

Probing Stringy Horizons with Pole Skipping in Nonmaximal Chaotic Systems

Ping Gao^{*}

Kavli Institute for Theoretical Sciences, *University of Chinese Academy of Sciences*, Beijing 100190, China
and School of Quantum, *University of Chinese Academy of Sciences*, Beijing 100190, China

Hong Liu[†]

MIT Center for Theoretical Physics—A *Leinweber Institute*, *Massachusetts Institute of Technology*,
77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA



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In this Letter, we study pole skipping in nonmaximally quantum chaotic systems. Using Rindler conformal field theories and the large- q Sachdev-Ye-Kitaev chain as illustrative examples, we argue that the pole skipping points of few-body operators organize into trajectories in the complex frequency-momentum plane, with the leading trajectory encoding the quantum Lyapunov exponent. We further propose that these trajectories admit a natural interpretation as Regge trajectories of stringy excitations in a dual stringy black-hole geometry. From this perspective, pole skipping for an individual operator can be viewed as tracking the stringy horizon through the response of a single excitation. Our results suggest that pole skipping reflects intrinsic properties of quantum chaotic systems and may be deeply connected to the structure of horizons in the stringy regime.

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Introduction—Despite significant progress in recent years, characterizing quantum chaos in many-body systems remains a central challenge. Random matrix behavior [1] provides one guiding framework, while the eigenstate thermalization hypothesis [2] offers another. In large- N systems with many internal degrees of freedom, quantities such as the quantum Lyapunov exponent and the butterfly velocity—extracted from out-of-time-ordered correlators (OTOCs)—have emerged as key diagnostics of chaotic dynamics at short timescales [3–8].

For large- N systems that saturate the maximal Lyapunov exponent, another manifestation of quantum chaos—known as pole skipping [9,10]—has been identified. This phenomenon refers to the observation that the *thermal* retarded function of the energy density in momentum space exhibits lines of poles and zeros in the complex frequency plane, which intersect at a special point $(\omega, k)_{\text{p.s.}}$ given by

$$(\omega, k)_{\text{p.s.}} = i2\pi/\beta(1, 1/v_B), \quad (1)$$

where $2\pi/\beta$ (with β the inverse temperature) gives the maximal Lyapunov exponent λ_{max} , and v_B is the butterfly

velocity [3,4]. At this point, a would-be pole on the pole line is skipped due to an exact cancellation with a zero.

In maximally chaotic systems, the pole skipping phenomenon can be understood as a consequence of the dual role played by the hydrodynamic mode associated with energy density: it simultaneously encodes energy conservation and the propagation of chaos [10]. On the gravity side, this behavior has been traced to properties of metric perturbations near the horizon [11] and further related to a class of near-horizon symmetries [12]. Pole skipping in the energy-density channel has since been observed in all known maximally chaotic systems (see Ref. [13] for a general discussion), including holographic models dual to higher-derivative gravity theories [14–18].

Interestingly, pole skipping has also been observed across a wide class of operators and also in the *lower-half* frequency plane [11,15–25]. For example, for a scalar field, pole skipping occurs only in the lower-half frequency plane at $\omega = -i2\pi n/\beta$ with $n = 1, 2, 3, \dots$, with the corresponding momenta k being model-dependent; for each n , there are $2n$ pole skipping points in total [19].

Pole skipping presents a range of open puzzles. A central question concerns its status in nonmaximally chaotic systems. It was shown in [26] that, in the large- q Sachdev-Ye-Kitaev (SYK) chain [27] model—which is nonmaximal—the pole skipping location of the energy density remains given by (1), which appears to be unrelated to the nonmaximal Lyapunov exponent λ . This raises the question of whether the connection between pole skipping and chaos is restricted to maximal chaos.

