$ [5], but in what follows we sloppily refer to $\tilde{E}(X_i)$ as the “entanglement wedge of A ” and drop the tilde from the notation. This avoids clutter and causes no confusion because all our analysis happens on the time-reversal-symmetric bulk slice.

Taking intersections of $E(X_i)$ and $\overline{E(X_i)}$ over all terms on the “greater-than” side (lhs) of the inequality, we partition the bulk slice into 2^l regions called $W(x)$. Here, $x \in \{0, 1\}^l$ is a bit string that summarizes whether $W(x)$ is inside ($x_i = 1$) or outside ($x_i = 0$) the entanglement wedges $E(X_i)$. By subregion duality, we can view x as a list of which boundary subsystems can ($x_i = 0$) or cannot ($x_i = 1$) be erased while preserving the bulk information in $W(x)$. For instance, the pattern of erasure correction featured in (5) is $W(10100)$.

To prove (8), we combine regions $W(x)$ into “candidate” entanglement wedges of subsystems Y_j such that

$$\sum_{i=1}^l \alpha_i S_{X_i} \stackrel{(b)}{\geq} \sum_{j=1}^r \beta_j S_{Y_j^{\text{candidate}}} \stackrel{(\#)}{\geq} \sum_{j=1}^r \beta_j S_{Y_j}. \quad (9)$$

Because the physically realized entanglement entropies of Y_j are minimal, step (#) in (9) is automatic. The clou of the proof is to select the right combinations of $W(x)$ to form

candidate entanglement wedges of Y_j such that $S_{Y_j}^{\text{candidate}}$ automatically satisfy step (b). That selection is encoded in a contraction map f .

The map takes bit strings $x \in \{0, 1\}^l$ into bit strings $f(x) \in \{0, 1\}^r$. From the definition of $W(x)$, observe that (the time slice of) the entanglement wedge of X_i is the union of all regions $W(x)$ such that the i th bit of x is 1,

$$E(X_i) = \bigcup_{x|x_i=1} W(x). \quad (10)$$

To form a candidate entanglement wedge of Y_j , we write an analogous expression with $f(x)$,

$$E(Y_j)^{\text{candidate}} = \bigcup_{x|f(x)_j=1} W(x). \quad (11)$$

To prove (8), map f must satisfy two conditions: (i) the contraction property

$$\sum_{i=1}^l \alpha_i |x_i - \tilde{x}_i| \geq \sum_{j=1}^r \beta_j |f(x)_j - f(\tilde{x})_j| \quad (12)$$

for all $x, \tilde{x} \in \{0, 1\}^l$. This validates step (b) in (9). (ii) Boundary conditions: for each indivisible subsystem A [in example (4) this includes the five subsystems A_s and the purifier O], form indicator bit strings x^A and f^A . The i th bit of x^A is 1 if $A \subset X_i$ and 0 otherwise, and likewise for f^A with respect to the Y_j . For all indivisible subsystems A we require

$$f(x^A) = f^A. \quad (13)$$

This ensures that the candidate entanglement wedges (11) satisfy the homology condition in the Ryu-Takayanagi proposal.

How these two conditions achieve their goals has been reviewed in many references, e.g., [13,30,33–35].

Candidate entanglement wedges violate entanglement wedge nesting—In Eq. (1) we highlighted a general feature of entanglement wedges: disjoint boundary subsystems have disjoint wedges. If we work with pure states, which we can always do by introducing a purifying boundary system O , then $E(X_i) = E(\overline{X_i})$ and the same fact can be expressed in another way,

$$X \subset Y \Rightarrow E(X) \subset E(Y). \quad (14)$$

This property, called “entanglement wedge nesting,” is necessary for subregion duality to hold or else tracing out $Y \setminus X$ would enhance our ability to reconstruct the bulk.

While physically realized entanglement wedges always obey (1) and (14), the candidate entanglement wedges employed in proofs by contraction need not do so. In fact, as two of us recently pointed out [35], the exact opposite is the case: properties (12) and (13) of contraction maps

directly imply that the candidate wedges defined in (11) cannot all obey (1) and (14). [The sole exception is the monogamy of mutual information [12], i.e., inequality (3) with $k = 1$, which can be proven without invoking nesting-violating candidate wedges.] A special class of instances where the tension between (12) and (13) and wedge nesting is most easily seen is given by Eqs. (18) and (19) below, but the phenomenon occurs more generally.

