

First Look at Quartic-in-Spin Binary Dynamics at Third Post-Minkowskian Order

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We compute the conservative and radiation-reaction contributions to classical observables in the gravitational scattering between a spinning and a spinless black hole to the fourth order in spin and third order in the gravitational constant. The conservative results are obtained from two-loop amplitudes for the scattering process of a massive scalar with a massive spin- s field ($s = 0, 1, 2$) minimally coupled to gravity, employing the recently introduced spin interpolation method to resolve all spin-Casimir terms. The two-loop amplitude exhibits a spin-shift symmetry in both probe limits, which we conjecture to be a sign of yet unknown integrability of Kerr orbits through the quartic order in spin and to all orders in the gravitational constant. We obtain the radial action from the finite part of the amplitude and use it to compute classical observables, including the impulse and spin kick. This is done using the recently introduced covariant Dirac brackets, which allow for the computation of classical scattering observables for general (nonaligned) spin configurations. Finally, employing the radiation-reaction amplitude proposed by Alessio and Di Vecchia, together with the Dirac brackets, we obtain radiation-reaction contributions to observables at all orders in spin and beyond the aligned-spin limit. We find agreement with known results up to the quadratic order in spin for both conservative and radiation-reaction contributions. Our results advance the state of the art in the understanding of spinning binary dynamics in general relativity and demonstrate the power and simplicity of the Dirac bracket formalism for relating scattering amplitudes to classical observables.

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Introduction—The LIGO-Virgo-KAGRA collaboration is steadily advancing our understanding of gravitational wave (GW) physics. During the current O4 run it is expected that the number of black hole and neutron star merger events detected will more than double. Furthermore, the next-generation GW observatories [1–3] will greatly increase detector sensitivity and scientific reach [4]. The data will challenge existing theoretical frameworks for producing correspondingly precise predictions [5]. Indeed, progress in the field hinges from detailed comparisons between experimental measurements and theoretical predictions, which allows for the inference of parameters of the detected events,

for example, the masses and spins of the interacting compact objects.

Recently it has been shown that the dynamics of binary compact systems interacting gravitationally have a natural description within the context of quantum field theory (QFT). Most notably, the so-called *post-Minkowskian* (PM) expansion mirrors the standard relativistic amplitude computation in QFT, where one expands perturbatively in a single small coupling while keeping full dependence on the velocity. In fact, relativistic QFT techniques have emerged as a new workhorse for computing PM corrections to the dynamics of spinless binaries through on-shell scattering amplitude methods, e.g., in Refs. [6–15], as well as worldline methods, e.g., in Refs. [16–26].

The effects of spin in compact bodies adds another level of complexity. Incorporating them into the PM expansion has required many novel developments, both in scattering amplitudes and worldline approaches; see, e.g., Refs. [27–84]. In this way, spectacular results have been achieved at $O(G)$ and $O(G^2)$, corresponding to tree-level and one-loop amplitude calculations, respectively. Furthermore, two-loop results, i.e., $O(G^3)$, from scattering amplitudes for observables at the second order in spin, for systems with one spinless and one spinning black hole, have been

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calculated in [44,81]. Worldline techniques have also been developed for the computation of two-loop corrections to observables with two powers of spin [67], three-loop corrections [$\mathcal{O}(G^4)$] to linear-in-spin observables [69], and, more recently, the framework has been extended to obtain results to fourth order in spin at the one-loop level [83,85].