^{*}Contact author: gaoping@ucas.ac.cn

[†]Contact author: hong_liu@mit.edu

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In holographic systems, this issue is also closely tied to the nature of the black-hole horizon in the stringy regime. In the Einstein regime, both the maximal Lyapunov exponent and the pole skipping phenomenon can be understood in terms of the local boost symmetry near the horizon, whereas deviations from maximal chaos in the stringy regime have often been interpreted as indications that the horizon becomes “fuzzy.”

In this Letter, we clarify the nature of pole skipping in general nonmaximally chaotic systems and use it as a probe of the stringy horizon in holographic settings. More specifically, we argue that (i) pole skipping should be viewed as a universal feature of chaotic systems; (ii) it provides a means to track both the leading bulk stringy Regge trajectory in a black-hole geometry and the associated Lyapunov exponent; and (iii) Rindler and black-hole horizons remain sharp even in the stringy regime, and their horizon symmetries persist.

Pole skipping of higher spin operators in a Rindler conformal field theory—To understand pole skipping for anti-de Sitter (AdS) black holes in the stringy regime from the perspective of the dual conformal field theory (CFT), one must analyze the corresponding phenomenon in the boundary theory on $\mathbb{R} \times S^{d-1}$ (or $\mathbb{R}^{1,d-1}$) in the large- N limit, at finite coupling, and above the Hawking-Page temperature. This, however, is a highly challenging problem over which we currently lack full theoretical control. As a first step, we therefore consider a simpler, solvable setting that—as we will argue below—may already capture key features relevant to more general systems.

Specifically, we study a CFT on $\mathbb{R}^{1,d-1}$ in its vacuum state and focus on the right Rindler wedge—hereafter referred to as the Rindler CFT—which corresponds to the CFT on $\mathbb{R} \times H_{d-1}$ at inverse temperature β , with H_{d-1} denoting a $(d-1)$ -dimensional hyperbolic space of radius $\beta/2\pi$. We set units such that $\beta = 2\pi$. Four-point functions of the original CFT on $\mathbb{R}^{1,d-1}$ in the Regge regime are mapped to OTOCs of the Rindler CFT in a nonmaximally chaotic regime [8,28,29]. Since two-point functions in the Rindler CFT are fixed by conformal symmetry, studying pole skipping in this setting provides an analytically tractable framework for probing this phenomenon in nonmaximally chaotic systems and, as we will see, for investigating the bulk AdS-Rindler horizon in the stringy regime.

Pole skipping of spin-1 and spin-2 (stress tensor) operators in the Rindler CFT has been previously studied in [30,31]. Here, we investigate pole skipping for general higher-spin operators with spin J , which, as we will see, uncovers a wealth of new and systematic information that cannot be captured from small-spin analyses or isolated higher-spin cases [32].

Consider a primary higher-spin operator $\mathcal{O}_{\mu_1 \dots \mu_J}$ with scaling dimension Δ and spin J . We focus on the component with all indices along the time direction, as

this component provides the most complete information about pole skipping. Introducing a polarization vector z^μ , the vacuum two-point function of the polarized operator $\mathcal{O}(x, z) = \mathcal{O}_{\mu_1 \dots \mu_J} z^{\mu_1} \dots z^{\mu_J}$ is given by

$$\langle \mathcal{O}(x, z) \mathcal{O}(x', z') \rangle = \frac{[-z \cdot I(x - x') \cdot z']^J}{[(x - x')^2]^\Delta}, \quad (2)$$

where $I_\nu^\mu(x) \equiv \delta_\nu^\mu - (2x^\mu x_\nu / x^2)$, and the overall normalization has been set to unity. When both x and x' lie in the Rindler wedge and are expressed in Rindler coordinates, (2) yields the thermal two-point function of the Rindler CFT. Fourier transforming this expression in Rindler coordinates allows us to obtain the corresponding retarded correlator in momentum space and identify the pole skipping points [33].

