

Perfect Superconducting Diode Effect in Altermagnets

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We investigate intrinsic superconducting diode effect in the unconventional superconducting state of d -wave altermagnets. We find large diode efficiencies in the wide-reaching finite-momentum pairing regimes of the phase diagram. Remarkably, even perfect diode efficiency of 100% can be obtained in the presence of an external magnetic field. We attribute the largest efficiencies to competition between multiple zero-momentum (BCS) and finite-momentum superconducting states, which is connected to a topological nodal-to-nodeless transition in altermagnets with magnetic field, making our results applicable to a wide range of altermagnets.

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Introduction—Broken symmetries often enhance functionalities of materials. One example is broken inversion symmetry in semiconductors, resulting in the conventional diode effect [1]. Recently it has been realized that a diode effect can also occur in superconductors with both broken time-reversal and inversion symmetries. In such a superconducting diode effect (SDE) a dissipationless current flows in one direction, while only a normal current may flow in the reverse direction [1,2]. Due to its potential for constructing energy-efficient electronic devices, the SDE has received enormous attention, theoretically in both bulk superconductors [3–21] and Josephson setups [22–33], and with many experimental findings [34–43], reaching a largest diode efficiency of 60% [42] in twisted flakes of cuprate d -wave superconductors [41,44,45].

Early theoretical work [3] already established the ubiquity of SDE in finite center-of-mass momentum superconductors. In particular, finite-momentum superconductivity in the Fulde-Ferrell (FF) [46] form, spontaneously breaks both time-reversal and inversion symmetries, hence

generating SDE [3]. Such a finite-momentum phase is found in nonmagnetic materials in the presence of a magnetic field, but only in a narrow window of magnetic field and temperatures [46,47]. Additionally, several correlated materials have experimentally been found to host finite-momentum superconductivity without applied magnetic field, then often called pair-density-waves (PDW) [48–55]. Finite-momentum superconductivity has also recently been theoretically discovered to be prevalent in altermagnetic materials for a wide range of magnetic fields, even starting at zero field [56–61] and also appearing as multiple phases interweaved with zero-momentum (or traditional BCS) superconductivity [58]. In contrast to ferro- and antiferromagnetism, altermagnetism breaks the Kramer’s spin degeneracy but with a momentum dependent band spin splitting such that the net magnetization is zero [62–72] and has already been observed in many materials [73–82].

Motivated by the prospects of finding finite-momentum pairing in altermagnets, we calculate in this Letter the SDE in altermagnets. We use a generic microscopic model of common d -wave altermagnetism, consider the simplest allowed superconducting states, and take into account the presence of an external magnetic field. We find large SDE efficiencies throughout the multiple finite-momentum pairing regions. Remarkably, we find diode efficiencies even reaching a perfect 100%. We further uncover a direct connection between the largest diode efficiencies and a topological nodal-to-nodeless transition occurring in the

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alters magnet band spin splitting with applied magnetic field [83]. This connection to topology makes our results generic and applicable across a wide range of altermagnets. Our results provide a pathway combining unconventional magnetism and superconductivity for developing extremely energy-efficient electronic devices.

Model and method—We consider a minimal two-band model for a d -wave altermagnetic metal given by the Hamiltonian

$$H_0 = \sum_{k,\sigma} (\xi_k - \sigma(t_{\text{am}}/2)(\cos(k_x) - \cos(k_y)) + \sigma B) c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c_{k+q\uparrow}^\dagger c_{-k+q\downarrow}^\dagger c_{-k'+q\downarrow} c_{k'+q\uparrow}, \quad (1)$$

where $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is the creation (annihilation) operator of an electron with spin σ and momentum k , $\xi_k = -2t[\cos(k_x) + \cos(k_y)] - \mu$ is the electron band dispersion, with $t = 1$ as the energy unit, and t_{am} is the strength of the d -wave altermagnetic spin splitting. We here only consider the two lowest energy bands in the prototype four-band tight-binding altermagnet model [65], since only they are relevant for superconductivity. These two bands encode a spin-sublattice locked dispersion, such that spin-up electrons only live on sublattice A, while spin-down electrons live on sublattice B [58,65,84]. Altermagnetism is generated from the combination of electric crystal field splitting and large spin-exchange field [65], where t_{am} in Eq. (1) mimics their combined effect. Further, μ is the chemical potential, which we tune to fix the average electron density $\rho = \sum_{k,\sigma} \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle$. We also add an in-plane external magnetic field B (with electron magnetic moment $\mu_B = 1$) for controlling a Zeeman spin splitting but with no orbital effects expected [85].