Going beyond the second order in spin at two or more loops has been a major challenge. The massive spinor helicity formalism [86] with all-order spin exponentiation suffers from the appearance of arbitrarily high-rank tensor integrals. The arbitrary-spin Lagrangian formalism [41] has the strength that it encompasses subtle effects such as spin magnitude changes [78], but requires careful choices of Wilson coefficients [46]. To make progress, we adopt the approach of using scattering amplitudes involving fixed spin- s fields, minimally coupled to gravity, to determine spin effects in classical two-body interactions up to $2s$ orders in the spin multipole expansion [87–90], specifically for the case of Kerr black holes. However such an approach requires subtle considerations. When performing calculations this way, an ambiguity appears in spin-Casimir terms, like S^2 , starting at one loop. This can be seen by the following power-counting argument. In the classical limit, the momentum q of the exchanged gravitons scales as $\mathcal{O}(\hbar)$ while the spin can become large. Then we see that, for fixed small values of s and employing natural units ($\hbar = 1$), the spin-Casimir $q^2 S^2 = -q^2 s(s+1)$ inevitably mixes with $q^2 \sim \mathcal{O}(\hbar^2)$ quantum corrections to the spin-independent part of the amplitude when performing an expansion in q .

In this Letter, we compute for the first time the classical amplitude for the scattering of a spinning black hole with a spinless one to fourth order in spin and to third order in the gravitational coupling, i.e., to $\mathcal{O}(G^3 S^4)$. Considering that for the Kerr black hole $|S| < Gm^2$, this physically can be counted as up to the seventh post-Minkowskian order (see Ref. [91]). We make use of the *spin interpolation* method [81], recently introduced by some of the present authors and M. S. Ruf, to systematically resolve ambiguities between quantum corrections of the scattering amplitudes and classical spin-Casimir terms, without resorting to arbitrary-spin formalisms that are challenging beyond one loop. In our amplitude we observe a spin-shift symmetry, previously established at one loop [45,51], as well as two loops truncated at lower orders in spin [81], up to the fourth order in spin in the limit when either the spinless or spinning black hole becomes a probe object. We also establish the equivalence of the radial action, obtained from the two-loop amplitudes via the amplitude-action relation [13,14,92], to known probe-limit results from direct classical calculations [93]. Then, we use the covariant Dirac brackets for spinning two-body dynamics recently introduced by Gonzo and Shi [93] (see also Refs. [94–96] for a related Poisson bracket approach), to compute the impulse and spin kick observables for general

(nonaligned) spin configurations. This simplifies the calculation compared with a direct application of the Kosower-Maybee-O’Connell formalism [7] generalized to include spin [90]. Finally, we combine our results with the known radiation-reaction results of Ref. [97] and find cancellation of a high-energy logarithmic divergence through quartic order in spin. We compute, for the first time, the radiation-reaction contribution to the observables mentioned before to all orders in spin beyond the aligned-spin limit using the covariant Dirac brackets.

Scattering amplitudes—As we are interested in computing observables through fourth order in spin, we consider the following Lagrangian:

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{2R}{\kappa^2} + \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_1^2\phi^2 + \mathcal{L}_{m_1}, \quad (1)$$

where we work in the mostly minus metric signature, $\kappa = \sqrt{32\pi G}$, $g_{\mu\nu}$ and g is the metric and its determinant, and R is the Ricci scalar. $\mathcal{L}_{m_1}$ denotes the Lagrangian of an additional minimally coupled massive field (with mass m_1), which can be either a scalar field Φ ($s = 0$) [98–100], a Proca field V^μ ($s = 1$) [101], or a Fierz-Pauli field $H^{\mu\nu}$ ($s = 2$) [102] (see Ref. [103] for more discussions). For the latter we have

$$\begin{aligned} \mathcal{L}_H = & g^{\mu\nu}g^{\alpha\beta}g^{\omega\sigma}\left\{\frac{1}{2}(\nabla_\omega H_{\mu\alpha})(\nabla_\sigma H_{\nu\beta}) - (\nabla_\mu H_{\alpha\omega})(\nabla_\beta H_{\nu\sigma}) \right. \\ & \left. + (\nabla_\mu H_{\nu\alpha})(\nabla_\beta H_{\omega\sigma}) - \frac{1}{2}(\nabla_\mu H_{\alpha\beta})(\nabla_\nu H_{\omega\sigma})\right\} \\ & - \frac{1}{2}m_1^2g^{\mu\nu}g^{\alpha\beta}(H_{\mu\alpha}H_{\nu\beta} - H_{\mu\nu}H_{\alpha\beta}), \end{aligned} \quad (2)$$