With ω and k denoting the frequency for the Rindler time and the magnitude of the spatial momentum [43], we find the pole skipping points are located at [44]

$$(\omega, k)_{\text{p.s.}} = \begin{cases} i(J - 2n - 1 - s, \gamma - s) & n \leq J \\ i(-n - 1 - s, \gamma + n - J - s) & n \geq J \end{cases}, \quad (3)$$

where $\gamma = \Delta - d/2$ and $n, s = 0, 1, 2, \dots$, together with another family obtained by taking $k \rightarrow -k$. We note the following features: (1) the highest pole skipping points for each J lies on the $(\Im \omega, \Im k)$ plane at

$$(\omega, k)_{\text{p.s.}}^{(\text{highest})} = i(J - 1, \Delta - d/2). \quad (4)$$

For the stress tensor, with $J = 2$ and $\Delta = d$, Eq. (4) can be related to (1), corresponding to the theory on $\mathbb{R}^{1,d-1}$ at finite temperature, where $v_B = 1/(d-1)$ by a shift $k \rightarrow k + i(d/2 - 1)$ [26,31]. (2) All these pole skipping points lie at the Matsubara frequencies (recall $\beta = 2\pi$ here),

$$\omega = 2\pi i(J - n)/\beta \quad \text{with} \quad n = 1, 2, 3, \dots, \quad (5)$$

which agrees with the results of [18] for bulk higher-spin fields in AdS black holes [45]. (3) For each J , writing $\omega = im$ with $m \leq J - 1$, we find that it is natural to separate the pole skipping points for each operator into two families based on the number N_m of pole skipping points for each m ,

$$N_m = \begin{cases} 2 \left\lfloor \frac{J-1-m}{2} \right\rfloor + 2 & -(J-1) \leq m \leq J-1 \\ 2|m| & m \leq -J \end{cases}. \quad (6)$$

The second group was motivated from the result of a scalar operator in holographic systems [19] where the number of point-skipping points at each ω_m are $2|m|$. For $J = 1$ and $J = 2$, the first group contains one and three

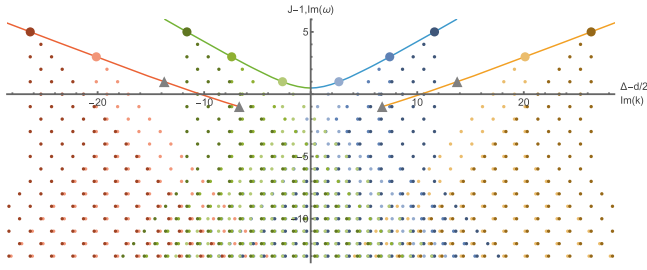


FIG. 1. The overlay of Regge trajectories and the pole skipping points. The horizon axis is $\Delta - d/2, \Im k$ and the vertical axis is $J - 1, \Im \omega$. The blue and green curve together is the leading Regge trajectory. The yellow and red curves are two symmetric subleading Regge trajectories. Each dot on the trajectories is a physical operator with even spin, which seeds a family of pole skipping points (3) (smaller dots with variant colors) below it. Pole skipping for odd spins likewise forms trajectories, but since they are not involved in the leading trajectory, we suppress them to avoid cluttering the plot. The triangles are missing or decoupling zeros on a subleading trajectory [50,53], where no physical operator exists at an integer spin, and are irrelevant to pole skipping.

rows of pole skipping points, respectively, which are consistent with previous works [30,31].

Recall that in a large- N CFT, operators can be organized into discretely spaced Regge trajectories on the (Δ, J) plane [35,46–49]. Each trajectory consists of an infinite sequence of physical operators whose scaling dimensions $\Delta(J)$ [or equivalently, $\gamma(J) = \Delta(J) - (d/2)$] take discrete values at integer spin. In principle, the full analytic Regge trajectory $\gamma(J)$, or its inverse function $j(\mu)$ (with $\gamma = i\mu$), can be reconstructed from these lattice points by analytic continuation in spin (see, e.g., Ref. [50] for a discussion). Among all the trajectories, the leading Regge trajectory $j_{\text{leading}}(\mu)$ —defined as the uppermost curve in the $j(\mu)$ versus μ plane, and hence the one with the largest intercept $j(0)$ —plays a particularly important role, as it governs the dominant behavior of conformal partial waves in the Regge limit of CFT correlators, much like its counterpart in flat-space S -matrix theory. In the Rindler CFT, the Lyapunov exponent extracted from an OTOC—equivalently, from the four-point function of the original CFT in the Regge regime—is determined by the intercept of $j_{\text{leading}}(\mu)$ at $\mu = 0$,