Superconductivity is generated by the interaction $V_{k,k'}$ in Eq. (1). Due to the spin-sublattice locking, uniform, or equivalently onsite, s -wave superconductivity is not present in a standard altermagnet [84]. We thus consider the simplest other pairing: spin-singlet pairing on nearest-neighbor bonds, giving either d -wave or extended s -wave superconductivity. Hence, we consider the generic effective attraction $V_{k,k'} = -V[\gamma(k)\gamma(k') + \eta(k)\eta(k')]$, with $\gamma(k) = \cos(k_x) + \cos(k_y)$ and $\eta(k) = \cos(k_x) - \cos(k_y)$ being the two form factors for nearest-neighbor interaction on the square lattice and V a constant attraction strength. To proceed, we mean-field decouple the interaction in Eq. (1) in the spin-singlet Cooper channel, resulting in

$$H_{\text{MF}} = \sum_{k,\sigma} \xi_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_k \left(\Delta_k^Q c_{-k+Q/2\downarrow} c_{k+Q/2\uparrow} + \text{H.c.} \right) + \text{constant}, \quad (2)$$

where $\xi_{k\sigma} = \xi_k + \sigma(t_{\text{am}}/2)[\cos(k_x) - \cos(k_y)] + \sigma B$ and Δ_k^Q is the spin-singlet superconducting order parameter obtained by the self-consistency equation

$$\Delta_k^Q = \sum_{k'} V_{k,k'} \langle c_{k'+Q/2\uparrow}^\dagger c_{-k'+Q/2\downarrow}^\dagger \rangle, \quad (3)$$

with Q being the finite center-of-mass momentum of the Cooper pair. We focus on both the $Q = 0$ BCS superconductivity and FF phases with finite Q , where the phase of the superconducting order parameter varies but the amplitude does not. We can further use the ansatz $\Delta_k^Q = \Delta_d^Q \eta(k) + \Delta_s^Q \gamma(k)$, with $\Delta_{d,s}^Q$ being the d - or extended s -wave superconducting order parameters [86], both parametrically depending on Q [87].

We proceed by solving the Hamiltonian H_{MF} in Eq. (2) fully self-consistently using Eq. (3) for fixed Q , and then obtain the true ground state by minimizing the ground state energy density, $E = \sum_{k,\sigma} \xi_{k\sigma} \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle - (\Delta_d^Q)^2/V - (\Delta_s^Q)^2/V + \mu\rho$ [87,88], with respect to Q , resulting at an optimum Q^* . We then calculate the current using [89–91] $I = \sum_k \zeta_k \langle c_{k+Q/2\uparrow}^\dagger c_{k+Q/2\uparrow} \rangle + \zeta_{-k+Q/2} \langle c_{-k+Q/2\downarrow}^\dagger c_{-k+Q/2\downarrow} \rangle$, where $\zeta_k = \sin(k_x) + \sin(k_y)$ and ignoring a universal prefactor. Importantly, our self-consistent minimization procedure ensures that the current in the ground state is always zero, due to Bloch's theorem [46,88,92]. This result is also evident from the current being, by definition, proportional to the derivative of the free energy with respect to Q [46,88,89]. However, an externally applied current produces a metastable state with Cooper pair momentum Q around the optimum Q^* , i.e., $Q = Q^* + \Delta Q$, with the sign of ΔQ set by the direction of current. We thus obtain the SDE by calculating the current for all Q around Q^* and use the thereby obtained maximum positive and negative currents to define the positive I_c^+ and negative I_c^- critical currents, respectively [3,93]. It is here sufficient to consider only $Q > 0$ due to the inversion symmetry present in the normal state [3,93,94]. Using the common convention [1,2], we finally define the SDE efficiency as $\eta = I_c^+ - I_c^- / (I_c^+ + I_c^-)$ [3,95].

Below we report results for $V = 2$ and $\rho = 0.6$ at zero temperature and on a square lattice of size 1000×1000 , enough to mimic the thermodynamic limit and capture relevant values of Q . Since the ground state energy minima occurs for a uniaxial Q [58], we only need results for Q along the x direction, since the critical currents are obtained for momenta (anti)parallel to the FF momentum [3]. For the parameters considered in this Letter, we find negligible Δ_s^Q and therefore focus on only reporting Δ_d^Q .