where ∇_μ is the covariant derivative defined with a Christoffel connection. We expand around a flat background metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where $h_{\mu\nu}$ is the graviton field. All scattering amplitudes are computed with the multiloop numerical unitarity method [104–108] as implemented in the CARAVEL framework [109]. In particular, CARAVEL has been employed in multiple applications to quantum and classical gravity calculations [44,59,81,110], and in this Letter we have extended it to include massive spin-2 bosons according to the Lagrangian in equation (2). Corresponding Feynman rules were derived with the `xAct` package [111,112] and external polarization states have been taken from [113]. As an important validation of our implementation, we have found that all three-, four-, and five-point tree-level amplitudes involved in the unitarity cuts of the two-loop amplitudes agree numerically with those obtained via the double copy [114–119] with dimensional reduction to massive amplitudes following, e.g., [9,103,120,121]. Furthermore, we have found agreement with analytic constructions of the relevant $2 \rightarrow 2$ tree-level and one-loop amplitudes. Additional technical details about

our calculations can be found in [81,109]. The scattering processes that we consider are

$$X(-p_1, \varepsilon_1) + \phi(-p_2) \rightarrow \phi(p_3) + X(p_4, \varepsilon_4^*), \quad (3)$$

where $X = \{\Phi, V^\mu, H^{\mu\nu}\}$ and $\varepsilon_1, \varepsilon_4^*$ are the polarizations associated to the massive spinning particle. We have the on-shell conditions $p_1^2 = p_4^2 = m_1^2$, $p_2^2 = p_3^2 = m_2^2$ and the transverse conditions $p_1 \cdot \varepsilon_1 = p_4 \cdot \varepsilon_4^* = 0$. The momenta are parametrized [122,123] as

$$\begin{aligned} p_1 &= -\bar{p}_1 + q/2, & p_4 &= \bar{p}_1 + q/2, \\ p_2 &= -\bar{p}_2 - q/2, & p_3 &= \bar{p}_2 - q/2, \end{aligned} \quad (4)$$

with $\bar{p}_i = \bar{m}_i u_i$, $\bar{p}_i \cdot q = 0$, the u_i are unit vectors and $\bar{m}_i^2 = m_i^2 - q^2/4$. We further define $y = u_1 \cdot u_2$, which is a measure of the rapidity difference between the two massive particles that scatter.

It is convenient to introduce a form factor decomposition for the scattering amplitudes that involve massive spinning fields. For the amplitudes involving $X = V^\mu$ we use the decomposition in Ref. [81], while for those involving $X = H^{\mu\nu}$ we write,

$$\mathcal{M} = \sum_{n=1}^9 M_n \varepsilon_{4\mu\nu}^* T_n^{\mu\nu\alpha\beta} \varepsilon_{1\alpha\beta}, \quad (5)$$

with

$$\begin{aligned} T_1^{\mu\nu\alpha\beta} &= \eta^{\mu\alpha} \eta^{\nu\beta}, & T_6^{\mu\nu\alpha\beta} &= q^\mu \bar{p}_2^\nu \bar{p}_2^\alpha q^\nu, \\ T_2^{\mu\nu\alpha\beta} &= \bar{p}_2^\mu \bar{p}_2^\nu \bar{p}_2^\alpha \bar{p}_2^\beta, & T_7^{\mu\nu\alpha\beta} &= q^\mu \eta^{\nu\alpha} q^\beta, \\ T_3^{\mu\nu\alpha\beta} &= \bar{p}_2^\mu \eta^{\nu\alpha} \bar{p}_2^\beta, & T_8^{\mu\nu\alpha\beta} &= 2q^\mu \bar{p}_2^\nu q^\alpha q^\beta, \\ T_4^{\mu\nu\alpha\beta} &= 2\bar{p}_2^\mu \bar{p}_2^\alpha \bar{p}_2^\nu q^\beta, & T_9^{\mu\nu\alpha\beta} &= q^\mu q^\nu q^\alpha q^\beta, \\ T_5^{\mu\nu\alpha\beta} &= 2\eta^{\nu\alpha} \bar{p}_2^\mu q^\beta, \end{aligned} \quad (6)$$