$$\lambda = j_{\text{leading}}(0) - 1. \quad (7)$$

The exponent λ encapsulates the collective contribution from the exchange of an infinite tower of higher-spin operators.

From (4), we see that the highest pole skipping point for each operator lies precisely on its Regge trajectory (see Fig. 1). Therefore, knowing the pole skipping points—which correspond to the locations of physical operators along their Regge trajectories—allows, in principle, the reconstruction of all the trajectories, including the leading

trajectory. This, in turn, enables us to extract all physical information encoded in the Regge data, including the nonmaximal Lyapunov exponent (7) [51]. Thus, while the pole skipping of the stress tensor or any single higher-spin operator may appear unrelated to the nonmaximal Lyapunov exponent λ , the collective set of pole skipping points across all operators reconstructs the full Regge trajectories and hence determines λ .

Stringy horizons and pole skipping for general theories—

The above discussion is specific to the Rindler CFT, where we established a connection between pole skipping and the nonmaximal Lyapunov exponent through the bridge of CFT Regge trajectories. In more general cases—such as a large- N CFT on $\mathbb{R}^{1,d-1}$ or $\mathbb{R} \times S^{d-1}$ at finite temperature and finite coupling (hence nonmaximally chaotic)—such a bridge does not appear to be available. In these theories, pole skipping need not be governed by the CFT Regge trajectories, and the Regge data need not determine the nonmaximal Lyapunov exponent.

We will now argue that the pole skipping structure observed here reveals features of the horizon geometry in the stringy regime, which in turn suggests a natural generalization to broader classes of theories, where the connection between pole skipping and the Lyapunov exponent is mediated by bulk stringy Regge trajectories.

Consider a large- N CFT with a gravitational dual. The boundary Rindler horizon is dual to the AdS-Rindler horizon, and the Rindler CFT is dual to the bulk theory in the corresponding AdS-Rindler region. Vacuum four-point functions of the boundary CFT in the Regge regime map to high-energy AdS scattering in the corresponding bulk Regge limit [35,46,49], effectively occurring across the AdS-Rindler horizon.

In the semiclassical gravity limit—dual to the boundary CFT at infinite coupling—only the graviton is exchanged, corresponding to the exchange of the boundary stress tensor, and yielding the maximal Lyapunov exponent $\lambda_{\text{max}} = 1$. If one were to include the exchange of a hypothetical higher-spin particle of spin J , the corresponding Lyapunov exponent would be $\lambda_J = J - 1$, although no such particles exist in the low-energy spectrum.

In the stringy regime, however, an infinite tower of bulk higher-spin string excitations appears, dual to higher-spin single-trace boundary operators whose dimensions become finite at finite coupling. *The Regge trajectories of these boundary operators coincide with those of the bulk stringy excitations.* In the corresponding AdS-Rindler scattering process, the intermediate states include all such higher-spin exchanges, whose collective contribution yields a nonmaximal Lyapunov exponent $\lambda < 1$.