SDE in altermagnets—In Fig. 1 we show the superconducting phase diagram with computed SDE efficiency η by varying both B and t_{am} . Before focusing on the SDE, we summarize the phase diagram, which almost takes the

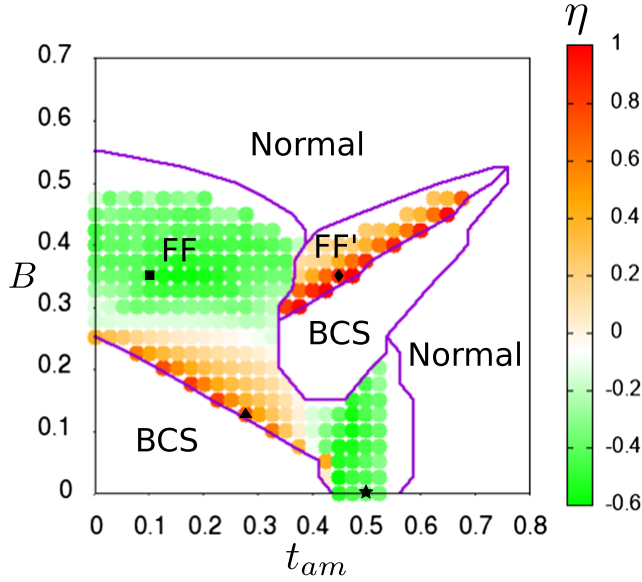


FIG. 1. SDE efficiency η in B - t_{am} phase diagram. Magenta lines demarcate different ground state phases, with the normal phase identified as $\Delta_d^Q < 0.0009$ for all Q . BCS and normal phase have inversion symmetry, and hence no diode effect. Calculations are performed on a set of discrete points spaced 0.025 apart in the B - t_{am} plane. Due to lack of numerical accuracy, η is not shown in small white regions near the FF to normal transition. Black symbols (star, square, triangle, diamond) marks parameters used in Fig. 2.

shape of a “Yoda-ear” [58]. The zero-momentum BCS state survives for low t_{am} and B , while finite-momentum FF superconductivity covers a large region of the phase diagram, including down to $B = 0$ for a range of finite t_{am} . Unexpectedly, the BCS state also reappears at higher B , as a field-induced BCS state, alongside with another finite-momentum state FF' with a different Q^* from the FF state. Both FF^(l) states in Fig. 1 spontaneously break time-reversal

and inversion symmetries [46,88,89] and are thus expected to host SDE [3], as is also evident from the color-intensity plot of η in Fig. 1. In contrast, inversion symmetry is intact in the BCS state, such that no diode effect is present there (or in the normal state).

First focusing on the already well-known FF phase without altermagnetism, i.e., $t_{am} = 0$, we find $\eta > 0$ for a small window of B close to the BCS transition, but then $\eta < 0$ for all higher B in the FF state, but never reaching beyond $|\eta| = 50\%$. Additionally, we also find $\eta \sim -50\%$ at zero applied field, $B = 0$, for a range of altermagnetism strengths. However, the more interesting behavior appears when both $B \neq 0$ and $t_{am} \neq 0$. Broadly, the diode behavior can be classified into two regions, characterized by the sign of η . Deep inside the FF state, we find $\eta < 0$ indicating $I_c^- > I_c^+$ and a reverse diode effect. We here find a maximum negative efficiency of 60%. However, in the FF phase near the BCS phase at both low B and in the FF' phase at higher B , we find $\eta > 0$, indicating a forward diode effect, and also significantly larger than in the reverse regime. Remarkably, we find the maximum η reaching a perfect SDE efficiency of 100% in the FF' phase near the transition to the field-induced BCS phase, but also the FF region at low B can achieve a large $\eta \approx 75\%$. Taken together, both low and high applied magnetic fields in a superconducting altermagnet give very large SDE efficiencies, including perfect SDE. Large SDE efficiencies also correspond to large absolute values of critical currents for low fields; see End Matter.

