

Thick Lunar Crust Amplifies Deci-Hertz Gravitational-Wave Signals

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Gravitational waves (GWs) in the 0.01 ~ 1 Hz band encode unique signatures of the early Universe and merging compact objects, but they are beyond the reach of existing observatories. Theoretical models suggest that the Moon could act as a resonant detector, but the unknown influence of its rugged surface and heterogeneous interior poses a challenge to the accurate modeling of its response. Here, we address this long-standing uncertainty by constructing the first high-resolution, two-dimensional model of the lunar GW response, more realistic than previous ones. We achieve this by combining high-fidelity spectral-element simulations with the analytical power of normal-mode perturbation theory, thereby resolving topographical effects down to 2 km grid spacing while maintaining the capacity to discern global free-oscillation patterns. This dual-methodology approach not only recovers the expected predominant quadrupole ($l = 2$) oscillation mode, but also exposes a systematic signal amplification in thick-crust regions. This enhancement is traced by our normal-mode analysis to a mode-coupling process, in which the original quadrupolar oscillation induced by the passing GW distributes energy into a series of higher-order modes, the hybridized eigenmodes of a laterally heterogeneous Moon. In certain narrow frequency ranges, we observe up to tenfold amplification spanning into the deci-hertz band, highlighting the power of numerical simulations in resolving these structurally fine-tuned features for designing future detectors. Our Letter establishes the Moon as a resonant GW detector albeit its complex topographical structures, and the resulting amplification maps provide a quantitative guide for the optimal landing site selection.

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Introduction—Gravitational waves (GWs) have been observed in the hundred-hertz band by ground-based interferometers [1–4] and in the nano-hertz band by pulsar timing arrays [5–8]. In between lies the middle band (0.01 ~ 1 Hz) where many intriguing sources exist, including the seeds of supermassive black holes, inspiraling binary stellar black holes or neutron stars, supernovae, as well as quantum fluctuations in the early Universe [9,10]. The detection of these sources defines the core science cases for future space-based GW detectors like LISA, TianQin, Taiji [10–12], and other projects [13,14].

An alternative proposal for detecting mid-band GWs is to find a celestial body that can act as a giant resonant detector [15–17]. In this spirit, the Moon offers a unique platform due to its quietness compared to the Earth [18].

Assuming spherical symmetry, earlier analytical works established that only the Moon's quadrupole ($l = 2$) mode resonates with GW [19,20], and the resonant peaks lie in the middle band [19,21–23]. These one-dimensional models also revealed the importance of lunar radial structure in determining the resonant frequencies [23,24].

In reality, the Moon is *not* a spherically symmetric body but has strong variations in both topography and crustal thickness, since altimetric surveys indicate tens of kilometers in topographic relief [25,26] and lunar gravity measurements reveal strong lateral variations in crustal thickness [27,28]. Many studies based on normal-mode perturbation theory [29,30] show that strong lateral velocity variations would severely impact the resonance of seismic wave propagation within the Earth, especially for the normal-mode coupling and energy redistribution [31–33]. However, the effect of such heterogeneity on the GW response of the Earth and the Moon remains unknown. If not properly understood, it may mislead our conclusion for lunar GW detection [34]. This issue is becoming even more pressing for the proposed projects like Lunar Gravitational-Wave Antenna [35,36], Laser Interferometer Lunar Antenna [37], and other lunar GW projects [38–40].

Using conventional normal-mode perturbation analysis [29,30] to address this issue presents distinct advantages

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and limitations. A major advantage arises from the simplicity of the GW source term: unlike earthquakes or moonquakes, which excite a broad spectrum of seismic modes, GWs interact almost exclusively with the Moon's quadrupole ($l = 2$) modes, drastically simplifying the analytical description. However, this approach becomes computationally demanding when accounting for the full mode mixing induced by the lateral heterogeneity, and it struggles to provide a fine-scale description of the resulting surface displacement field. To complement the perturbation analysis, in the past few years we have developed high-fidelity numerical simulations based on the high-order finite-element method, specifically the spectral-element method (SEM) [41,42], which enables precise modeling of wave propagation across realistic lunar topography [42–44]. Although computationally intensive, the SEM provides a direct numerical test bed for verifying key analytical predictions, particularly in regions of strong topographic and crustal variations. However, three-dimensional (3D) simulations remain computationally prohibitive for long-duration GW signals. Therefore, it is essential to develop a novel calculation framework that combines the high resolution of the finite-element method and the generality of the normal-mode analysis to reveal both the fine details of the lunar response and the underlying physical mechanism.

