

How Lorentz Boosts Reshape Relaxation Spectra

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In relativity, relaxation processes are often assumed to undergo time dilation under Lorentz boosts. We show that this intuition fails generically. Because of relativity of simultaneity, Lorentz boosts can split a single relaxation mode into a continuum of excitations, with a width set by the maximal signal propagation speed. Focusing on linearized relativistic (kinetic or rheological) theories with an Onsager-type symmetry, we derive rigorous bounds on relaxation spectra in arbitrary inertial frames, expressed solely in terms of rest-frame spectral data at zero wave number. As a consequence, nonhydrodynamic gaps, maximal relaxation rates, and the convergence radii of hydrodynamic modes obey nontrivial Lorentz-covariant constraints. These results provide a unified framework for understanding how relativity constrains relaxation dynamics in many-body systems.

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Introduction—Much of what we know about fluids follows from the study of their mode structure. Starting from a homogeneous equilibrium state, one introduces a small perturbation, and expands it in Fourier modes $e^{ikx-i\omega t}$ (with $k \in \mathbb{R}$). The resulting complex frequencies $\omega(k)$ determine how wave packets propagate and decay, and thereby characterize the fluid’s response properties. Consequently, considerable effort has been devoted to rigorously analyzing and classifying mode spectra in hydrodynamics, kinetic theory, and holography [1–20].

Such analyses are usually carried out in the reference frame where the equilibrium state is at rest. In Newtonian physics, this entails no loss of generality, since the dispersion relations $\omega(k)$ and $\omega'(k')$ measured by two observers $\mathcal{O}$ and $\mathcal{O}'$ moving at relative velocity v satisfy

$$\begin{aligned}\omega' &= \omega + vk \\ k' &= k\end{aligned}\quad (\text{Galilei boost}). \quad (1)$$

Since k is invariant (and real, for Fourier modes), phases and group velocities simply shift by v , while the damping rate is untouched ($\text{Im } \omega' = \text{Im } \omega$).

In relativity, the picture changes qualitatively [21–24]. Relativity of simultaneity causes k' to depend on ω :

$$\begin{aligned}\omega' &= \gamma(\omega + vk) \\ k' &= \gamma(k + v\omega)\end{aligned}\quad (\text{Lorentz boost}), \quad (2)$$

with $\gamma = (1 - v^2)^{-1/2}$. As a result, the mapping between $\omega(k)$ and $\omega'(k')$ is now implicit, and a mode that appears as a Fourier excitation to $\mathcal{O}$ (i.e., $k \in \mathbb{R}$) no longer corresponds to a Fourier mode for $\mathcal{O}'$ (i.e., $k' \notin \mathbb{R}$) [22]. This can change most qualitative features of the spectrum, even for the simplest systems. Consider, for instance, the diffusive mode $\omega = -ik^2$: a single-valued, stable dispersion relation (i.e., $\text{Im } \omega \leq 0$ for $k \in \mathbb{R}$ [10,21]), gapless, and analytic with infinite radius of convergence around $k = 0$. After a boost with, say, $v = 1/2$, one instead finds two branches, $\omega'_\pm = 2k' + \sqrt{3}i(1 \pm \sqrt{1 - \sqrt{3}ik'})$, one of which is unstable and gapped, and both of which possess only a finite radius of convergence about $k' = 0$.

Recently, the appearance of the unstable branch ω'_+ was shown to be connected to the ability of the diffusion equation $\omega = -ik^2$ to propagate information superluminally [22]. It was further established that, if the underlying theory is causal, “stability” (or the lack thereof) is promoted to a Lorentz-invariant property [25]. This prompts a broader question: are there additional, physically motivated assumptions (besides causality) that ensure Lorentz invariance of other spectral features, like the existence of gaps or the analyticity properties of $\omega(k)$?