The large diode efficiencies, including perfect efficiency, can be understood by looking at the evolution of the energy density E as a function of Q . In Fig. 2 we plot E (relative to the normal state) with varying $Q = (Q_x, 0)$ (a)–(d) and the calculated currents I (e)–(h) for four representative points in the phase diagram, indicated by black symbols ($\star, \blacksquare, \blacktriangle, \blacklozenge$) in Fig. 1. We see directly that for all panels $I = 0$ at the optimum Q^* where E reaches its global

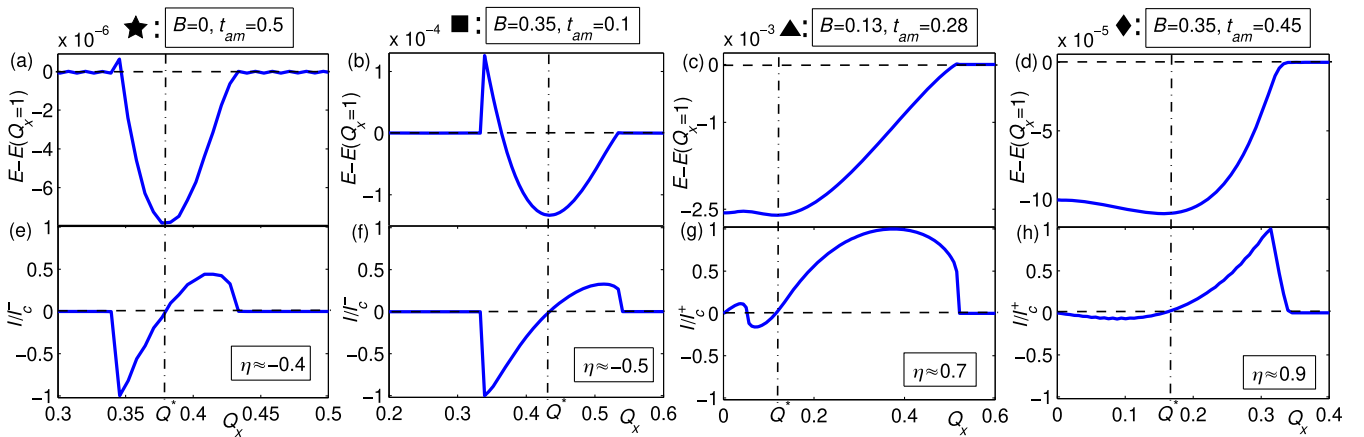


FIG. 2. Energy density (a)–(d) and current I (e)–(h) for four representative points in the phase diagram Fig. 1, indicated by black symbols (star, square, triangle, diamond), as a function of $Q = (Q_x, 0)$. Energies given relative to the normal state by subtracting $E(Q_x = 1)$. I normalized by $\max[I_c^-, I_c^+]$.

minima, fully consistent with $Q = Q^*$ corresponding to the ground state [46,88,89]. The critical currents, and hence η , is then obtained by finding the maximum current around this ground state. Thus the slope of E as a function of Q around Q^* becomes an essential property. In Figs. 2(a) and 2(b) the slope of E for $Q_x > Q^*$ is less than the slope for $Q_x < Q^*$. Consequently, $I_c^- > I_c^+$ in Figs. 2(e) and 2(f), giving $\eta < 0$. In contrast, in Figs. 2(c) and 2(d) the slope of E for $Q_x > Q^*$ is much larger than the slope for $Q_x < Q^*$. Hence, $I_c^- \gg I_c^+$ in Figs. 2(g) and 2(h), giving large positive η .

In both Figs. 2(c) and 2(d), the FF^(l) states are very close to a BCS state, reflected in the small energy difference between $E(Q = 0)$ to $E(Q = Q^*)$ and also seen in Fig. 1. However, the nature of large diode effect in these two regions are still quite different. In the FF phase in Figs. 2(c) and 2(g) both $E(Q = 0)$ and $E(Q = Q^*)$ features local minima, with $|E(Q = Q^*)| > |E(Q = 0)|$, resulting into the FF ground state being the global minima. The transition into the nearby BCS state occurs when $|E(Q = Q^*)| < |E(Q = 0)|$, but notably both local minima at $E(Q = 0)$ and $E(Q = Q^*)$ remain also in the BCS state. As a result, I_c^- never fully approaches zero, and hence we always find $\eta \leq 80\%$, albeit this is already a substantial efficiency. In contrast, the transition of the FF' phase in Figs. 2(d) and 2(h) to its nearby field-induced BCS phase is different. As seen in Fig. 2(d), $E(Q = 0)$ is a now maxima and thus the transition to the nearby BCS state occurs by this maxima becoming a minima, while, importantly, the local minima at $E(Q = Q^*)$ no longer remains. Hence, I_c^- asymptotically must reach zero near the FF' to BCS transition and thus η reaches a perfect 100% at the FF-BCS transition point found in close vicinity to the $\blacklozenge$ point in the phase diagram in Fig. 1.