For the above reason, we present the first structurally realistic model of the lunar GW response: high-precision two-dimensional (2D) SEM simulations employed to resolve the effects of rugged topography and strong nonuniform crustal structures, while a 3D mode-coupling approach characterizes the global low-order response. Our results show that the Moon's crustal asymmetry acts as a potent amplifier, efficiently redistributing energy from the GW-excited quadrupole modes into a rich spectrum of higher-order spheroidal modes. This mode-mixing process not only enhances signal detectability in specific frequency bands but also indicates the potential of GW-induced seismic responses to constrain the Moon's 3D internal structure.

Numerical simulation of lunar response to GWs—We first perform 2D numerical simulations to quantify how lunar topography and crustal structure modulate the GW-induced surface displacement and velocity fields. Details of modeling and parameter setting can be found in Supplemental Material [45]. Our lunar model integrates topographic and crustal-thickness data from the LOLA [25] and GRAIL [26] missions, incorporating major structural features such as the South Pole-Aitken (SPA) basin, Mare Imbrium, and Mare Humboldtianum (Fig. 1). To reduce the extremely heavy computational cost, we extract a typical 2D profile along a selected great circle of the real 3D

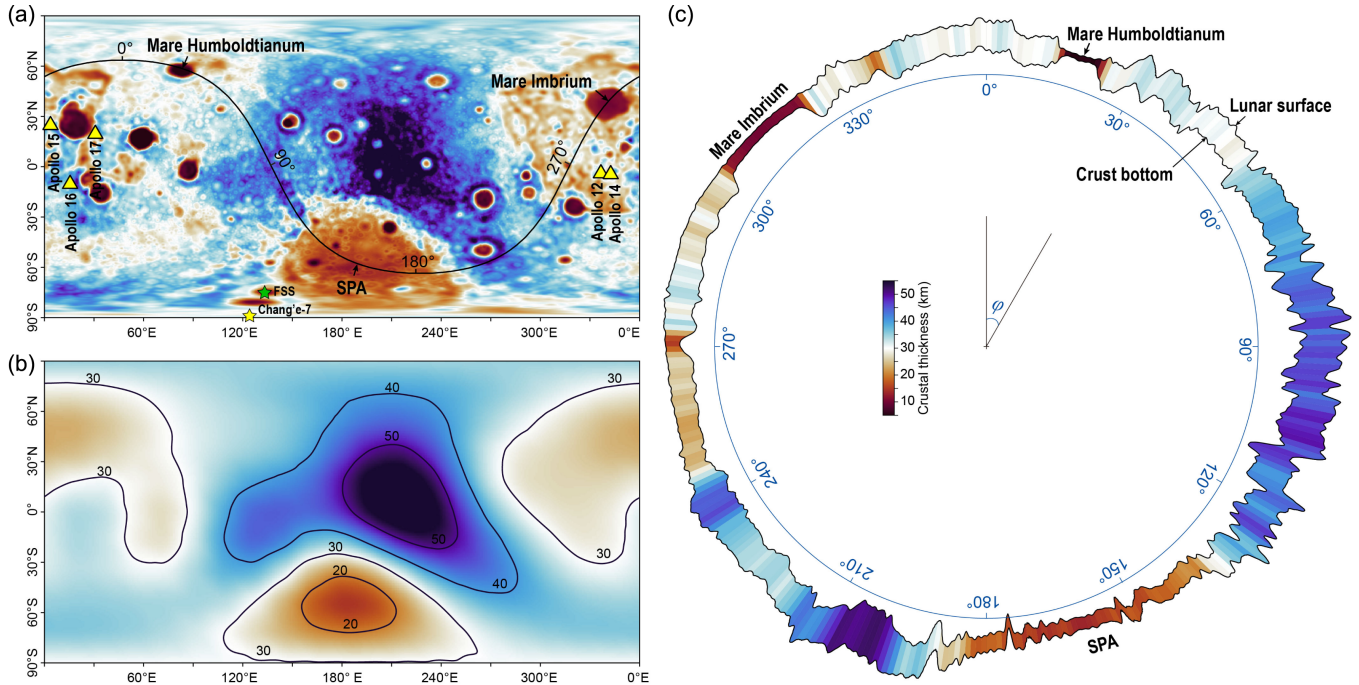