where we use $x^{[\mu} y^{\nu]} = (x^\mu y^\nu - x^\nu y^\mu)/2$. In practice, there is one more crossing symmetric tensor which, however, is evanescent, i.e., linearly dependent on the previous structures in *four* space-time dimensions. The form factors M_n in (5) contain all the theory-dependent information. They are given as linear combinations of 2781 Feynman integrals with various propagator structures and including irreducible scalar products in the numerator. The integrals are expanded in the soft region and reduced to master integrals via integration-by-parts (IBP) relations [124,125]. To isolate conservative dynamics, the master integrals are evaluated in the potential region following Ref. [123]. The IBP reduction was performed using a private version of the FIRE program which builds upon Refs. [126–128]; this was previously used for four-loop spinless binary dynamics calculations in toy model theories [92,129]. We used an

approach based on modular arithmetics [130,131] running multiple reductions with different values of variables. It was followed by a functional reconstruction procedure [131] based on Thiele's interpolation formula for the first variable and on a balanced Zippel approach for the second variable [128,132], taking into account that denominators have factorized dimension and kinematic dependence in our reduction results [133,134]. The FIRE program takes care both of the reduction and the reconstruction, and can be used on HPC systems using MPI for the task distribution. While initially aimed for problems with more variables, the balanced Zippel approach is also efficient for problems with two variables, automatically exploiting sparsity of the polynomials in kinematic variables. The soft-expanded integrals requiring IBP reduction has up to 13 powers of irreducible scalar products and up to ten additional propagator powers.

The radial action—The radial action for spinning amplitudes can be obtained via the amplitude-action relation, schematically,

$$i\mathcal{M} = |\mathbf{p}| \int d^{D-2} \mathbf{b} e^{i\mathbf{b} \cdot \mathbf{q}} (e^{iI_r} - 1), \quad (7)$$

where $\mathbf{b}$ and $\mathbf{q}$ are the transverse spatial parts of the impact parameter and transfer momentum, respectively, and $\mathbf{p}$ is the three-momentum at past infinity. Here I_r is the radial action, which is defined classically through the radial momentum, p_r , along the scattering trajectory; see Refs. [13,14,81,92,129,135] for more details. An equivalent formulation, at least in the spinless case, is the exponential representation of the S matrix proposed in Refs. [15,136]. In the context of perturbation theory, the amplitude and radial action are expanded in powers of the coupling, G ,

$$\mathcal{M} = \sum_k G^k \mathcal{M}_k, \quad \tilde{I}_r = \sum_k G^k \tilde{I}_{r,k}, \quad (8)$$

$$\tilde{I}_r = |\mathbf{p}| \int d^{D-2} \mathbf{b} e^{i\mathbf{b} \cdot \mathbf{q}} I_r. \quad (9)$$

In this way, the first few orders of equation (7) read,

$$\begin{aligned} \mathcal{M}_1 &= \tilde{I}_{r,1}, \\ \mathcal{M}_2 &= \tilde{I}_{r,2} + \int_e \frac{\tilde{I}_{r,1} \tilde{I}_{r,1}}{Z_1}, \\ \mathcal{M}_3 &= \tilde{I}_{r,3} + \int_e \frac{\tilde{I}_{r,2} \tilde{I}_{r,1}}{Z_1} + \int_e \frac{\tilde{I}_{r,1} \tilde{I}_{r,1} \tilde{I}_{r,1}}{Z_1 Z_2}, \end{aligned} \quad (10)$$

where Z_i are three-dimensional propagators whose explicit forms are not relevant here. The important point is that the lower-order iterations precisely correspond to the superclassical divergences coming from the amplitudes, such that the radial action is simply the finite part of loop-level

TABLE I. Spin structures for the ansatz (15), with $n^\mu \equiv \epsilon^{\mu\alpha\beta} q_\nu \bar{p}_{1\alpha} \bar{p}_{2\beta}$ and the $\Omega_\pm$ structures defined in equation (16).