In this Letter, we demonstrate that, when the equations governing relaxation to equilibrium obey a certain “Onsager-type symmetry” (which is a built-in feature of kinetic theory and rheology), a wide variety of spectral properties become Lorentz invariant. In particular, one can place bounds on both magnitudes and convergence radii of the frequencies measured by a moving observer solely in terms of rest-frame spectral features at $k = 0$.

Conventions—We adopt the metric signature $(-, +, +, +)$, and work in natural units, $c = \hbar = k_B = 1$.

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Symmetry assumption—Consider a homogeneous equilibrium state, and let $\Psi(x^\mu) \in \mathbb{C}^D$ (with $D \in \mathbb{N}$) be a list of linearized fields describing small perturbations about it. These may represent, for example, fluctuations of the moments of the kinetic distribution function [26], or a set of transient hydrodynamic variables. We assume that their linear evolution is governed by a first-order system of coupled partial differential equations,

$$\mathbb{E}^\mu \partial_\mu \Psi = -\sigma \Psi, \quad (3)$$

where $\mathbb{E}^\mu$ and σ are constant background matrices. Our key structural assumption is that all these matrices are *Hermitian*: $\mathbb{E}^\mu = (\mathbb{E}^\mu)^\dagger$ and $\sigma = \sigma^\dagger$. Furthermore, we require that σ be non-negative definite, and $\mathbb{E}^0$ be positive definite in *every* inertial frame. It follows that the quadratic forms $\Psi^\dagger \mathbb{E}^\mu \Psi$ constitute a future-directed timelike (or null) vector, which implies causality [28,29] (if $\mathbb{E}^\mu$ were non-Hermitian, one would need to compute the characteristics explicitly, to determine causality).

The above assumptions seem rather restrictive, but are satisfied by a very broad class of matter models [12,30–32], including relativistic kinetic theory [16,33], radiation transport [34,35], Israel-Stewart theory [36–38], divergence-type theories [28,39], GENERIC [40,41], Carter’s multi-fluid formulation [42–44], and theories of (super)solidity [32,45]. Indeed, the constraints on the structure of $\mathbb{E}^\mu$ and σ can be traced back to Onsager’s theory [46–48]: $\mathbb{E}^0$ and σ are Hessians of the free-energy density and the entropy production rate, respectively (and so are positive definite in all frames), while Hermiticity arises as a reciprocal relation related to *PT* symmetry [16,31] (see Supplemental Material for more details [49]).

We stress that, in the present Letter, we are mostly interested in models whose nonhydrodynamic sector is physical. Hence, frameworks like Israel-Stewart should be regarded here not just as causal extensions of hydrodynamics [50], but as mesoscopic “quasihydrodynamic” [3,51,52] models of non-Newtonian fluids [53–58].

Some basic invariant statements—To warm up, we derive a few elementary Lorentz-invariant features of the excitation spectrum that follow immediately from our assumptions. To this end, consider perturbations of the form $\Psi \propto e^{ik_\mu x^\mu}$, with complex k^μ . Then, Eq. (3) reduces to

$$k_\mu \mathbb{E}^\mu \Psi = i\sigma \Psi. \quad (4)$$

Multiplying both sides by $\Psi^\dagger$, we obtain

$$k_\mu \Psi^\dagger \mathbb{E}^\mu \Psi = i\Psi^\dagger \sigma \Psi. \quad (5)$$

Taking the real and imaginary parts of Eq. (5), and noting that $\Psi^\dagger \mathbb{E}^\mu \Psi$ and $\Psi^\dagger \sigma \Psi$ are real, we obtain

$$\begin{aligned} (\Psi^\dagger \mathbb{E}^\mu \Psi) \text{Re} k_\mu &= 0, \\ (\Psi^\dagger \mathbb{E}^\mu \Psi) \text{Im} k_\mu &= \Psi^\dagger \sigma \Psi \geq 0. \end{aligned} \quad (6)$$