FIG. 1. Lunar model with topography and crustal-thickness variations. (a) Lunar crustal thickness model. The yellow triangles mark the locations of Apollo seismographs. The black line indicates the great circle passing through Mare Humboldtianum, Mare Imbrium, and the SPA basin. The potential landing sites of Chang'e-7 and Farside Seismic Suite (FSS) are marked as stars around the south pole. (b) Crustal thickness model truncated at $l = 4$. Solid lines represent crustal thickness contours in km. (c) The surface topography and crustal thickness along the great circle shown in (a). The outer black curve is the surface topography, exaggerated by a factor of 30 for visibility, and the inner black curve marks the crust bottom, whose radial position is drawn by exaggerating the crustal thickness by a factor of 5 inward from the surface, both relative to the mean lunar radius. The color fill encodes the local crustal thickness, highlighting its pronounced lateral variations.

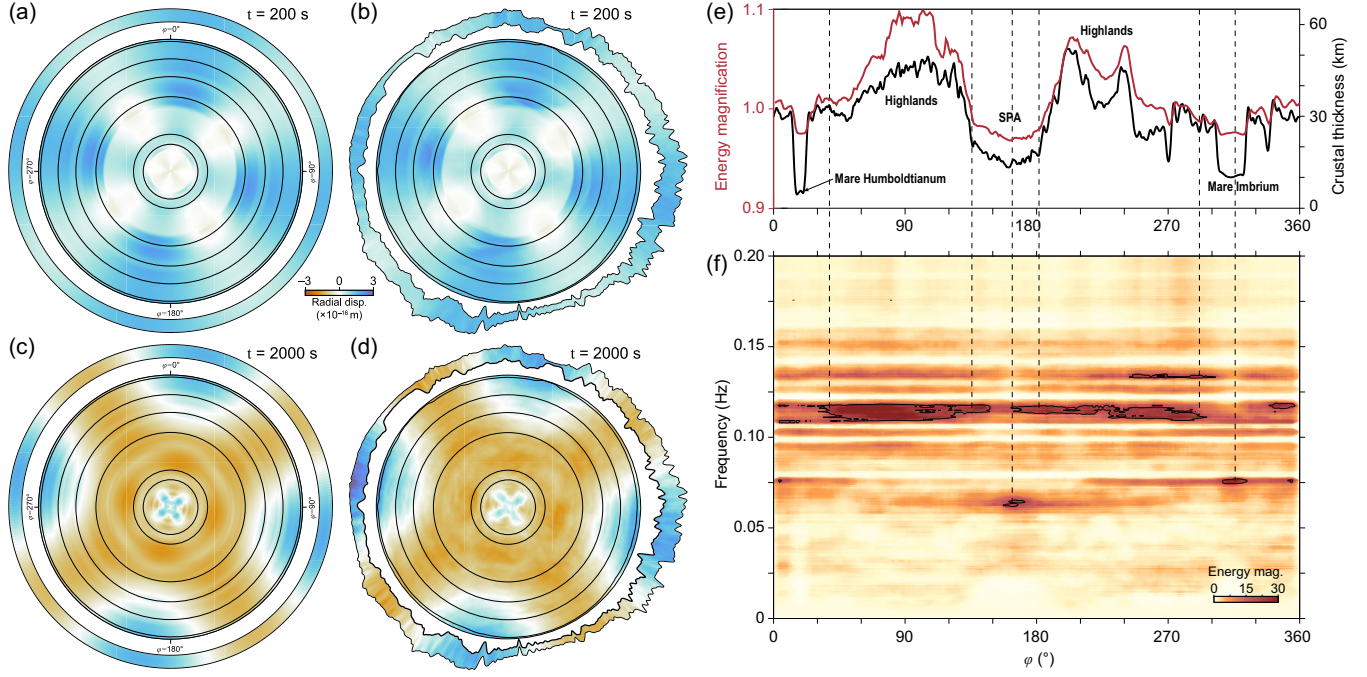


FIG. 2. Comparison between layered and heterogeneous models and the simulated location-frequency distribution of amplification ratios. (a) The snapshot of lunar response to GWs for a spherically layered model at $t = 200$ s. (b) The same as (a) but for the model with varying topography and crustal thickness shown in Fig. 1(c). In (a) and (b), the topography and crustal thickness are magnified by 30 and 5 times in the outside annulus, respectively. (c) The same as (a) but at $t = 2000$ s. (d) The same as (b) but at $t = 2000$ s. More snapshots can be found in [48]. (e) The averaged amplification ratio M_E (red line), integrated up to 0.2 Hz. The black line is the crustal thickness for reference. (f) The amplification ratio $m_E(f)$ in the frequency band up to 0.2 Hz, where the black contour lines indicate the amplification ratio of 20.