Topological transition—The large, and even perfect, SDE in Fig. 1 may at first seem fortuitous, but we are able to directly connect it to a recently identified topological nodal-to-nodeless transition present in altermagnets when driven by an external magnetic field [83]. In altermagnets, the Fermi surface gets spin-split with a momentum dependent splitting, see lower inset in Fig. 3 for a d -wave altermagnet, with nodes along $|k_x| = |k_y|$ for $B = 0$ where the Fermi surface is spin degenerate. With an applied magnetic field the nodes move toward $k_x = 0$, due to the Zeeman effect. At a critical B , the nodes coalesce and for B even larger there are no nodes (i.e. nodeless) as the two spin Fermi surfaces do not intersect; see the upper inset in Fig. 3, where it is also clear that the spin Fermi surfaces now instead look more similar to a ferromagnet than an altermagnet. This nodal-to-nodeless transition is a topological transition, which has been shown to be independent of the considered band structure [83]. The main plot in Fig. 3 shows the magnetic fields where nodal-to-nodeless transition occurs (blue dots) in our system.

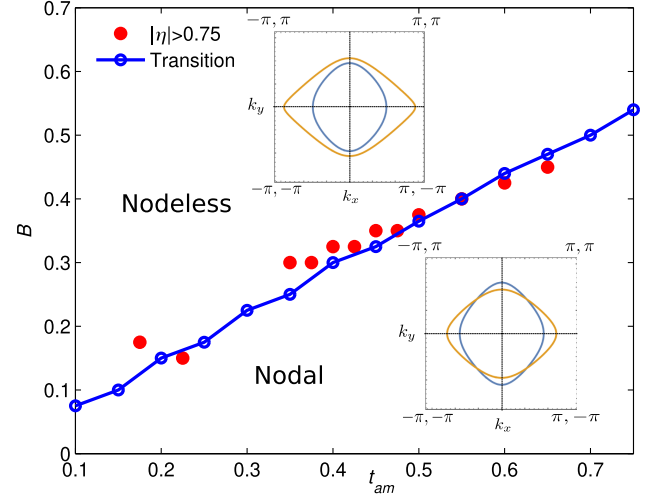


FIG. 3. Critical B (blue open circles, with line guide to the eye) for topological nodal-to-nodeless transition of the normal-state altermagnet band structure as a function of t_{am} . Red dots indicate regions of phase space with $|\eta| > 75\%$. Critical B is obtained for a fixed $\mu = -1.12$. Insets: contour plots of the normal-state electronic bands $\xi_{k\uparrow} = 0$ (yellow) and $\xi_{k\downarrow} = 0$ (blue) mapping the spin-split Fermi surfaces, with nodes for $B = 0.1$, $t_{am} = 0.5$ (lower inset) and no nodes for $B = 0.55$, $t_{am} = 0.5$ (upper inset).

Near the nodal-to-nodeless transition the two spin Fermi surfaces of the altermagnet become near degenerate around $k_x = 0$. This is also the region where a d -wave superconducting order parameter has its maximum values. As a result, the BCS state can most effectively compete with finite-momentum states in this transition region. This competition is also visible in the energy density plots in Figs. 2(c) and 2(d) reporting parameter values close to the nodal-to-nodeless transition, where $E(Q = 0)$ is indeed found to be close to the finite-momentum $E(Q = Q^*)$. It is due to this presence of this nearby BCS state that I_c^- is extremely small in the FF^(l) phases. Hence we find large diode efficiencies, including the perfect efficiency, near the topological transition. We illustrate this in Fig. 3 by plotting all areas in the phase diagram with $|\eta| > 75\%$ (red dots). The closeness of the red dots to the blue topological transition line clearly highlights the close connection of the large diode effect and the topological transition. We here also emphasize that large η is not achieved in the FF phase near the BCS transition when $t_{am} = 0$ or $B = 0$, see End Matter. This is because this BCS to FF transition is a first-order transition, which is different from the ones described above.