$\mathcal{O}_{(n,m)}$	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
$n = 0$	$\mathbb{1}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$n = 1$	$i(n \cdot S)$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$n = 2$	Ω_-	Ω_+	$q^2(\bar{p}_2 \cdot S)^2$	$\cdots$	$\cdots$	$\cdots$
$n = 3$	$i(n \cdot S)\Omega_-$	$i(n \cdot S)\Omega_+$	$iq^2(n \cdot S)(\bar{p}_2 \cdot S)^2$	$\cdots$	$\cdots$	$\cdots$
$n = 4$	Ω_-^2	Ω_+^2	$\Omega_+\Omega_-$	$q^2(\bar{p}_2 \cdot S)^2\Omega_-$	$q^2(\bar{p}_2 \cdot S)^2\Omega_+$	$q^4(\bar{p}_2 \cdot S)^4$

amplitudes. In practice, this is obtained through two loops by simply dropping the double- and triple-iteration ladder-like master integrals, as was done in the spinless case in Ref. [137] and the quadratic-in-spin case in Ref. [81]. We follow the proposal of Ref. [93] that a single radial action encodes complete information about scattering observables even beyond the aligned spin limit, and apply the covariant Dirac bracket formalism for spinning two-body dynamics developed in the same article to calculate such observables to $\mathcal{O}(G^3 S^4)$.

Resolving spin structures—The spin vector for a massive bosonic spin- s particle as an operator in spin space takes the form

$$\mathbb{S}_{ij}^\mu = \frac{(-1)^s}{2m} \epsilon^{\mu\lambda\alpha\beta} p_\lambda \epsilon_{i|s}^\star(p) (\Sigma_{\alpha\beta})_{J(s)}^{I(s)} \epsilon_j^{J(s)}(p), \quad (11)$$

where i, j are little group indices, $(-1)^s$ accounts for the normalization of the polarization tensors, and where the Lorentz generators in the spin- s representation, with indices $I(s)$ and $J(s)$, are defined as

$$(\Sigma_{\alpha\beta})_{J(s)}^{I(s)} = 2is \delta_{[\alpha}^{I_1} \eta_{\beta](J_1} \delta_{J_2}^{I_2} \cdots \delta_{J_s}^{I_s)}, \quad (12)$$

where symmetrizations over n indices includes a factor of $1/n!$. To make a connection with the scattering amplitudes, we define the spin vector in the frame of an average momentum $\bar{p}'_1 = \bar{p}_1(m_1/\bar{m}_1)$ and relate $\epsilon_4^{*I}, \epsilon_1^J$ to polarizations $\bar{\epsilon}^{*I}, \bar{\epsilon}^J$ in the aforementioned frame through Lorentz transformations [87,89,90] (see Ref. [81] for more details). The classical spin vector then follows as an expectation value over all external spin states

$$S^\mu = \sum_{i,j} \omega_i^\star \mathbb{S}_{ij}^\mu \omega_j, \quad \sum_i \omega_i^\star \omega_i = 1, \quad (13)$$

where ω_i are arbitrary complex weights for polarizations of different spin states. Products of the classical spin vector are mapped to symmetrized products of spin operators

$$S^\mu S^\rho \cdots S^\sigma S^\nu = \sum_{i,k,\dots} \omega_i^\star \mathbb{S}_{ik}^{(\mu} \mathbb{S}_{k\ell}^\rho \cdots \mathbb{S}_{rm}^\sigma \mathbb{S}_{mj}^{\nu)} \omega_j. \quad (14)$$

To disentangle ambiguous spin-Casimir contributions to the scattering amplitudes we employ the spin interpolation

method introduced in Ref. [81] as follows. Given that we know what spin structures to expect in an amplitude involving a spin- s particle, we can write an ansatz valid for all representations that we consider, that is for $s = 0, 1$ and 2 . Then we interpret spin universality as the statement that the coefficients appearing in the ansatz—classical or quantum—are independent of the spin representation. The ansatz is naturally organized as an expansion in S and q . In this way, we can truncate the ansatz to a desired order in spin by retaining only terms up to the same order in q , as terms of higher orders in spin are always accompanied by higher orders in q due to the classical power counting.