model. Dyson-type force density is applied to model the GW sources [17]. To accurately capture short-wavelength scattering in the deci-hertz regime, this simulation utilizes a high-resolution topographic model with a grid spacing of approximately 2.0 km. Our model achieves a maximum resolved frequency of 0.36 Hz with a time step constrained to 5×10^{-3} s, ensuring both numerical stability and computational tractability. Then we use the SEM [41] to simulate the excitation of seismic activity by GWs, employing a broadband source time function centered near 0.1 Hz. We focus on the response in the frequency band up to 0.2 Hz. Figures 2(a)–2(d) show that the lateral heterogeneity of structures has a significant impact on the lunar response to GWs. Compared with the results obtained by spherically layered models [22,41], the wavefronts in the realistic model are evidently distorted, especially in Mare Imbrium, Mare Humboldtianum, and the SPA basin.

To quantify the impact of lunar topography and crustal thickness variation, we define an amplification ratio M_E as the ratio of local displacement energy ($|\vec{s}(\vec{r}, \omega)|^2$) [49] with or without lateral heterogeneity (marked with “P” or “NP”), integrated over frequency

$$M_E(\vec{r}) = \frac{\int_{\omega_{\min}}^{\omega_{\max}} \langle |\vec{s}^P(\vec{r}, \omega)|^2 \rangle_{\text{pol}} d\omega}{\int_{\omega_{\min}}^{\omega_{\max}} \langle |\vec{s}^{\text{NP}}(\vec{r}, \omega)|^2 \rangle_{\text{pol}} d\omega}. \quad (1)$$

Here, $\langle \rangle_{\text{pol}}$ denotes an average over all polarization in order to separate the amplification due to lunar structure from polarization-induced variations in the GW amplitude. More details of this equation are discussed in Supplemental Material [45].

Figure 2(e) presents the distribution of M_E along the typical great circle (integrated up to 0.2 Hz). The spatial characteristics generally show strong correlations with both topography and crustal thickness. Notably, thick-crust regions (e.g., highlands) show an average signal amplification [red curve in Fig. 2(e)]. However, in some places such as Mare Imbrium, where the topography has almost no relief, the amplification ratios still vary dramatically with the varying crustal thickness. This key observation demonstrates that crustal thickness rather than topography is the dominant factor of governing the amplification effect. Hence, we should pay more attention to the crustal thickness when evaluating lunar local response to GWs in future works.

To find the optimal frequency band for GW search, we plot the frequency-dependent amplification ratio $m_E(f)$, defined as

$$m_E(\vec{r}, f) = \frac{\langle |\vec{s}^P(\vec{r}, 2\pi f)|^2 \rangle_{\text{pol}}}{\langle |\vec{s}^{\text{NP}}(\vec{r}, 2\pi f)|^2 \rangle_{\text{pol}}}, \quad (2)$$

in Fig. 2(f), where we have applied a median-filter smoothing to reveal fundamental patterns. The plot shows that the structurally induced signal enhancement is a broadband phenomenon. In the lower frequency regime (≤ 0.04 Hz), the amplification exhibits rich spectral peaks with narrow bandwidths (also see Figs. S3 and S4 in Supplemental Material [45]). These peak frequencies align closely with the resonant frequencies of the lunar response function [41], suggesting a direct link to the normal modes of the Moon. Crucially, near the optimal sensitivity band of proposed lunar antennas (around 0.1 Hz), this localized energy magnification $m_E(f)$ not only persists but further intensifies. The broadband spatial variations in this deci-hertz regime remain tightly correlated with the thick-crust topography. For example, on the highlands surrounding the SPA basin (between $\varphi = 70^\circ$ and 120°), $m_E(f)$ can reach tenfold, with peak amplifications often exceeding those observed at lower frequencies.

