

Rise of Linear Regge Trajectories

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(Received 23 October 2025; revised 20 May 2026; accepted 21 May 2026; published 24 July 2026)

In this Letter, we consider constraints on the low-energy spectrum of scalar amplitudes with higher-spin exchanges. Assuming unitarity, crossing symmetry, and superconvergent high energy behavior, we demonstrate that the spectrum that maximizes the leading higher spin couplings of the second and third resonances is consistently given by a linear trajectory. We work strictly in ten dimensions although we expect analysis in other dimensions to be similar. For gravitational theories, the optimal spectrum is the linear trajectory defined by the mass and spin of the graviton and the lightest spin-4 resonance.

DOI: [10.1103/byzm-k9lw](https://doi.org/10.1103/byzm-k9lw)

Introduction—Motivated by the success of the numerical conformal bootstrap, where unitarity and crossing symmetry are recast as a convex optimization problem [1], substantial progress has been made in adapting semidefinite programming techniques to the S -matrix bootstrap [2–6]. Within this framework, unitarity and analyticity constraints translate into sharp bounds on Wilson coefficients of effective field theories (EFTs), and on the couplings of low-lying resonances [5–35]. These developments open the tantalizing prospect of isolating the minimal set of physical principles under which the string-theory S matrix emerges as the unique, self-consistent solution.

In the early S -matrix program, the zero-width approximation and channel duality played a crucial role in leading to the Veneziano amplitude [36] and reproducing Regge asymptotics [37]. In this approximation, the S matrix is “tree level” and the exchanged states are stable particles of different spin and mass. A spectrum involving higher-spin states is infinite and highly constrained. For example, the gap above the massive spin-2 [28], as well as higher spin states [26,35], must be finite. Furthermore, the mass of the lightest spin- ℓ state must be smaller than that of the lightest spin- $(\ell + 2)$ state, and satisfy convexity properties [30,33,38].

In this Letter, we study the low lying spectrum that can be exchanged at tree-level in scalar amplitudes, consistent with unitarity, crossing symmetry and twice-subtracted dispersion relations at fixed momentum transfer, which

apply to gapped, as well as gravitational theories [39]. In this context, it was recently shown that the additional high-energy (UV) assumption of ultrasoftness and patterns of zeros, uniquely fix the four-point amplitude to that of Veneziano and Virasoro–Shapiro [40], while related arguments show that sufficiently soft high-energy behavior in the presence of infinite higher-spin towers enforces Veneziano-like amplitudes with asymptotically linear Regge trajectories [41]. Here, we instead focus on low-energy (IR) properties of the amplitude, and ask what further assumptions lead to stringlike configurations.

We focus on the coupling of the lightest spin- ℓ state, and search for spectra that maximize it [42]. The couplings of spin-0 and spin-2 states cannot be bounded. This can be traced to the existence of amplitudes that satisfy bootstrap constraints but, in addition to spin-0 or spin-2 states, have an unphysical accumulation point at m_∞ [6],

$$\mathcal{M}(s, t) = \frac{\lambda}{2} \frac{G_\ell^D \left(1 + \frac{2t}{m^2}\right)}{m^2 - s} \frac{m_\infty^2}{m_\infty^2 - u} + \text{perm.}, \quad (1)$$

with $\ell = 0, 2$ and G_ℓ^D the Gegenbauer polynomial in D dimensions.

Here we circumvent this by instead considering -2 and 0 subtractions, generally referred to as superconvergence. This arises, for instance, in maximal and half-maximal supersymmetry, respectively, [see discussion around Eq. (4)], or in the scattering of spin-2 and -1 helicity states in $D = 4$ [21,22,45]. In this Letter we focus on $D = 10$, where Type-IIA/B string theory operates, but we expect the result to be more general.

Remarkably, by studying the couplings of the lightest few states as a function of masses, we find that a linear spectrum consistently maximizes the couplings and singles

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out a linear leading Regge trajectory. In the presence of gravity, the maximal couplings correspond to a linear trajectory with zero intercept, passing through the graviton pole, marking a sharp distinction to nongravitational theories. Linear Regge trajectories have long been central to dual resonance models and hadronic phenomenology, beginning with the Chew-Frautschi plot [46] (see Ref. [47] for a concise recount of the history). Our result gives a modern bootstrap principle for their emergence: higher-spin coupling maximization.