Our discussion of pole skipping in the Rindler CFT relies only on conformal symmetry and therefore extends to the boundary CFT at finite coupling, which is dual to the bulk theory in the stringy regime. In particular, the pole skipping of a single-trace spin- J boundary operator in the Rindler

CFT maps to that of the corresponding bulk stringy field in the AdS-Rindler region. Consequently, all the features discussed in the previous section translate directly into properties of bulk stringy fields relative to the AdS-Rindler horizon: (1) from the conformal partial wave analysis, in the stringy regime, each higher-spin field still contributes individually to AdS Regge scattering with an exponent $\lambda_J = J - 1$. This “field-dependent Lyapunov exponent” is still captured by the highest pole skipping point of the field. (2) The highest pole skipping points of bulk stringy fields collectively trace out the Regge trajectories of *bulk* string excitations. (3) The full analytic Regge trajectories of the bulk string theory can be reconstructed from the pole skipping points via analytic continuation, including in particular the leading trajectory $j_{\text{leading}}(\mu)$ —which corresponds to the highest curve in the $(\Im k, \Im \omega)$ plane—together with the associated nonmaximal Lyapunov exponent λ .

In semiclassical gravity, it has been argued that for $J = 2$ (metric perturbations), the highest pole skipping point (4) can be attributed to certain horizon symmetries [12]. It appears reasonable to expect that this argument can be extended to fields of general spin [54]. Therefore, Eq. (4) suggests that (4) the horizon symmetries underlying pole skipping persist into the stringy regime for each individual bulk field.

At strong coupling, where the bulk dynamics is described by classical gravity—including both Einstein and higher-derivative theories—there is strong evidence that the pole skipping frequencies $\omega = [2\pi i(J - n)/\beta]$, $n = 1, 2, \dots$ for each bulk field depend only on the near-horizon geometry [11,18], irrespective of whether the horizon corresponds to a black hole, an AdS-Rindler patch, or other background details [55]. Thus we conjecture that *the statements in items 1–4 above apply to a stringy black hole horizon as well.*

Now consider a boundary CFT on $\mathbb{R} \times S^{d-1}$ at a temperature $1/\beta$ above the Hawking-Page transition and at finite coupling, dual to an eternal black hole in the stringy regime. An OTOC in the boundary CFT maps to a stringy Regge scattering process in the black-hole geometry [6]. Translating items 1–4 above into the boundary description then leads to the following *conjectures*: (1) in a general nonmaximally chaotic theory at a finite temperature, the highest pole skipping points at imaginary Matsubara frequencies align along certain trajectories, among which there is a leading trajectory $\Im \omega = \lambda(k)$, whose intercept $\lambda(0)$ gives the Lyapunov exponent. (2) For systems with a bulk stringy dual, the trajectories of pole skipping points correspond to the Regge trajectories of stringy excitations in the associated black-hole geometry.

This picture—that in the stringy regime, at leading order in the g_s expansion (the free-string limit), the horizon remains sharp for each individual higher-spin bulk stringy field—is also consistent with recent discussions in [56,57]. Stringy fuzziness arises only in observables that involve

summing over the full Hagedorn density of modes or that incorporate interactions.

In the Einstein regime, the Lyapunov exponent appears to be directly related to the horizon boost symmetry. However, when viewed within the broader stringy regime, this relation seems accidental: there, $\lambda = (2\pi/\beta)$ is determined by the stress tensor, which itself is fixed by the horizon symmetry. In the stringy regime, although the same horizon symmetries persist, the Lyapunov exponent emerges from the collective contribution of an infinite tower of higher-spin modes—each individually respecting the same symmetry—yet their sum is no longer directly governed by it. In other words, each discrete point on the stringy Regge trajectory reflects a horizon symmetry, but the intercept does not.

Given its close connection to horizon structure, pole skipping should be viewed as a universal feature of quantum chaotic systems—one that is distinct from, and perhaps even more fundamental than, the Lyapunov exponent itself, since the Lyapunov exponent is merely one of several pieces of information extractable from it.

In the next section, we provide evidence for the above conjectures using the large- q SYK chain model, even though it is neither conformal nor possesses a notion of spin.