Discussion—In summary, we find a large intrinsic superconducting diode effect in the finite-momentum superconducting state of altermagnets, especially in the presence of external magnetic fields. The finite-momentum phase is classified into two broad regions, depending on sign of the diode efficiency. Already this finding can be used to experimentally detect different natures of the finite-momentum

phase. We further show that the maximum diode efficiency in the traditional magnetic field-induced finite-momentum phase (i.e., in the absence of altermagnetism) is around 50%, as is the maximum value achieved in altermagnets in the absence of magnetic field. In contrast, when both altermagnetism and magnetic field are present, we often find much larger diode efficiencies, including the remarkable possibility of reaching a perfect diode efficiency of 100%, by tuning the external magnetic field.

We identify the mechanism behind the large diode efficiencies by investigating the total energy with varying Cooper pair momentum. Our analysis reveals that a close competition between zero-momentum BCS and finite-momentum superconducting ground states generates the largest diode efficiencies. We further connect this competition, and hence large diode efficiencies, to a topological nodal-to-nodeless transition present in altermagnets in an external magnetic field. The connection to a topological transition makes our results generic and not tied to specific altermagnet models or parameters. As an example, considering much weaker superconducting pairing strengths, the BCS region at low magnetic fields shrinks in size in the phase diagram, while the field-induced BCS state makes up a larger part [58]. As a consequence, the region with perfect diode efficiency, which occurs near the topological transition of the finite-momentum pairing state to the field-induced BCS state, is enhanced. This example illustrates large diode efficiencies do not require using high magnetic fields and is also achievable in a wide range of altermagnetic materials. Furthermore, our findings connecting the topological nodal-to-nodeless transition and large SDE can also have implications for other non-reciprocal effects in superconductors such as magnetoelectric effects [96,97], which may be a prospective future direction.

We further emphasize that our results are reached using a fully self-consistent treatment of superconductivity, essential for correctly extracting the supercurrents and therefore the SDE. Self-consistently calculating the chemical potential, assuming a fixed filling, is also required to reach the correct finite-momentum state. Due to a strong spin-sublattice coupling in altermagnets, care must also be taken when considering possible superconducting pairing symmetries [84].

Note added—When we were finalizing this Letter, a preprint [60] appeared investigating the SDE in an altermagnet in the absence of external magnetic field within an s -wave superconducting state using Ginzburg-Landau theory. While these zero-field results find moderate efficiencies similar to ours in the zero-field limit, our self-consistent results are valid for all magnetic fields, deep within the superconducting phase, and, importantly, find perfect SDE at finite fields. Moreover, the s -wave superconducting state considered in [60] is not likely in

altermagnets [84] and a zero-field FF state is predicted to not be stable for s -wave superconductivity [56,58].

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End Matter

Variation of critical currents—Here, we discuss how the values of the critical currents I_c^+ and I_c^- vary through the phase diagram. In particular, we show in Figs. 4(a)–4(d) the critical currents along four different fixed-B line cuts in the phase diagram of Fig. 1, along with the associated SDE efficiency η in Figs. 4(e)–4(h). For lower values of B (a) and (b), increasing t_{am} results in BCS to FF to BCS to FF to normal transitions. For these values of B , near the first BCS to FF transition, the large SDE efficiencies $|\eta|$ (e) and (f) are also associated to large values of I_c^+ . Further increasing t_{am} , I_c^+ decreases, while I_c^- increases, resulting in decrease of $|\eta|$ to zero when eventually transitioning to the high-field

BCS phase. Further increase in t_{am} marks re-appearance of the FF phase. In this region, both I_c^+ and I_c^- decrease with increasing t_{am} . However, the rate of decrease of I_c^+ and I_c^- is now different and consequently $|\eta|$ decreases. Hence in this low-field regime, the largest SDE efficiencies are associated with the largest I_c^+ . Next for larger B (c) and (d), increasing t_{am} leads to FF to FF' to BCS to normal transitions. For these values of B , I_c^+ , I_c^- , and $|\eta|$ all behave similarly with increasing t_{am} until the FF to FF' transition. For $B = 0.33$ (c), further increase in t_{am} leads to increase in I_c^+ but direct decrease in I_c^- to zero. Subsequently, $|\eta|$ quickly reaches the perfect value of 1. For $B = 0.4$ (d), I_c^+ increases