Normal-mode perturbation method—The observed close correspondence between crust thickness and GW signal amplification in our numerical simulations can be understood in the theoretical framework of normal-mode perturbation and mixing. In the ideal case where the Moon is spherically symmetric, nonrotating, elastic, and isotropic (SNREI), its free oscillation can be described by a series of orthonormal normal modes $|k\rangle \equiv |\sigma, n, l, m\rangle$, which are eigenfunctions of the secular equation [30]

$$\mathcal{V}|k\rangle = \omega_k^2 \mathcal{T}|k\rangle. \quad (3)$$

Here, $\sigma = S/T$ indicates the mode type (spheroidal or toroidal), n is the radial quantum number, and (l, m) are the angular momentum quantum number and magnetic quantum number. $\mathcal{V}$ and $\mathcal{T}$ are potential and kinetic operators, respectively. The presence of surface topography and crustal variation perturbs the operators by $\delta\mathcal{V}$ and $\delta\mathcal{T}$, restructuring the eigenmodes. Given that the perturbation is small, the new modes can be approximated by a linear combination of the basic SNREI modes, so that

$$|i\rangle = \sum_k C_{ki}|k\rangle. \quad (4)$$

For this reason, the new modes are also known as “hybridized modes.” The transformation matrix $\mathbf{C}$ satisfies the perturbed equation [33]

$$\sum_{k'} (\omega_k^2 \delta_{kk'} + V_{kk'}) C_{k'i} = \Omega_i^2 \sum_{k'} (\delta_{kk'} + T_{kk'}) C_{k'i}, \quad (5)$$

where Ω_i denotes the perturbed eigenfrequency, $V_{kk'} = \langle k|\delta\mathcal{V}|k'\rangle$, and $T_{kk'} = \langle k|\delta\mathcal{T}|k'\rangle$ are the elements of the coupling matrices that can be calculated using Woodhouse kernels [29].

GWs exert a tidal force $\vec{f}_{\text{GW}}$ on the Moon. Owing to their quadrupole character, this force couples almost exclusively

to the unperturbed $l = 2$ modes. In a laterally heterogeneous Moon, however, these unperturbed SNREI modes are no longer the true eigenmodes. Instead, they hybridize according to Eq. (4). Consequently, the energy initially in the quadrupole modes is systematically redistributed into the hybridized mode spectrum. Following the standard Green-function approach [19,21] as well as perturbation formalism introduced above, this mode mixing process is captured by the following equation:

$$\begin{aligned} \vec{s}^{\text{P}}(\vec{r}, \omega) &= \sum_i \frac{\langle \vec{r}|i\rangle \langle i|\vec{f}_{\text{GW}}\rangle}{\Omega_i^2 - \omega^2} \\ &= \sum_i \frac{\sum_k C_{ki} \vec{u}_k(\vec{r})}{\Omega_i^2 - \omega^2} \left(\sum_{k' \in \{\sigma l=S2\}} C_{k'i}^* f_{k'} \right), \end{aligned} \quad (6)$$

where $\vec{s}^{\text{P}}(\vec{r}, \omega)$ is the seismic displacement field at location $\vec{r}$, $\vec{u}_k(\vec{r}) = \langle \vec{r}|k\rangle$ is the unperturbed free-oscillation pattern, and $f_{k'}$ is the projection of the GW force onto the unperturbed mode $|k'\rangle$. Other details of this core equation can be found in Supplemental Material Secs. C, D, and F [45].

We implement this framework using the topography model derived from NASA’s GRAIL Model 2 data [26], which is already visualized in Fig. 1(a). For comparison, the one-dimensional SNREI model is adopted from our previous work with slight modifications [48]. To maintain computational tractability while preserving key large-scale asymmetries, notably the farside-nearside dichotomy, we truncate the spherical-harmonic expansion of the Moon at $l_{\text{max}} = 4$, yielding the approximate lunar model shown in Fig. 1(b). In the hybridized mode calculation, we allow free-oscillation modes up to $l = 6$ and $n = 100$, reflecting both the selection rule and numerical feasibility. Toroidal modes are excluded because GWs couple predominantly to spheroidal modes, and toroidal eigenfunctions exhibit lower accuracy in standard solvers such as MineOS [50]. While computing resource limits this perturbation analysis to a relatively narrow frequency band under ~ 0.02 Hz [51], the physical mechanism of energy redistribution validated here provides the fundamental explanation for the broadband amplification observed in our deci-hertz numerical results.