Trajectories of pole skipping for large- q SYK chain—We will now examine pole skipping for large- q SYK chain [27], which is believed to be dual to a “string-type” gravity theory. The SYK chain consists of M sites labeled by $z = 0, \dots, M - 1$ to form a spatial chain, and N Majorana fermions ψ_j^z ($j = 1, \dots, N$) on each site with an SYK Hamiltonian plus a nearest neighbor coupling term of similar type (see Sec. S-III of Supplemental Material [34]). q denotes the number of fermions in each interaction term. In the large- q limit, this model admits a simple Liouville-like effective action [26], allowing many quantities to be computed analytically. We set units such that the inverse temperature is $\beta = 2\pi$, and the Lyapunov exponent $\lambda_0 < 1$ can be obtained explicitly [see Eq. (S72) in Supplemental Material [34]].

Since it is a lattice model, the spatial momentum $p = 2\pi k/M$ is discretized with $k \in \mathbb{Z}_M$. We will take the continuum limit $M \rightarrow \infty$ and regard p as a real number in the first Brillouin zone $p \in [-\pi, \pi]$. In the discussion of pole skipping, we will analytically continue p to complex values. At order $1/N$, pole skipping in this model was analyzed in [26] for the two-point function of the composite operator $\varepsilon_z(t) = (1/iq) \sum_i \psi_i^z(t) \partial_i \psi_i^z(t)$.

Before discussing their results, we note that the two-point function of ε_z corresponds to the time-ordered four-point function of SYK fermions. Consequently, the associated pole skipping structure should reduce to that of the intermediate operators exchanged in the four-point function, together with any additional *ad hoc* contributions arising from the specific definition of ε_z itself. In other

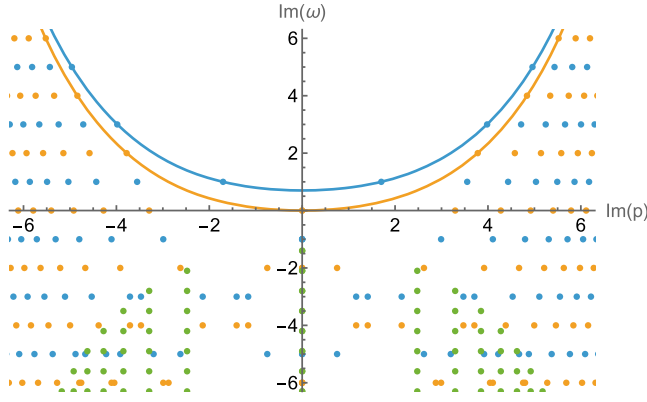


FIG. 2. The pole skipping points of the large- q SYK chain on the $(\Im\omega, \Im p)$ plane with parameters $\eta = 0.8$ and $\lambda_0 = 0.7$. The blue and yellow dots are (8) with odd and even n respectively, and the green dots are (11). The pole skipping points without pure imaginary p are not included in this plot. The blue and yellow curves are the two bounds in (9) respectively.

words, by examining the pole skipping behavior of such composite operators, we can in principle extract the pole skipping points of a large class of operators in the theory—namely, those bilinear in ψ [58]. In [59], we show that the two families identified in [26] (shown in Fig. 2 and described below) are the only ones that are universal and independent of the specific choice of composite operator. They can therefore be identified with the full universal set of pole skipping points associated with bilinear operators. Note that, in contrast to the AdS-Rindler case, we can no longer separate the pole skipping points associated with individual operators, as those operators are unknown. What we obtain instead is the full universal collection.

The first family of pole skipping points lies at

$$(\omega, h(p))_{\text{p.s.}} = \left(in, \frac{n}{\lambda_0} + \frac{1 + (-)^n}{2} + 2k + 1 \right), \quad (8)$$

where $n \in \mathbb{Z}$, $k = 0, 1, \dots$, $h(p)$ is a function of p defined as $h(h-1)/2 = 1 + (\eta/2)(\cos p - 1)$ [60], and $\eta \in [0, 1]$ is a constant that can be expressed in terms of the couplings of the theory. Note that all the points have imaginary Matsubara frequencies. For each n , there are infinitely many pole skipping points as k is unbounded from above. The special choice with $n = 1$ and $k = 0$ corresponds to the pole skipping point (1) related to the maximal Lyapunov exponent.