Since $\Psi^\dagger \mathbb{E}^\mu \Psi$ is timelike future directed, it follows that $\text{Re} k^\mu$ is spacelike, and $\text{Im} k^\mu$ lies outside the future light cone (Ref. [59], Sec. 1.4.2) (it is either spacelike or past directed). To see what this implies in practice, take $k^\mu = (\omega, k, 0, 0)$. The real-part equation then gives $|\text{Re} \omega| \leq |\text{Re} k|$, which means that the phase velocity $\text{Re} \omega / \text{Re} k$ never exceeds the speed of light in any of these models (although the group velocity can). This behavior is well known from both hydrodynamics [2] and kinetic theory (Ref. [60], Sec. 9.3.1) [61]. We also recover the standard result that $\omega \in i\mathbb{R}$ at $k = 0$, a universal feature of kinetic and transient hydrodynamic theories [12]. Because of the manifest covariance of (6), we see that these statements hold in all reference frames.

The imaginary-part equation instead yields $\text{Im} \omega \leq |\text{Im} k|$, which expresses the fact that these models are stable in all inertial frames [25,62] and that their dispersion relations respect microcausality [63].

Conservation laws—Before presenting our main result, we must introduce a few relevant concepts.

Let Φ_I ($I = 1, 2, \dots, N$) be a basis of $\ker(\sigma)$. Since $\sigma = \sigma^\dagger$ and $\sigma \Phi_I = 0$, it follows that $\Phi_I^\dagger \sigma = (\sigma \Phi_I)^\dagger = 0$. Contracting (3) with $\Phi_I^\dagger$ then yields a set of linearized local conservation laws:

$$\partial_\mu J_I^\mu = 0, \quad (7)$$

for the currents $J_I^\mu = \Phi_I^\dagger \mathbb{E}^\mu \Psi$. This set is complete.

Indeed, suppose that there exists another conserved current of the form $J^\mu = \mathbb{A}^\mu \Psi$, where $\mathbb{A}^\mu \in (\mathbb{C}^D)^\dagger$ are constant. Introduce $\mathbb{B}^\mu = \mathbb{A}^\mu (\mathbb{E}^0)^{-1}$, which is well defined because $\mathbb{E}^0$ is positive definite. Then,

$$\partial_\mu J^\mu = 0 \Rightarrow \mathbb{B}^\mu \mathbb{E}^0 \partial_\mu \Psi = 0. \quad (8)$$

Subtracting $\mathbb{B}^0 \times (3)$, we get $(\mathbb{B}^j \mathbb{E}^0 - \mathbb{B}^0 \mathbb{E}^j) \partial_j \Psi = \mathbb{B}^0 \sigma \Psi$. For this relation to hold for arbitrary initial data $\Psi(0, x^j)$, we must require $\mathbb{B}^0 \sigma = 0$, which implies $\mathbb{B}^0 = \sum_I \lambda^I \Phi_I^\dagger$ for some constants λ^I , and $\mathbb{B}^j \mathbb{E}^0 = \mathbb{B}^0 \mathbb{E}^j$, which then gives

$$J^\mu = \mathbb{B}^0 \mathbb{E}^\mu \Psi = \sum_I \lambda^I J_I^\mu. \quad (9)$$

Hence, all linearized conserved currents are linear combinations of those generated by the basis Φ_I .

Nearby equilibria—The vectors Φ_I play an additional key role. One immediately verifies that any constant state of the form

$$\Psi(x^\mu) = \sum_I c^I \Phi_I, \quad c^I = \text{const} \in \mathbb{C} \quad (10)$$

solves (3). Such configurations have the property that the associated conserved density,

$$\sum_I (c')^* J_I^0 = \sum_I (c')^* \Phi_I^\dagger \mathbb{E}^0 \Psi = \Psi^\dagger \mathbb{E}^0 \Psi, \quad (11)$$

is nonzero for $\Psi \neq 0$. Since Ψ parametrizes linear departures from the background equilibrium, the states in (10) may be viewed as nearby equilibria obtained by infinitesimally shifting the conserved densities.