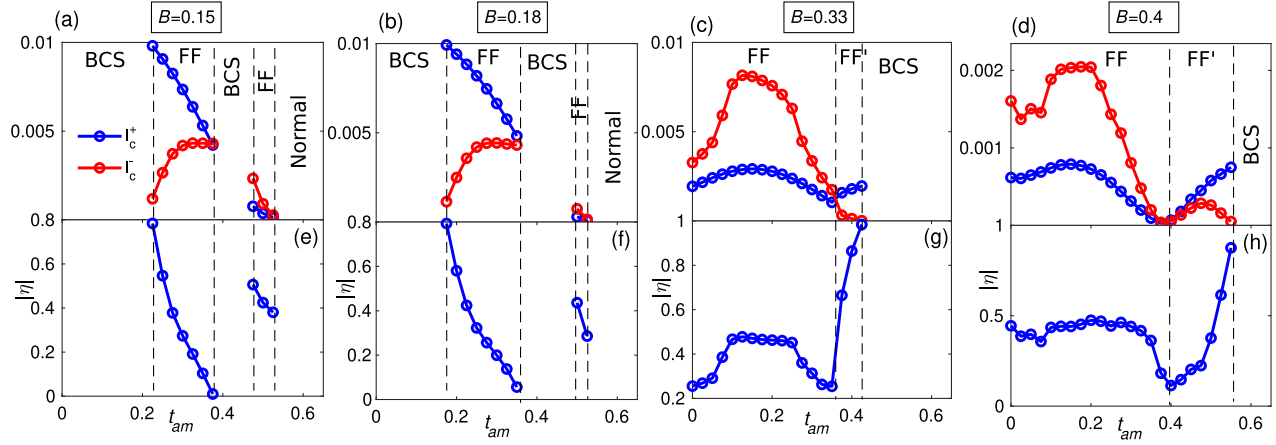


FIG. 4. Values of critical currents (a)–(d) for fixed- B line cuts and the corresponding SDE efficiencies η (e)–(h). Critical currents are in units of $-e \times t$, where e is the electronic charge and t is the energy unit. Also, note that by the convention used in the main text, maximum positive and negative currents are defined as I_c^+ and I_c^- , respectively, without any signs. Different phases, BCS, FF, and FF', separated by vertical dashed lines. Since $\eta = 0$ in the BCS phases no currents are shown in regions.

with t_{am} beyond the FF to FF' transition, but I_c^- first increases and only later decreases to zero, thus only at a higher t_{am} resulting into perfect SDE. As a consequence, the perfect SDE is associated with still substantial but more moderate values of I_c^+ . All regimes of large and even perfect SDE should thus have experimentally measurable critical currents in the current-carrying direction.

SDE near the low-field BCS-FF transition—Here, we detail the connection of the topological transition and large SDE efficiencies η near the BCS-FF transition at low B field. As seen in Figs. 1 and 3 in the main text, the high field FF' to BCS transition coincides with the topological nodal-to-nodeless transition and hence results in very large, essentially perfect, η . However, large η is achieved only near some points of the low-field BCS-FF transition. Here we show that all those points also coincide with the topological transition. In Fig. 5, we show the absolute value of the SDE efficiency $|\eta|$ (b) in the FF phase following the path shown in (a) tracing closely the BCS-FF transition on the FF side. Close to both the $t_{am} = 0$ and $B = 0$ BCS-FF transitions, $|\eta| \approx 40\text{--}50\%$. However, in between

these two limits, $|\eta|$ increases as the topological transition is approached, reaching a maximum at the nodal-to-nodeless topological phase transition. Hence, even for the low-field BCS-FF transition, the largest SDE values are closely connected to the topological nodal-to-nodeless transition.

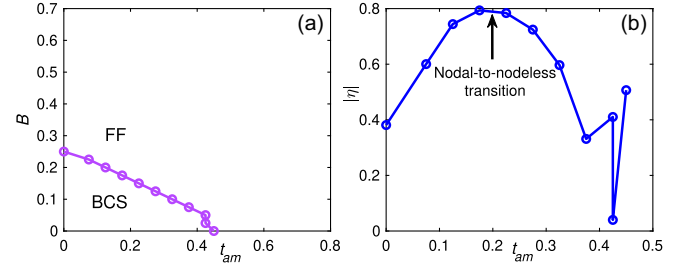


FIG. 5. (a) Path in B - t_{am} coordinate space tracing out the BCS-FF transition at low fields on the FF side. (b) Absolute SDE efficiency $|\eta|$ for the values of B and t_{am} in (a). Topological nodal-to-nodeless transition point indicated by black arrow. There is a sign-change in η for $t_{am} = 0.45$ resulting in an apparent jump when the absolute value $|\eta|$ is plotted in (b).