Since SYK chain does not have conformal symmetry nor even Lorentzian symmetry, there does not exist a notion of “higher-spin” operator in this model. However, from Fig. 2, we see that, for this family of pole skipping, all pole skipping points have $\Im\omega$ bounded from above by

$$\Im\omega \leq \begin{cases} \lambda_0[h(p) - 1], & \Im\omega = n \text{ odd} \\ \lambda_0[h(p) - 2], & \Im\omega = n \text{ even} \end{cases}. \quad (9)$$

Each upper bound is saturated by infinitely many pole skipping points with nonnegative integer $n \geq 0$. We thus identify the first line of (9) as giving the leading trajectory [61]

$$j_{\text{leading}}(p) = [h(p) - 1]\lambda_0 + 1, \quad (10)$$

which indeed recovers the Lyapunov exponent with $j_{\text{leading}}(0) - 1 = \lambda_0$ [$h(0) = 2$]. The large- q SYK chain thus verifies beautifully the conjecture 1. Note that we have essentially extracted the Lyapunov exponent from the time-ordered four-point function. Although we cannot separate the pole skipping points associated with individual operators, we have nevertheless extracted the leading trajectory, which can be interpreted as the leading Regge trajectory of the “string theory” dual to the SYK chain. It would be interesting to construct each individual operator on this trajectory and check if its Regge scattering contribution matches with the conjecture 2.

There is a second family of pole skipping points at

$$(\omega, p)_{\text{p.s.}} = \left(-i\lambda_0 n, \pm \arccos \left[1 + \frac{k(k-1)-2}{\eta} \right] \right), \quad (11)$$

where $k = 1, \dots, n$, and $n = 1, 2, \dots$. In contrast with the first family, this family exhibits two new features: (i) their locations are not at the imaginary Matsubara frequencies of the physical temperature, but instead at the Matsubara frequencies of the so-called fake temperature $T_{\text{f.d.}} = \lambda_0/(2\pi)$ [62]. (ii) They appear only in the lower-half frequency plane, and the total number of pole skipping points at fixed n is $2n$. The second feature suggests that these are the pole skipping points associated with a (bulk stringy) “scalar” operator, while the first feature indicates that, unlike other operators that “experience” the physical temperature, this operator effectively “sees” the fake temperature.

Conclusion—Using Rindler CFTs and the large- q SYK chain as illustrative examples, we have argued that in generic nonmaximally chaotic systems, the pole skipping points of few-body operators organize into trajectories in the complex frequency-momentum plane. The leading trajectory encodes the quantum Lyapunov exponent. We further proposed that these trajectories admit a natural interpretation as Regge trajectories of stringy modes in a dual stringy black-hole geometry. From this viewpoint, pole skipping for an individual operator can be seen as “tracking” the stringy horizon via the response of a single stringy excitation.

We also emphasized the power of analyzing pole skipping for generic composite operators. This approach allowed us to extract the universal set of pole skipping

points associated with essentially all operators, without requiring a detailed analysis of each individual field. Notably, this information alone was sufficient to reconstruct the leading trajectory that determines the Lyapunov exponent. Moreover, such an analysis could potentially reveal pole skipping associated with nonlocal operators.

Overall, our results suggest that pole skipping reflects intrinsic and universal features of quantum chaotic systems. Developing a more microscopic understanding of its origin in generic chaotic models would be highly desirable, both to clarify its connection to horizon structure and to place our conjectures on firmer ground. In particular, Ref. [56] proposed defining stringy horizons in terms of half-sided modular inclusions in the boundary theory, which capture quantum ergodicity and chaoticity at the algebraic level. It is tempting to speculate that the emergence of pole skipping is rooted in this underlying modular structure.

Progress along these lines would open new avenues for understanding quantum chaos and black-hole dynamics, and yield fresh insights into the structure of horizons in the stringy regime.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [33] We relegate the details to Sec. S-I of Supplemental Material [34] and present only the final results here.
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