Nonhydrodynamic spectrum—Fix an observer $\mathcal{O}$ with four-velocity u^μ , and look for exponential solutions $\Psi \propto e^{ik_\mu x^\mu}$ that are spatially uniform (i.e., have $k = 0$) in the frame of $\mathcal{O}$, so that $k^\mu = \omega u^\mu$, with ω the frequency measured by $\mathcal{O}$. Substituting this ansatz into (4) yields

$$\omega(-u_\mu \mathbb{E}^\mu) \Psi = -i\sigma \Psi. \quad (12)$$

This can be solved by working on a basis in Ψ space [64] such that $-u_\mu \mathbb{E}^\mu = 1$ and $\sigma = \text{diag}(\sigma_n)$, with $\sigma_n \geq 0$. In this basis, (12) reduces to $\sigma \Psi = i\omega \Psi$. The eigenmodes are therefore the unit vectors $\Psi_n = (0, \dots, 0, 1 \text{ at } n, 0, \dots, 0)^T$, with eigenfrequencies $\omega_n = -i\sigma_n$. Among them, N modes have $\omega = 0$, and correspond to the equilibria (10). The remaining $D - N$ modes have $i\omega_n = \sigma_n > 0$, and define the *nonhydrodynamic spectrum* (at $k = 0$) relative to $\mathcal{O}$.

Main theorem—We can finally state our main result.

Theorem 1—Consider two inertial observers $\mathcal{O}$ and $\mathcal{O}'$ moving at relative speed $v \geq 0$. Suppose that the nonhydrodynamic spectrum (at $k = 0$) relative to $\mathcal{O}$ is confined within some interval $a \leq i\omega \leq b$ (with a non-negative). Then, the nonhydrodynamic spectrum (at $k' = 0$) relative to $\mathcal{O}'$ is confined within the interval

$$\frac{a(1-v)}{\gamma} \leq i\omega' \leq \frac{b}{\gamma(1-v)}. \quad (13)$$

Proof—We carry out the analysis in the rest frame of $\mathcal{O}$, and choose a basis in Ψ space such that $\mathbb{E}^0 = 1$ in such a frame. Then, the nonhydrodynamic modes relative to $\mathcal{O}$ fulfill the equation

$$i\omega \Psi = \sigma \Psi \quad (\text{with } i\omega > 0). \quad (14)$$

Multiplying both sides by $\Phi_I^\dagger$ ($I = 1, \dots, N$), we find that $\Phi_I^\dagger \Psi = 0$. Hence, the nonhydrodynamic frequencies are the eigenvalues of σ restricted to $(\ker \sigma)^\perp$. A theorem of spectral theory (Ref. [65], Theorem 2.19) states that the extremal values of $\Psi^\dagger \sigma \Psi / (\Psi^\dagger \Psi)$ are eigenvalues of σ , so that

$$\begin{aligned} \inf_{(\ker \sigma)^\perp} \frac{\Psi^\dagger \sigma \Psi}{\Psi^\dagger \Psi} &= \inf_{(\ker \sigma)^\perp} \text{Spectrum}(\sigma) \geq a, \\ \sup_{(\ker \sigma)^\perp} \frac{\Psi^\dagger \sigma \Psi}{\Psi^\dagger \Psi} &= \sup_{(\ker \sigma)^\perp} \text{Spectrum}(\sigma) \leq b. \end{aligned} \quad (15)$$