We spell out the conditions under which candidate wedge $E(Y_j)^{\text{candidate}}$ violates (1) or (14),

$$Y_j \subset X_i \quad \text{and} \quad f(x)_j = 1 \quad \text{and} \quad x_i = 0, \quad (15)$$

$$Y_j \cap X_i = \emptyset \quad \text{and} \quad f(x)_j = 1 \quad \text{and} \quad x_i = 1, \quad (16)$$

$$Y_j \supset X_i \quad \text{and} \quad f(x)_j = 0 \quad \text{and} \quad x_i = 1. \quad (17)$$

These conditions follow from substituting (11) for $E(Y_j)$ in the wedge nesting theorems (1) and (14).

Observe that whenever circumstances (15)–(17) occur and $W(x)$ is nonempty, $E(Y_j)^{\text{candidate}}$ is necessarily distinct from the physical entanglement wedge $E(Y_j)$. To wit, when (15) or (16) applies, including a nonempty $W(x)$ in $E(Y_j)^{\text{candidate}}$ according to (11) causes the latter to violate entanglement wedge nesting, which the physical $E(Y_j)$ is guaranteed to respect. Likewise, when (17) applies, excluding a nonempty $W(x)$ from $E(Y_j)^{\text{candidate}}$ according to (11) forces the latter to violate entanglement wedge nesting, which the physical $E(Y_j)$ respects.

Argument and result—Consider a holographic entropy inequality (8) and a contraction map f , which proves it. (1) If x satisfies one of conditions (15)–(17) and $W(x)$ is nonempty, then $E(Y_j)^{\text{candidate}} \neq E(Y_j)$. (2) If (8) is saturated, then we have equality in step (‡) in (9) and, therefore, $E(Y_j)^{\text{candidate}} = E(Y_j)$. (3) Thus, if x satisfies one of conditions (15)–(17) and inequality (8) is saturated, then $W(x) = \emptyset$.

Point (3) above is our main result. When introducing regions $W(x)$, we pointed out that the bit string x lists which boundary erasures are protected against in the holographic erasure-correcting code. Thus, the nonsaturation of inequality (8) is a necessary condition for the scheme of erasure correction characterized by x .

In step (2) above, equality in (9)(‡) directly implies $S_{Y_j}^{\text{candidate}} = S_{Y_j}$ for all j because the latter are global minima. One might worry that $E(Y_j) \neq E(Y_j)^{\text{candidate}}$ even though $S_{Y_j}^{\text{candidate}} = S_{Y_j}$. But this could only occur if the entanglement wedge of Y_j were right at a phase transition between $E(Y_j)^{\text{candidate}}$ and $E(Y_j)$, a circumstance in which the S_{Y_j} incur enhanced geometric corrections [36,37]. In SM [18] we show that our main result—if properly applied—continues to hold even in this fine-tuned scenario.

Applications—Applied to the presently known holographic entropy inequalities, our result gives rise to a plethora of necessary conditions for various schemes of erasure correction. In fact, as discussed in detail in [35], contraction proofs of *all* known inequalities except the monogamy of mutual information feature multiple instances of (15)–(17). For generic examples, in SM [18] we randomly choose two inequalities from [21] and inspect particular contraction maps that prove them. We find that saturating the inequality eliminates 340 (585) out of 569 (1145) *a priori* admissible erasure correction schemes. We also present an example application based on one inequality drawn from the infinite families [15–17]; see inequality (20) below.

Our result has broad applications because it relies on the *existence of one* contraction map f with properties (15)–(17). In particular, we do not need the nesting violation to occur in all contraction maps, only in one. This makes it algorithmically easy to check if a given inequality constrains a specific erasure-correcting scheme x . One lists all potential choices of $f(x)$ that satisfy (15) or (16) or (17)—call them $f^q(x)$ —and looks for a contraction map f with $[x, f^q(x)]$ imposed as an additional boundary condition. If such an f exists, then the saturation of the inequality guarantees that $W(x)$ is empty.