Therefore, we write an ansatz for the finite part of the amplitude in terms of all possible spin structures that appear through $\mathcal{O}(S^4)$,

$$\mathcal{M}_{\text{fin}}^{(L)} = \frac{\kappa^{2+2L}}{(-q^2)^{1-L/2+L\epsilon}} \sum_{n=0}^4 \sum_m \alpha^{(L,n,m)} \mathcal{O}^{(n,m)}, \quad (15)$$

where the coefficients $\alpha^{(L,n,m)}$ contain both classical and quantum contributions, L denotes the number of loops, n denotes the spin orders, and m organizes each spin contribution within a given order. Furthermore, we include the appropriate prefactor for an L -loop amplitude that is regularized using dimensional regularization, $D = 4 - 2\epsilon$. The spin structures $\mathcal{O}^{(n,m)}$ are given in Table I. To study the so-called spin-shift symmetry [45,51,81,138] of the amplitude under the transformation $S^\mu \rightarrow S^\mu + \xi q^\mu$ for arbitrary ξ , the ansatz is written in a basis involving

$$\Omega_\pm = (q \cdot S)^2 \pm q^2 S^2, \quad (16)$$

where Ω_- preserves the spin-shift symmetry, whereas Ω_+ breaks it.

To fix the coefficients in the ansatz, we first evaluate (15) in the $s = 0, 1$ and 2 representations and expand to $\mathcal{O}(q^4)$. In the ansatz, this means writing the $\alpha^{(L,n,m)}$ as an expansion in q . In this way, we find 14 classical coefficients—given by the leading q contribution of each structure in Table I—as well as 6 quantum-suppressed coefficients. Then, we project onto the appropriate form factor basis for each representation [see, e.g., (5)], and solve a linear system of equations by demanding equality with the corresponding spinning amplitude. This matching

requires that the amplitudes are also expanded to the same order in q . In the end, we find a system of 31 linear relations, of which 11 are linearly dependent. This linear dependence is an explicit manifestation of spin universality, since terms fixed by lower-spin amplitudes, such as spinless terms and spin-orbit interaction terms, reappear in higher-spin amplitudes. The 20 linearly independent relations then are the final product of our spin interpolation method, which we use to compute the 20 spin coefficients in terms of amplitude coefficients. In the end we obtain a set of linear relations which determine the ansatz coefficients to fourth-order in spin in terms of the finite part of the L -loop amplitude coefficients in any theory.

With this we reproduce the known classical amplitudes in the literature through $\mathcal{O}(G^2 S^4)$ and, for the first time, obtain the classical amplitude to $\mathcal{O}(G^3 S^4)$. Our results are explicitly given in the Supplemental Material [139]. We find the spin-shift symmetry to be present in the probe structures at $\mathcal{O}(G^3)$ through fourth order in spin. Since we have extracted the classical amplitude, from now on we can ignore quantum corrections, for example, in the difference between $\bar{m}_i$ and m_i .

Then, using our classical spinning amplitude through two loops, the radial action follows as

$$I_r = \int \frac{d^4 q}{(2\pi)^2} \delta(p_1 \cdot q) \delta(-p_2 \cdot q) e^{ib \cdot q} \mathcal{M}_{\text{fin}}. \quad (17)$$

To validate our results for I_r , we find agreement with the direct classical calculation in Ref. [93] through $\mathcal{O}(G^3 S^4)$ in the limit of a spinless probe in a Kerr background.

Observables—A new method to compute observables from the radial action was recently proposed in Ref. [93], which makes use of covariant Dirac brackets. Here we show how these brackets, together with the radial action obtained from the finite piece of the amplitude, can be used to compute nonaligned spin observables.