Recent works on the numerical bootstrap for large- N QCD, show that the spectrum inferred from the optimal functional that yields the maximal coupling of the spin-2 resonance in the presence of a lighter spin-1 state also exhibits a single (almost) linear trajectory [6,25,48]. However, it is known that single trajectories cannot be self-consistent [49], hence the interpretation of such extremal spectra remains an open question. Here, by directly studying the coupling of a spin- ℓ resonance as a function of the leading spectrum, we are able to put the emergence of linear trajectories on a stronger footing, independently of the existence of subleading trajectories.

Setup—We consider the $2 \rightarrow 2$ scattering amplitude of massless scalars $\mathcal{M}(s, t)$, where $s = (p_1 + p_2)^2$ and $t = (p_1 - p_4)^2$. Assuming that the amplitude involves only tree-level exchanges, $\mathcal{M}(s, t)$ is a meromorphic function with simple poles reflecting s , t and $u = -s - t$ channel exchanges. Near these poles, the amplitude behaves as

$$\mathcal{M}(s, t) \Big|_{s \rightarrow m_i^2} = - \frac{\sum_{\ell} \lambda_{i,\ell} G_{\ell}^D \left(1 + \frac{2t}{m_i^2}\right)}{s - m_i^2}, \quad (2)$$

with $\lambda_{i,\ell} \geq 0$, by unitarity. The amplitude is bounded by s^2 in the Regge limit, i.e.,

$$\lim_{s \rightarrow \infty} \frac{\mathcal{M}(s, t)}{s^2} = 0, \quad \text{fixed } t, \quad (3)$$

which is guaranteed in gapped theories, and follows from the emergence of classical general relativity at a large impact parameter in gravitational theories [39].

For ten-dimensional theories with $\mathcal{N} = 2$ maximal (32 supercharges) and $\mathcal{N} = 1$ half-maximal (16 supercharges) supersymmetry (SUSY), the amplitude takes the form

$$\mathcal{M}(s, t) = \delta^{8\mathcal{N}}(Q) f_{\mathcal{N}}(s, t) \quad (4)$$

with $\delta^{8\mathcal{N}}(Q)$ the fermionic delta function that solves the SUSY Ward identities at four-point (see Ref. [26] for a review). Since $\delta^{8\mathcal{N}}(Q) \sim s^{2\mathcal{N}}$, this implies an improved UV behavior for $f_{\mathcal{N}}$,

$$\lim_{s \rightarrow \infty} f_{\mathcal{N}}(s, t) s^{2(\mathcal{N}-1)} = 0, \quad \text{fixed } t. \quad (5)$$

This is reminiscent of graviton ($\ell = 2$) and photon ($\ell = 1$) scattering in four dimensions, where, for specific helicity configurations, the amplitude is proportional to $s^{2\ell}$ and leads to an identical improved UV behavior [22]. Indeed, for $\mathcal{N} = 2$, the massless multiplet contains the graviton in addition to the scalar, while for $\mathcal{N} = 1$ it contains the photon.

In this context, we study fixed $t = -p^2$ dispersion relations. These relate integrals,

$$\alpha_n^{\mathcal{N}}(p^2) \equiv \oint \frac{ds}{2\pi i} \frac{f_{\mathcal{N}}(s, -p^2)}{s [s(s - p^2)]^{n/2}}, \quad n \geq 0, \quad (6)$$

along a circle of radius $M^2 + t/2$ around $-t/2$, and integrals along the real s axis,

$$\alpha_n^{\mathcal{N}}(p^2) = \sum_{\ell \text{ even}} n_{\ell}^D \int_{M^2}^{\infty} \frac{ds}{2\pi} \frac{(2s - p^2) \rho_{\ell}(s) s^{\frac{4-D}{2}}}{[s(s - p^2)]^{n/2+1}} G_{\ell}^D(s, -p^2) \quad (7)$$

for even $n \geq -2$ or $n \geq 0$ for $\mathcal{N} = 2, 1$, respectively. Here, $\rho_{\ell}(s)$ is the spectral density, positive by unitarity. For narrow resonances at mass m_i , $n_{\ell}^D \rho_{\ell}(s) = 2\pi \lambda_{i,\ell} \ell s^{[(D-4)/2]} \delta(s - m_i^2)$, see Eq. (2).

Beautifully, the spectral density must further enable a description of the amplitude as a sum of dual t resonances, implying a series of null constraints on the spectral density itself [4,10]. These can be constructed in the forward limit via a double contour integral [48]:

$$\chi_{n,k} = \text{Res}_{t=0} \left[\left(\frac{1}{s^{n-k-1} t^{k+1}} - \frac{1}{t^{n-k-1} s^{k+1}} \right) - (s \rightarrow -s - t) \right] \mathcal{P}_{\ell} \left(1 + \frac{2t}{s} \right), \quad (8)$$

where $\langle \chi_{n,k} \rangle = 0$ for $n = 3, 4, 5, \dots$, and, $k = 0, 1, \dots, (n-2)/2$ in the case without gravity—with gravity, $n = 4, 5, 6, \dots$ instead [50].