Now, if we orient the axes so that the four-velocity of $\mathcal{O}'$ is $u'^\mu = (\gamma, -\gamma v, 0, 0)$, then the nonhydrodynamic modes

relative to $\mathcal{O}'$ fulfill the equation $i\gamma\omega'(1+v\mathbb{E}^1)\Psi = \sigma\Psi$ (with $\omega' > 0$). Multiplying both sides by either $\Psi^\dagger$ or $\Phi_I^\dagger$ ($I = 1, \dots, N$), we obtain

$$i\gamma\omega' = \frac{\Psi^\dagger \sigma \Psi}{1 + v\Psi^\dagger \mathbb{E}^1 \Psi}, \quad \Phi_I^\dagger \Psi = -v\Phi_I^\dagger \mathbb{E}^1 \Psi, \quad (16)$$

where we have normalized Ψ such that $\Psi^\dagger \Psi = 1$. Since $\Psi^\dagger \mathbb{E}^\mu \Psi$ is timelike or null, we have that $|\Psi^\dagger \mathbb{E}^1 \Psi| \leq \Psi^\dagger \mathbb{E}^0 \Psi = 1$. Hence, the denominator in (16) is bounded between $1 - v$ and $1 + v$. To bound the numerator, we take Φ_I to be orthonormal, and define the matrix $\mathbb{P} = 1 - \sum_I \Phi_I \Phi_I^\dagger$, which is the orthogonal projector ($\mathbb{P}^2 = \mathbb{P}^\dagger = \mathbb{P}$) onto $(\ker \sigma)^\perp$. Clearly, $\Psi^\dagger \sigma \Psi = \Psi^\dagger \mathbb{P}^\dagger \sigma \mathbb{P} \Psi$, so we can use (15) to derive the following inequalities:

$$a\Psi^\dagger \mathbb{P} \Psi \leq \Psi^\dagger \sigma \Psi \leq b\Psi^\dagger \mathbb{P} \Psi \leq b. \quad (17)$$

The upper bound in (13) is immediately recovered. To obtain the lower bound, we invoke the definition of $\mathbb{P}$ and the second condition in (16):

$$\begin{aligned} \Psi^\dagger \mathbb{P} \Psi &= 1 - \sum_I |\Phi_I^\dagger \Psi|^2 = 1 - v^2 \sum_I |\Phi_I^\dagger \mathbb{E}^1 \Psi|^2 \\ &\geq 1 - v^2 \|\mathbb{E}^1 \Psi\|^2 \geq 1 - v^2. \end{aligned} \quad (18)$$

Here, we used Bessel's inequality and the spectral norm identity $\|\mathbb{E}^1\| \equiv \sup_{\|\Phi\|=1} |\Phi^\dagger \mathbb{E}^1 \Phi| \leq 1$ [Ref. [65], Eq. (2.65)]. ■

Note that, to simplify the presentation, we have been working with a finite-dimensional Ψ (see Ref. [26]). The argument, however, extends directly to the case where Ψ lives in a Hilbert space, up to standard functional-analytic subtleties [65,66]. For illustration, Supplemental Material provides a complete proof of Theorem 1 within linearized kinetic theory [49].

Two quick examples—To test the bounds, we examine two representative models with $a = b = 1$, in some units. Our first example is Cattaneo's theory of diffusion [67], which in the rest frame takes the form

$$\begin{aligned} \partial_t T + w \partial_j q^j &= 0, \\ \partial_t q^j + w \partial^j T &= -q^j, \end{aligned} \quad (19)$$

and satisfies all of our hypotheses, provided $|w| \leq 1$. In a boosted frame, this model features three nonhydrodynamic modes, with eigenfrequencies

$$i\omega'_1 = \frac{1}{\gamma(1-v^2w^2)}, \quad i\omega'_{2,3} = \frac{1}{\gamma}. \quad (20)$$

Our second example is kinetic theory in the relaxation time approximation (RTA), $p^\mu \partial_\mu \delta f = p^\mu u_\mu (\delta f - \delta f_{\text{eq}})$, where $\delta f(x^\mu, p^\alpha)$ is the perturbed distribution function. In a