Let us first discuss the derivation of the brackets for the case of two spinning massive particles following Ref. [93]. Although our discussion so far has involved only one spinning black hole, we will consider the more general case with two spinning black holes (at the end, we can set one spin to zero). Note that we work with normalized four-momenta, u_i , and spin tensors $S_i^{\mu\nu}$.

The relevant dynamics for the radial action is in the asymptotic past, where we have straight line trajectories,

$$x_i^\mu(\tau_i) = u_i^\mu \tau_i + b_i^\mu, \quad i = 1, 2, \quad (18)$$

where τ_i is the proper time along the trajectory and b_i is the initial trajectory. The nonvanishing Poisson brackets read,

$$\begin{aligned} \{b_i^\mu, u_i^\nu\}_{\text{P.B.}} &= -\eta^{\mu\nu}/m_i, \\ \{S_i^{\mu\nu}, S_i^{\alpha\beta}\}_{\text{P.B.}} &= S_i^{\mu\alpha} \eta^{\nu\beta} - S_i^{\mu\beta} \eta^{\nu\alpha} - (\mu \leftrightarrow \nu). \end{aligned} \quad (19)$$

Next we impose all second class constraints of the system, which promotes Poisson brackets to Dirac brackets [93]. More specifically, there are two spin-related constraints, known as the spin-supplementary and the conjugate gauge conditions,

$$V_i^\mu = u_{i\nu} S_i^{\mu\nu} = 0, \quad \chi_i^\mu = \Lambda_{i,0}^\mu - u_i^\mu = 0, \quad (20)$$

where Λ_0^μ is the timelike component of the body-fixed frame (see Ref. [140] for more details). Moreover, there are four constraints following from transversality and on-shellness,

$$\begin{aligned} \phi_1 &= b \cdot u_1 = 0, & \phi_2 &= u_1^2 = 1, \\ \phi_3 &= b \cdot u_2 = 0, & \phi_4 &= u_2^2 = 1, \end{aligned} \quad (21)$$

where $b^\mu = b_2^\mu - b_1^\mu$ is the impact parameter. In this way, the nonvanishing Dirac brackets read

$$\{b^\mu, u_i^\nu\} = -\text{sgn}_i \frac{(y^2 - 1)b^\mu b^\nu + l^\mu l^\nu}{m_i b^2 (y^2 - 1)}, \quad (22)$$

$$\{b^\mu, a_i^\nu\} = \text{sgn}_i \frac{u_i^\nu ((y^2 - 1)b^\mu (b \cdot a_i) + l^\mu (l \cdot a_i))}{m_i b^2 (y^2 - 1)}, \quad (23)$$

$$\begin{aligned} \{b^\mu, b^\nu\} &= \frac{(b^2 (y u_2^\mu - u_1^\mu) - l^\mu (a_1 \cdot u_2)) b^\nu - (\mu \leftrightarrow \nu)}{m_1 b^2 (y^2 - 1)} \\ &+ (1 \leftrightarrow 2, b \rightarrow -b), \end{aligned} \quad (24)$$

$$\{a_i^\mu, a_i^\nu\} = \frac{\epsilon_{\rho\sigma}^{\mu\nu} u_i^\rho a_i^\sigma}{m_i}, \quad (25)$$

where $\text{sgn}_1 = -1$ and $\text{sgn}_2 = 1$. To simplify the expressions we defined

$$l^\mu = \epsilon_{\nu\rho\sigma}^\mu b^\nu u_1^\rho u_2^\sigma, \quad a_i^\mu = \frac{1}{2m_i} \epsilon_{\nu\rho\sigma}^\mu u_i^\nu S_i^{\rho\sigma}. \quad (26)$$

Note that there are sign differences compared to Ref. [93], which is a consequence of an opposite metric signature alongside an additional sign in defining the spin vector.