At energies lower than the mass of the exchanged massive states, the amplitude is well described by an EFT with,

$$f_1^{\text{IR}}(s, t) = -8\pi G_N \left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u} \right) + g_0 + g_2 \sigma_2 + \dots, \\ f_2^{\text{IR}}(s, t) = \frac{8\pi G_N}{stu} + g_0 + g_2 \sigma_2 + \dots \quad (9)$$

where $\sigma_2 = s^2 + t^2 + u^2$ and $\dots$ representing higher derivative terms. For $\mathcal{N} = 1$, g_0 and g_2 can be identified with the Wilson coefficients of the operators F^4 and $D^4 F^4$ in the photon sector, while for $\mathcal{N} = 2$, it is maximal supergravity. Substituting Eq. (9) into Eq. (6), we find for example,

$$a_{-2}^2 = \frac{8\pi G_N}{p^2}, \quad a_0^1 = g_0 + 2g_2 p^4 + \dots \quad (10)$$

In the rest of this Letter we shall be interested in how the null constraints Eq. (8) shape spectra with narrow resonances. The EFT coefficients in Eq. (9)—which have been the focus of most of the EFT bootstrap literature—will only serve us as normalization factors.

Zero subtractions—We begin with the 0-subtracted scenario, where the amplitude *must* include the exchange of infinitely many resonances. An example of a UV complete amplitude satisfying these properties is the open superstring completion of the Dirac–Born–Infeld action [55], where the four-point amplitude is given by

$$f_1(s, t) = -\frac{\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)} - \frac{\Gamma(-s)\Gamma(-u)}{\Gamma(1+t)} - \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(1+u)}, \quad (11)$$

and where we normalize the string scale $\alpha' = 1$. This function has s -channel poles at $s = 1, 3, 5, \dots, 2k + 1$, where at $s = 2k + 1$ lies the highest-spin $\ell = 2k$ state [the factor of 2 comes from the $\delta^8(Q)$ in Eq. (4)].

We assume the existence of a finite number of isolated narrow resonances at scales below M . Above M , on the other hand, we remain agnostic about the spectrum and refer to this region as the UV. We aim to bound the couplings of the resonances normalized by the low-energy data, such as the EFT Wilson coefficients or G_N . Note, however, that since the DBI amplitude is already self-consistent without a graviton pole, there cannot be any bound when normalized against G_N . Thus, we normalize the couplings with respect to g_0 and set $G_N = 0$ for simplicity.

Two-state system: We begin with an ansatz for the spectrum which involves two sets of isolated states with masses $0 < m_1^2 < m_2^2 \leq M^2$ and spins $\ell_1 = 0, \ell_2 \leq 2$, in addition to the massless scattered particles. For this setup, the precise ansatz that enters Eq. (7) is

$$\tilde{\rho}_\ell^{\text{state}}(s) = \delta_{\ell,0}\lambda_{1,0}\delta(s - m_1^2) + \sum_{\ell'=0}^2 \delta_{\ell,\ell'}\lambda_{2,\ell'}\delta(s - m_2^2) + \tilde{\rho}_\ell(s)\theta(s - M^2), \quad (12)$$

where $\tilde{\rho}_\ell(s) \equiv n_\ell^D s^{[(4-D)/2]} \rho_\ell(s) / 2\pi$, and we use $\lambda_{i,\ell}$ to represent the coupling of the i th state with spin ℓ .

Fixing m_2 and M^2 , we then maximize the coupling $\lambda_{2,2}$ (more precisely, the dimensionless ratio $\lambda_{2,2}/g_0 m_2^2$) as a function of m_1 . We repeat this for different values of M , and show the results in Fig. 1. We observe that as we increase m_1 , the maximal value tracks an approximate plateau until an M -dependent critical value, after which it sharply drops to zero.