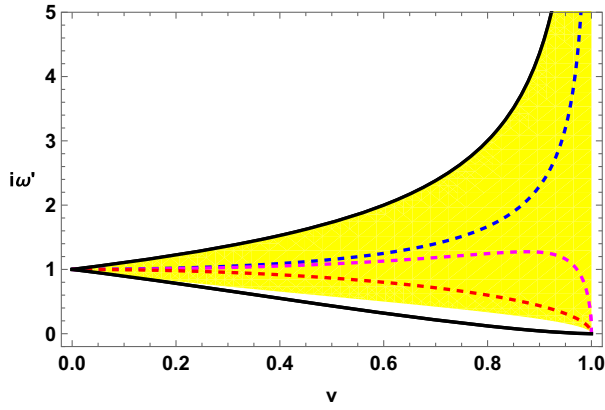


FIG. 1. Splitting of a single nonhydrodynamic frequency $i\omega = 1$ into multiple frequencies $i\omega'$ induced by a boost of speed v . The black curves show the bounds (13), with $a = b = 1$, and they delimit the allowed region. The dashed curves refer to Cattaneo's eigenfrequency $i\omega'_1$, see Eq. (20), with $w = 0$ (red curve), 0.9 (magenta curve), and 1 (blue curve); the red curve also captures the modes $i\omega'_{2,3}$. The yellow region covers the continuum of modes (21) of RTA.

boosted frame, this model exhibits a continuous family of nonhydrodynamic excitations of the form

$$i\omega' = \frac{1}{\gamma(1 + vw)} \quad (\text{with } w \in [-1, 1]). \quad (21)$$

This splitting arises due to relativity of simultaneity: in the rest frame, particles travel different intervals $\Delta x = w\Delta t$ in the same amount of time $\Delta t \sim \tau$. When we boost, we have $\Delta t' = \gamma(\Delta t + v\Delta x) \sim \gamma(1 + vw)\tau = 1/(i\omega')$.

In Fig. 1, we graph the modes of these two models against bounds (13). RTA saturates the upper bound. We are not aware of a theory saturating the lower bound.

Some applications—We now discuss some important consequences of Theorem 1, starting with two corollaries that are particularly relevant for kinetic theory.

Corollary 1—If the nonhydrodynamic spectrum possesses a fastest relaxation timescale at $k = 0$ in one inertial frame, then the same is true in all inertial frames.

Corollary 2—If the nonhydrodynamic spectrum is gapped at $k = 0$ in one inertial frame, then the same is true in all inertial frames.

Both results follow directly from Theorem 1, by choosing $a = 0$ and finite b in the first case, and finite a with $b = \infty$ in the second.

Corollary 2 has an important implication. In [68] it was shown that, in kinetic theory, a gap in the nonhydrodynamic spectrum guarantees that all hydrodynamic dispersion relations admit expansions as power series in k with a finite radius of convergence. The proof does not rely on the background being at rest, and therefore the result persists under boosts. Combining this with Corollary 2, we obtain the following statement.

Corollary 3—If the nonhydrodynamic spectrum of a kinetic theory is gapped at $k = 0$ in one inertial frame, then the hydrodynamic dispersion relations possess nonvanishing convergence radii in all inertial frames.

This means that, if a genuine separation of scales exists, density-frame hydrodynamics [69–71] is well defined up to infinite order in derivatives in every inertial frame [72].

Convergence radius of boosted hydrodynamics—Consider a kinetic description of a system admitting a single conserved current, $\partial_\mu J^\mu = 0$. Assume that its nonhydrodynamic spectrum is gapped in the rest frame, with slowest relaxation rate a . In a frame boosted with velocity v , the corresponding spectral gap is $a' \geq a(1 - v)/\gamma$. In that boosted frame, the unique hydrodynamic mode $\omega'(k')$ propagating along the x direction solves