The power of these brackets becomes apparent when computing observables. Given a radial action, I_r , we may compute the change in an observable via [93]

$$\Delta \lambda^\mu = \sum_j \frac{1}{j!} \underbrace{\{I_r, [I_r, \dots, \{I_r, \lambda^\mu\} \dots]\}}_{j \text{ times}}, \quad (27)$$

where λ^μ can be either the normalized four-momenta or spin vectors. Notice that this approach bypasses the need for explicit cut diagrams, since these contributions appear in the iterative structures of equation (27). For our purposes, we simply set $a_1 = a$ and $a_2 = 0$ when computing

the impulse and spin kick through $\mathcal{O}(G^3 S^4)$. In doing so, we find exact agreement with known results at $\mathcal{O}(G^3 S^2)$ including both probe structures and self-force corrections [44,67,81]. The results at $\mathcal{O}(G^3 S^3)$ and $\mathcal{O}(G^3 S^4)$ are new and we provide them in Supplemental Material [139]. We have also found agreement with the post-Newtonian velocity expansion results for the scattering angle in the aligned-spin limit through $\mathcal{O}(G^3 S^4)$ [141].

We combine our result with those of Alessio and Di Vecchia [97] to find that the logarithmic divergence—appearing in the high-energy limit of the two-loop amplitude—cancels through fourth order in spin. The cancellation was previously observed at $\mathcal{O}(G^3)$ in the spinless case [142,143] and through the second order in spin [68]. Moreover, treating their results as a radiation-reaction contribution to the radial action—which is a unique finite contribution at $\mathcal{O}(G^3)$ —we compute the impulse and spin kick to all orders in spin, finding exact agreement with the known results of Ref. [68] to quadratic order in spin. These results can be found in Supplemental Material [139].

Conclusion—In this Letter we have computed for the first time the classical two-loop amplitude for the conservative scattering process between a spinning and a spinless black hole through fourth order in spin. We have done this by computing scattering amplitudes between a scalar massive particle and a spin- s particle ($s = 0, 1$ and 2) and using the recently introduced spin interpolation method [81] to extract all spin structures in the classical amplitude. We find that the spin-shift symmetry [45,81,138] is present in all probe structures. It is natural to conjecture that this is related to a hidden symmetry, and hence integrability, of probe orbits in a Kerr background through fourth order in spin of either the background or probe object. This goes beyond the quadratic order in spin where integrability of binary Kerr systems has already been established [144–148]. To compute observables, we use the amplitude-action relation, and we find agreement with the radial action known in the literature for the case of a probe scalar. We then compute non-aligned spin observables by using the covariant Dirac brackets introduced in Ref. [93]. With this we reproduce known observables to $\mathcal{O}(G^3 S^2)$, giving further evidence for equation (27). To complete the non-dissipative picture, we have used the results of Ref. [97] to show the cancellation of the logarithmic divergence in the high-energy limit, and we have further obtained observables for the radiation-reaction dynamics to all orders in spin with the covariant Dirac brackets. Though we have yet to rigorously establish the radial action and Dirac bracket formalisms for nonaligned spin observables, the various checks that we have performed provide strong evidence for the correctness of the formalisms, likely to all orders, as the two-loop case remains unchanged from the one-loop case. This provides a simple and compelling answer to the quest of finding an eikonal-like formula for spinning binary dynamics [41,61–63,66].

Our techniques can be further exploited in several directions. We can complete the two-loop picture for quartic-in-spin observables by studying the scattering of two spinning black holes. We can also extend the conservative picture by incorporating dissipative effects, which requires including a small number of additional diagrams that vanish in the potential region in our scattering amplitude calculation as well as using the values of master integrals in the full soft region. We would also like to go beyond the quartic order in spin by matching with black hole perturbation theory [52] and study more general spinning objects such as neutron stars. Another important direction would be to explore higher orders in G . Finally, it would be interesting to explicitly connect the spin-shift symmetry observed here with the integrability of probe orbits in Kerr backgrounds, e.g., by recasting the symmetry of the amplitude as a certain classical Ward identity, and this could potentially offer a novel on-shell amplitude perspective on hidden symmetries of classical systems.

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