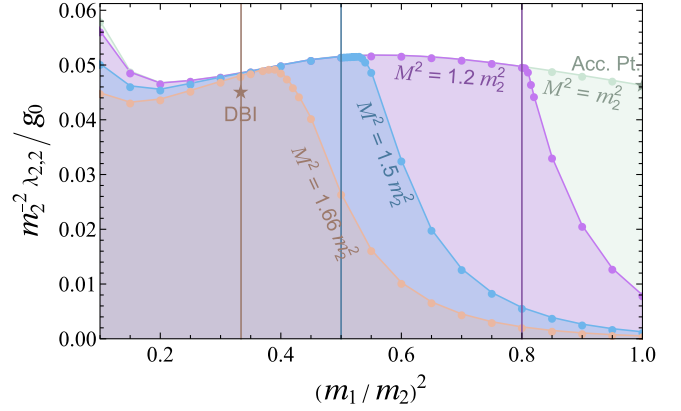


FIG. 1. Maximal value of $\lambda_{2,2}/(g_0 m_2^2)$ for two resonances below threshold, as a function of m_1^2/m_2^2 and for several cutoffs M^2 . All our results are in $D = 10$ dimensions. See Sec. IA of Supplemental Material [56] for details on numerics. Vertical lines mark the critical values $m_1 = m_{1,c}$ in Eq. (13). The orange and green stars denote the DBI and accumulation-point examples $[(s - M^2)(t - M^2)(u - M^2)]^{-1}$, respectively.

The sharp departure from the plateau occurs when the assumed low-lying spectrum can no longer be completed into a linear trajectory with the next spin-4 state above threshold. This gives

$$m_1^2 \leq 2m_2^2 - M^2 \equiv m_{1,c}^2. \quad (13)$$

We call $m_{1,c}^2$ the critical value that saturates this inequality—see vertical lines in Fig. 1, as well as solid lines in Fig. 2. The sharp drops in Fig. 1 are in the proximity of $m_{1,c}^2$. On the plateau, both the maximal coupling and the extracted spectrum are essentially independent of M , indicating that once such a linear completion is allowed, it is selected by

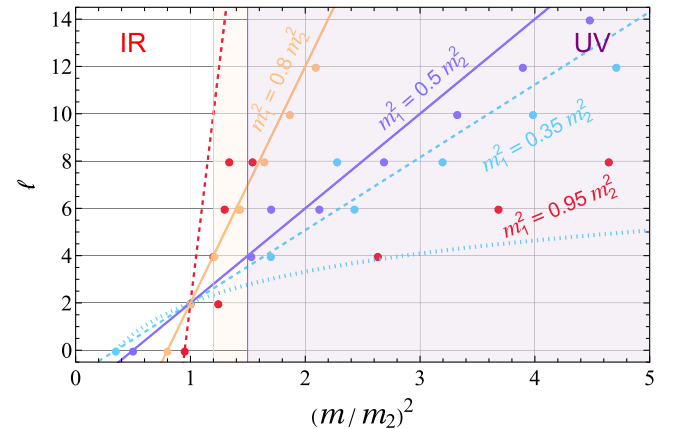


FIG. 2. Spectra maximizing the couplings in Fig. 1, showing only states with $\lambda_{i,\ell} \geq 10^{-5} g_0 m_1^2$. Shaded regions denote the UV for the indicated cutoffs. Solid lines are the linear trajectories fixed by the two lightest states; the dotted curve shows the spectrum saturating the analytic bounds of Refs. [30,38].

the optimization. Figure 2 shows that the corresponding trajectories persist into the UV.

One might suspect that the sharp fall-off is controlled by the analytic meromorphic-spectrum bounds of Refs. [30,38]. This is not the case: the fall-off occurs near the linear-trajectory threshold $m_{1,c}^2$, rather than the analytic bound $m_{1,a}^2$, and the extremal spectra differ qualitatively from the spectra saturating those bounds. Details and convergence are given in Sec. IB of Supplemental Material [56].

Three-state system: To further establish the prominence of linear trajectories, we bring m_3^2 into the IR spectrum where we now have $(m_1^2, \ell_1 = 0)$, $(m_2^2, \ell_2 \leq 2)$ and $(m_3^2, \ell_3 \leq 4)$, with $m_1^2 \leq m_2^2 \leq m_3^2 \leq M^2$. The ansatz for the spectral density now takes the form

$$\begin{aligned} \tilde{\rho}_\ell^{\text{3state}}(s) = & \delta_{\ell,0} \lambda_{1,0} \delta(s - m_1^2) + \sum_{\ell'=0}^2 \delta_{\ell,\ell'} \lambda_{2,\ell'} \delta(s - m_2^2) \\ & + \sum_{\ell''=0}^4 \delta_{\ell,\ell''} \lambda_{3,\ell''} \delta(s - m_3^2) + \tilde{\rho}_\ell(s) \theta(s - M^2). \end{aligned} \quad (14)$$

In this setup, we search for maxima of the coupling of the spin-4 state $\lambda_{3,4}/g_0 m_3^2$, as a function of m_2 [58]. The results are shown in Fig. 3, for $m_1^2 = 1$, $m_3^2 = 5$ and for different values of the cutoff M . The linear trajectory hypothesis is made manifest by the sharp peak located close to $m_2^2 = (m_1^2 + m_3^2)/2$. Furthermore, for values of M that are compatible with higher states lying on the linear trajectory, i.e., an $\ell = 6$ state at $m_4^2 = (3m_3^2 - m_1^2)/2 \geq M^2$, this sharp peak is almost unchanged, as illustrated by the purple and orange curves in the figure. Instead for larger values of M that do not allow for a linear trajectory, the coupling is suppressed—see blue curve in Fig. 3.