$$(\sigma + \mathbb{E}^1 ik')\Psi = i\omega'\Psi, \quad (22)$$

where we work on a basis for which $\mathbb{E}^0 = 1$. Upon complexifying k' , this equation defines an eigenvalue problem for the perturbed family $\sigma(ik') \equiv \sigma + ik'\mathbb{E}^1$. Since both σ and $\mathbb{E}^1$ are Hermitian (with $\|\mathbb{E}^1\| \leq 1$), and 0 is a nondegenerate eigenvalue of σ , standard results from analytic perturbation theory apply (Ref. [73], Secs. 3 and 5). In particular, there exists a unique, analytic hydrodynamic mode $\{\omega'(k'), \Psi(k')\}$ in a neighborhood of $k' = 0$, whose radius of convergence R' satisfies the lower bound [74]:

$$R' \geq \frac{a'}{2\|\mathbb{E}^1\|} \geq \frac{a(1 - v)}{2\gamma}. \quad (23)$$

About time dilation—It is natural to expect that, as the velocity of the fluid approaches the speed of light, all nonhydrodynamic modes should effectively “freeze” due to time dilation. From this viewpoint, one would predict that the upper bound on $i\omega'$ should vanish as $v \rightarrow 1$. Figure 1 shows that the opposite happens: the upper bound diverges. Where does our intuition fail?

The time-dilation argument implicitly assumes that signals are advected with the fluid element. Indeed, if, in the rest frame, a signal propagates from $(t, x) = (0, 0)$ to $(t, x) = (\Delta t, 0)$ before decaying, then in a frame where the medium moves at velocity v , the signal travels from $(t', x') = (0, 0)$ to $(t', x') = (\gamma\Delta t, \gamma v\Delta t)$. Under this assumption, one finds that $i\omega' \sim 1/(\gamma\Delta t)$, which tends to zero as $v \rightarrow 1$, in line with expectations. The problem appears if the signal has a nonzero rest-frame propagation speed w . If in the rest frame it moves from $(t, x) = (0, 0)$ to $(t, x) = (\Delta t, -w\Delta t)$, then in the boosted frame it travels to $(t', x') = (\gamma\Delta t - \gamma vw\Delta t, -\gamma w\Delta t + \gamma v\Delta t)$, which leads to $i\omega' \sim 1/[\gamma(1 - vw)\Delta t]$. Differentiating with respect to v then shows that, for $v < w$, the boosted frequency *increases* with v , in sharp contrast with the simple time-dilation picture. Only once $v > w$ does time dilation dominate,

forcing the frequency downward. In the limiting case of a luminal signal ($w = 1$), we recover $i\omega' \sim 1/[\gamma(1-v)\Delta t]$, which has the same v dependence as the upper bound in (13), and diverges as $v \rightarrow 1$.

The argument above suggests that, if the medium supports signal propagation only inside an “acoustic cone” narrower than the light cone, then the boosted nonhydrodynamic spectrum should obey bounds sharper than (13). This is made precise in the following theorem.

Theorem 2—Consider a medium whose $k = 0$ nonhydrodynamic spectrum lies within some interval $a \leq i\omega \leq b$ (with $a \geq 0$) in the rest frame. Assume further that signal propagation in the rest frame is bounded by a maximal speed $w \leq 1$. Then, for an observer moving at speed $v \geq 0$ relative to the medium, the corresponding $k' = 0$ nonhydrodynamic frequencies are confined within the interval

$$\frac{a(1-vw)}{\gamma} \leq i\omega' \leq \frac{b}{\gamma(1-vw)}. \quad (24)$$

Proof—The characteristic propagation speeds v_n of signals traveling along the x direction are determined by the condition $\det(\mathbb{E}^1 - v_n \mathbb{E}^0) = 0$ [37]. In a basis where $\mathbb{E}^0 = 1$, the v_n coincide with the eigenvalues of $\mathbb{E}^1$, so imposing that no signal exceed the maximal speed w is equivalent to the operator bound $\|\mathbb{E}^1\| \leq w$. With this identification, the derivation of Theorem 1 carries over unchanged when $\mathcal{O}$ is chosen to comove with the medium. The only modification is that, in Eq. (16), the denominator is now bounded between $1 - vw$ and $1 + vw$. Likewise, in the final step of (18), we can replace $1 - v^2$ with $1 - v^2 w^2$. ■

In Fig. 2, we plot the bounds (24) with $a = b = 1$, for various values of w . In Supplemental Material we test such bounds with some randomly generated models [49].