Figure 3(a) shows the band-integrated total magnification $M_E(\vec{r})$ across the lunar surface. This map reveals a robust and systematic pattern, with values deviating from unity by approximately 10%, consistent with the crustal-thickness dependence observed in our 2D numerical simulations. This spatial pattern exhibits a clear correlation with the underlying crustal structure shown in Fig. 1(b): regions of thin crust, such as the nearside maria, consistently exhibit deamplification ($M_E < 1$), whereas the thick-crust terrains of the farside highlands show pronounced amplification. This correlation confirms the role of crustal thickness in modulating the Moon’s global seismic

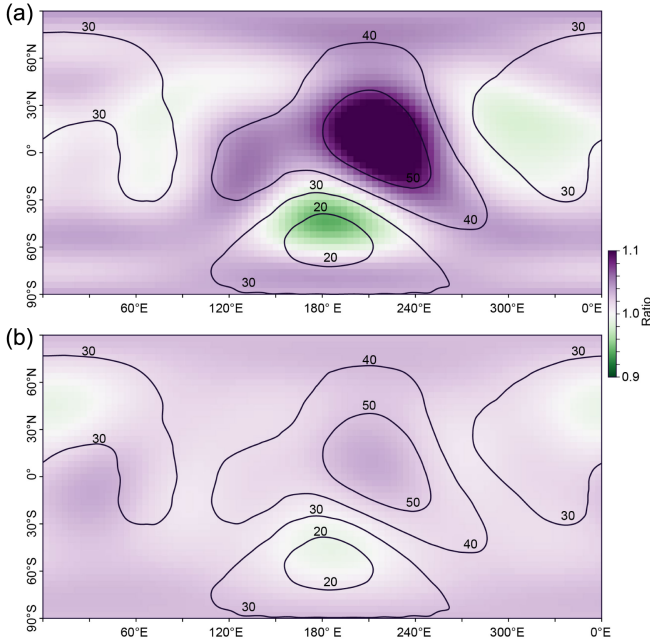


FIG. 3. Amplification according to the normal-mode perturbation theory. (a) Total energy magnification M_E , integrated between 0.01 and 0.012 Hz (same as frequency range in Fig. S3 [45]) and averaged over polarization. Here, all the free-oscillation modes up to $l = 6$ are included. (b) Contribution from the $l = 2$ free-oscillation mode alone. For comparison, the crustal thickness contours from Fig. 1(b) are overlaid.

response to GWs. It also points to a direct link between lateral heterogeneity and surface observables.

To more clearly trace the cause of the amplification, Fig. 3(b) isolates the contribution to M_E from the fundamental quadrupole mode $\vec{u}_{l=2}$ alone. The resulting map shows values near unity, with much smaller variation ($\sim 4\%$) and weaker correlation to crustal thickness. This result implies that the amplification pattern seen in Fig. 3(a) is not primarily due to modulation of the $l = 2$ mode, but rather emerges from the collective response of higher-order modes ($l > 2$) excited through mode mixing.

Discussions and implications—In this Letter, we have elucidated how the rugged surface and heterogeneous crust of the Moon can modulate the surface seismic signals triggered by GWs. We achieve this by developing a calculation framework that combines the advantages of two complementary approaches: a perturbation theory provides an efficient and transparent physical picture of global mode mixing, whereas the SEM delivers the high spatial resolution required to capture complex local wave-field distortions at the expense of greater computational cost.

Our SEM simulation not only recovers the GW-induced quadrupole mode of the lunar free oscillation but also reveals a close link of the seismic amplitude to the crustal thickness. On average, the seismic signal can be amplified by 10% in thick-crust regions. In some narrow frequency ranges, the amplification can even exceed one order of