There are special instances of our result, which one can write down by hand after glimpsing a given inequality [35]. To do so, fix a subsystem A and rewrite the inequality such that A appears in every term, trading X_i or Y_j for their complements where necessary. In this convention, $x^A = 1\dots 1$ and $f^A = 1\dots 1$ (all ones) is the boundary condition (13) for A . Suppose that, after the rewriting, (8) contains a region X_{i^*} such that the combined coefficient of all its subsets is negative,

$$\sum_{i|X_i \subseteq X_{i^*}} \alpha_i - \sum_{j|Y_j \subseteq X_{i^*}} \beta_j < 0. \quad (18)$$

Then for x defined by

$$x_i = \begin{cases} 0 & \text{if } X_i \subseteq X_{i^*} \\ 1 & \text{otherwise} \end{cases}, \quad (19)$$

imposing (12) with $\tilde{x} = x^A$ implies that $f(x)$ must contain at least one instance of (15). Thus, whenever (8) is saturated the $W(x)$ defined in (19) is empty. Example (4), (5) belongs to this class, with $A = A_3$ and $X_{i^*} = A_2^+$.

Applications to the infinite families of inequalities—The necessary conditions on holographic erasure correction imposed by the infinite families of inequalities proven in [16] are particularly convenient to state and use. Because of the regular structure of the inequalities and the contraction maps, the necessary conditions pertain to intersections of only a few entanglement wedges [35]; we provide examples with six or seven of them. This is in

contrast to a generic holographic inequality, whose nesting violations typically concern specific combinations of *all* terms on the “greater-than” side of (8).

The toric inequalities are defined for pure states on systems A_s ($1 \leq s \leq 2k_A + 1$) and B_t ($1 \leq t \leq 2k_B + 1$) where, like in inequalities (3), the indices are cyclically ordered. Extending notation (6) to consecutively indexed tuples of B -type regions, we have

$$\sum_{s=1}^{2k_A+1} \sum_{t=1}^{2k_B+1} (S_{A_s^+ B_t^-} - S_{A_s^- B_t^-}) \geq S_{A_1 \dots A_{2k_A+1}}. \quad (20)$$

In addition to these “toric” inequalities, we also have the family of “projective plane” inequalities, which are discussed in SM [18]. The ensuing analysis is phrased with reference to (20) but it applies to the projective plane inequalities without modifications.

Reference [16] proved inequalities (20) by contraction. To conduct the proof, they presented the terms in the double sum in (20) as faces ($X_i \equiv A_s^+ B_t^-$) and vertices ($Y_j \equiv A_s^- B_t^-$) of a graph embedded on a torus; see Fig. 2. A vertex and a face are incident if and only if $Y_j \subset X_i$ or $Y_j \cap X_i = \emptyset$ or $Y_j \supset X_i$ —that is precisely when the entanglement wedge nesting theorem applies. This has the benefit that instances of (15)–(17) are locally detectable on the graph.

For a visual representation of bit strings $x \in \{0, 1\}^{(2k_A+1)(2k_B+1)}$, on every face X_i draw a pair of vertical (if $x_i = 1$) or horizontal ($x_i = 0$) lines; see Fig. 2. The rationale for this recipe is that setting $f(x)_j = 1$ avoids (15)–(17) only if vertex Y_j is surrounded by a minimal loop. (This fact is explained in Sec. D of Supplemental Material [18] and, in greater detail, in Ref. [35].) In all other circumstances, setting $f(x)_j = 1$ incurs a nesting violation and our result applies.

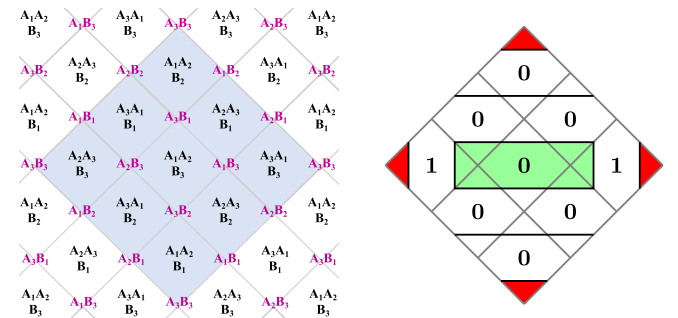


FIG. 2. Left: terms in the sum in (20) presented as faces and vertices of a graph embedded on a torus. Right: visual representation of a bit string x . Pairs of vertical (horizontal) lines are drawn on faces X_i if the corresponding bit is $x_i = 1$ ($x_i = 0$). We color the interiors of contractible loops; because the green loop is nonminimal, $f(x)$ satisfies one of conditions (15)–(17). Section C of SM [18] shows an example application of our result, which is based on this setup.