Nonrelativistic media—In the regime where the microscopic dynamics are nonrelativistic in the rest frame (so that $w \ll 1$), the usual time-dilation intuition is recovered: the bounds collapse to $a/\gamma \leq i\omega' \leq b/\gamma$. Indeed, in this limit, the boosted eigenvalue equation $i\gamma\omega'(1 + v\mathbb{E}^1)\Psi = \sigma\Psi$ simplifies to $i\gamma\omega'\Psi = \sigma\Psi$, implying that $\gamma\omega'$ matches a corresponding rest-frame nonhydrodynamic frequency.

Conclusions—We have set rigorous foundations for a relativistically covariant spectral theory of moving fluids, which applies to a wide variety of substances, including gases, solids, superfluids, and supersolids [32,45]. The resulting physics departs sharply from naive time-dilation expectations: in fully relativistic media, the equilibration timescale of some modes can become infinitely fast under large boosts (see Fig. 1). This divergence, however, is avoided if the medium has a strictly subluminal signaling speed w (see Fig. 2). Moreover, the usual time-dilation intuition is recovered in substances that are nonrelativistic in the rest frame, for which $w \ll 1$.

Our results point to several promising directions for future investigation. A particularly natural extension

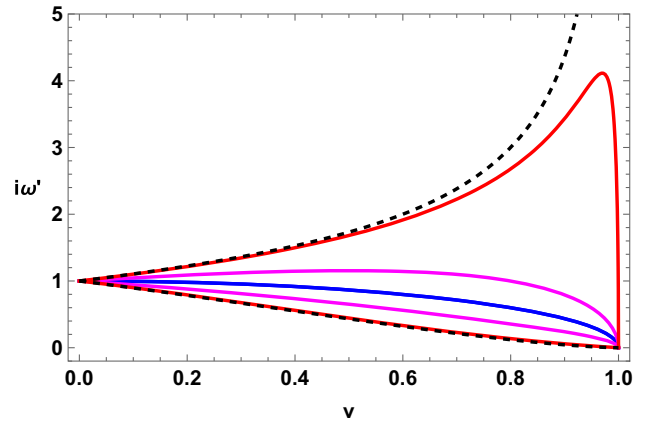


FIG. 2. Bounds (24) on the nonhydrodynamic spectrum of a moving fluid, assuming a single rest-frame nonhydrodynamic frequency $i\omega = 1$, and rest-frame signaling speed $w = 0$ (blue curve), 0.5 (magenta curve), 0.97 (red curve), and 1 (black dashed curve). For $w = 0$, the admissible region collapses to the time-dilation line $i\omega' = 1/\gamma$, while for $w = 1$ the result reduces to (13), which is the only case in which $i\omega'$ may diverge as $v \rightarrow 1$.

concerns the applicability of quasihydrodynamic [51,52,57,58,75] at large boosts. One can show that there exist systems where a parametrically slow mode is absorbed into the continuum past a critical velocity (see Supplemental Material [49]), raising the question of which additional assumptions are required for quasihydrodynamics to exist in all reference frames. Another important question is whether the bounds derived here extend to electrically charged fluids or to holographic quasinormal modes, neither of which obey the symmetry principle assumed in this Letter [12,19]. It is tempting to conjecture that conclusions such as Corollaries 1 and 2 remain valid in these broader settings, but establishing them without symmetry constraints is likely to require substantially more sophisticated techniques.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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