magnitude. Crucially, our high-frequency simulations confirm that this strong amplification persists around 0.1 Hz, agreeing well with the optimal sensitivity window of proposed lunar seismometer arrays. These findings establish a direct link between crustal structure and GW detectability. Regions characterized by thicker crust, particularly the farside highlands, consistently exhibit an amplification of the GW signal, therefore offer better opportunities for GW detection. However, the relationship between topography and signal amplification is highly complex and frequency-dependent. While broadband integrated magnification correlates positively with crustal thickness, our high-resolution simulations reveal that in certain narrow frequency bands (e.g., around 0.06 Hz), local topography can induce strong focusing effects probably caused by trapped waves in basins, leading to significant signal amplification even in regions with a relatively thin crust. This structural complexity underscores the importance of our 2D and 3D modeling in real application scenarios, such as the Lunar Gravitational-Wave Antenna or Laser Interferometer Lunar Antenna. Optimal site selection for seismometers cannot rely solely on macroscale crustal thickness; it requires mission-specific, high-resolution numerical simulations tuned to the detectors' target sensitivity bands to fully exploit these localized, frequency-dependent amplification zones.

Our simulations also pinpoint the strongest signal amplification in a couple of frequency bands that can be as narrow as ~ 0.1 mHz (Fig. S3 [45]). Resolving such narrow features requires long-duration, low-noise observations since a bandwidth of 0.1 mHz implies a minimum observational period of $T \sim 1/(0.1 \text{ mHz}) = 10^4 \text{ s} \approx 2.8 \text{ h}$. Therefore, continuous seismic data collection spanning one day or longer is recommended.

The correlation between signal amplification and crust thickness is also verified by our analytical normal-mode perturbation model. The analysis further identifies mode mixing as the origin of the enhancement: the Moon's crustal asymmetry reshapes its eigenmodes, enabling the energy from a purely quadrupolar perturbation, such as that imposed by a passing GW, to transfer into an ensemble of higher-order free-oscillation modes. Such a correlated, structured signal across the lunar surface differs fundamentally from the stochastic, spatially incoherent background of seismic noise. By employing a spatial correlation approach (e.g., see Ref. [52]) that incorporates our theoretical model of the hybridized modes, we can in principle statistically separate the GW-induced seismic response from the spectrally and spatially diffuse lunar seismic background.

Several caveats are worth mentioning. Although computational costs restricted our SEM simulations to 2D, previous 2D-vs-3D comparisons [53] support the reliability of this approach. Furthermore, despite differing dimensionalities, the 2D SEM and fully 3D mode-coupling analysis yield consistent, crustal-thickness-dependent amplification

patterns at low frequencies (Figs. 2 and 3). This agreement suggests the 2D great-circle model captures the dominant GW response, since the $l = 2$ driving force and leading hybridized modes project strongly onto this plane, implying that 3D SEM simulations will largely preserve these qualitative trends. Analytically, the mode-coupling calculation excluded toroidal modes and was truncated at a relatively low spherical harmonic degree. Furthermore, our current model does not resolve the km- to sub-km-scale heterogeneity or the pervasive fracturing of the lunar megaregolith [54–56]. Including them may affect the lunar response at the upper end of the deci-hertz band, where the seismic wavelength becomes comparable to these small scatterers. In this regime, multiple scattering randomizes the phase of the GW-induced wave field and, as a result, prolongs the seismic signal [57,58].

Future work should bridge these gaps by performing fully 3D SEM simulations and extending the mode-coupling basis to higher l_{max} . Additionally, with finer topographical models and expanded computational resources, subsequent simulations could probe higher-frequency bands, where small-scale scattering and fracturing are expected to enhance the spatial and spectral variability of the local response. Finally, while this study focuses on polarization-averaged global correlations, individual GW events may exhibit sharper local resonances due to constructive interference; modeling these complex, event-specific interactions will be crucial for precise source parameter estimation in future observations.

Finally, we emphasize that the integration of global mode-coupling theory with high-resolution SEM simulations provides the framework necessary to reach the target of calibrating the Moon as a resonant GW antenna. Beyond enabling the detection of mid-band GW sources, this approach also works reciprocally. In principle, observed GW signals could be analyzed to constrain the Moon’s 3D internal structure [59], complementing traditional seismic surveys. This interdependence hints at a promising future for lunar GW seismology as a dual-purpose diagnostic tool.

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J. Z. and X. C. conceived and designed the research. L. Z. performed the numerical SEM simulations and

amplification analysis. H. Y. developed the theoretical mode-coupling framework and conducted the semianalytical calculations. L. Z. and H. Y. contributed equally to the data analysis and drafted the manuscript. All authors contributed to the interpretation of results and the revision of the paper.

Data availability—The data that support the findings of this article are openly available [48].

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