Yet the contraction proof of [16] places a 1 in *every* collection of vertices Y_j , which are surrounded by a loop—minimal or not. Since only minimal loops avoid nesting violations (15)–(17), we conclude that whenever the visual representation of x contains a nonminimal contractible loop, one of conditions (15)–(17) arises and the saturation of (20) implies $W(x) = \emptyset$. (Note that noncontractible loops fail to surround vertices.) This result is strong because its applicability is not conditioned on all bits of x but only on those that assemble a nonminimal loop. For example, the loop with the green interior in Fig. 2 involves only seven bits of x ; see SM [18] for detailed examples. Moreover, multiple nonminimal contractible loops can be drawn in Fig. 2, so the saturation of (20) constrains many distinct erasure-correcting schemes x .

Discussion—To our knowledge, these results are the first instance in which holographic entropy inequalities are interpreted in terms of a task—in this case, erasure correction. Prior work used the $k = 1$ inequality (3), i.e., the monogamy of mutual information [12], for detecting topological order [38] and for amplitude computations [39], but other inequalities have been harder to interpret [40–42]. A loosely related result is that—subject to a genericity assumption—the entanglement wedge of a composite system is connected if and only if a certain combination of its constituents’ entanglement entropies vanishes [21]. Unlike [21], we focus on holographic entropy inequalities (8), exploit properties of contraction maps, and interpret the result in terms of erasure correction.

Our examples in Fig. 1 and SM [18] focus on maximally tight holographic inequalities—often called facets of the holographic entropy cone [13]. However, convex combinations of inequalities such as

$$\text{lhs}_1 + \text{lhs}_2 \geq \text{rhs}_1 + \text{rhs}_2, \quad (21)$$

where

$$\text{lhs}_1 \geq \text{rhs}_1 \quad \text{and} \quad \text{lhs}_2 \geq \text{rhs}_2 \quad (22)$$

are two individually true inequalities, give rise to additional, *independent* applications of our result. That is because a contraction proof of (21) may contain additional nesting violations (15)–(17) that do not originate from nesting violations in contraction proofs of (22). Thus, nonmaximally tight inequalities extend the corpus of constraints on holographic erasure correction in further ways, which we intend to characterize elsewhere.

The Ryu-Takayanagi prescription is subject to quantum corrections from entanglement among bulk (logical) degrees of freedom [43]. We have ignored them in this Letter because our motivation is to design holographic codes, which protect logical information from specific erasures. From this viewpoint, it is never helpful to entangle two logical degrees of freedom before encoding and

protecting them against distinct erasure types, which is how bulk entanglement might contaminate our results. In other words, we consider applying holographic erasure-correcting codes to product states in the logical Hilbert space and, therefore, all entropies in this Letter should be understood as surface areas.

While we have assumed a time-reversal-symmetric setup in this Letter, it is interesting to contemplate how dynamics may affect our result. This offers a potentially new perspective on the still unsettled question of whether holographic entropy inequalities for time-dependent states are weaker than those for static states [30–32]. Our result also applies to entanglement wedges of bulk regions [44,45] because the latter obey all holographic entropy inequalities proven by contraction [46].

Note added—We note that some figures that appear in this Letter were previously used in Refs. [16,35], of which we are coauthors.

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Data availability—The data that support the findings of this article are openly available [20].

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- [18] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/dl3c-h3hg>, which includes Refs. [19,20,22–24]. Section A of SM inspects a special nongeneric circumstance in which step (2) of our “Argument” requires extra care. Section B illustrates our result by applying it to two specific contraction maps, which prove two holographic entropy inequalities randomly chosen from [24]. Section C illustrates our result by applying it to one toric inequality (20) and the configuration shown in Fig. 2. In Sec. D, we review the projective plane inequalities of Ref. [16] and confirm that our general result applies to them in the same way as it does to the toric inequalities (20).
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