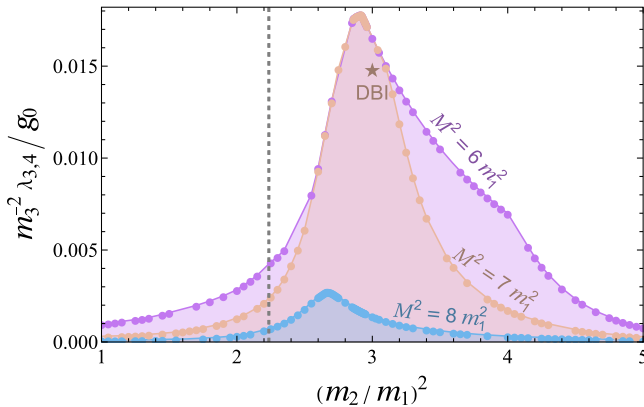


FIG. 3. Maximal coupling of the $\ell = 4$ state, as a function of m_2 for $m_3^2 = 5m_1^2$, and for different values of $M^2/m_1^2 = 6, 7, 8$, with 7 the reference DBI value, and the DBI result denoted by a star. Analytic lower bound shown by the dashed line [30,38].

Away from the peak, the optimizer instead aligns the spin-2, spin-4, and putative spin-6 states on a different linear trajectory. This is made manifest by the sharp kink on the purple curve at $m_2^2 = 4m_1^2$, which is the uppermost value for such a trajectory without m_1 . This behavior is confirmed by the extremal spectra in Sec. IC of Supplemental Material [56].

Our results have deeper implications than the exact shape of the maximal curve. The sharp suppression away from linear alignment suggests that linear trajectories do not merely describe extremal solutions; they are required for sizable low-energy higher-spin couplings.

Gravity and -2 subtractions—We now consider the case with gravity and maximal supersymmetry, captured by a -2 subtracted dispersion relation. In this case, a relevant UV completion is [8],

$$f_2(s, t) = \frac{-\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} \times \left(1 - \epsilon \frac{stu}{(1+t)(1+u)(1+s)} \right). \quad (15)$$

This amplitude is unitary for $0 \leq \epsilon \leq 1$, [59] and reduces to the Virasoro–Shapiro amplitude for $\epsilon = 0$. In addition to the graviton pole, this function has s -channel poles at $s = 1, 2, \dots$, where at $s = k$ lies the highest-spin $\ell = 2k - 2$ state.

Two-state system: We start with two isolated states below the gap M^2 and maximize the spin-2 coupling $\lambda_{2,2}$ for different values of m_1^2/m_2^2 . The result is shown in Fig. 4.

Compared to the 0-subtraction analysis in Fig. 1, we see that here there is no plateau at $m_1^2 < m_2^2/2$, independently of M^2 . Moreover, for $M^2 < 3m_2^2/2$, the curves peak at $m_1^2 = m_2^2/2$ and are followed by a slow-roll slope at $m_2^2/2 \leq m_1^2 \leq m_{1,c}^2$. Instead, for $M^2 > 3m_2^2/2$, the coupling is drastically suppressed.

We interpret this as follows: as in the 0-subtracted case, the sharp drop at $m_1^2 > m_{1,c}^2$ reflects the impossibility of completing the low-lying states into a linear trajectory. Gravity adds a new constraint: the graviton pole and the first massive scalar define a critical trajectory. Slopes below this trajectory cannot maximize the spin-2 coupling, explaining both the suppression for $m_1^2 < m_2^2/2$ and the loss of the peak for $M^2 > 3m_2^2/2$. The optimum therefore lies on the graviton trajectory and selects the Virasoro–Shapiro point.

Therefore, the graviton pole leaves a distinctive imprint, with the graviton trajectory providing the optimal solution for maximizing the spin-2 coupling. The vertical line in Fig. 4 denotes the values spanned by Eq. (15). Maximization indeed selects the Virasoro–Shapiro amplitude. We provide details on the numerical implementation and convergence in Sec. II of Supplemental Material [56].

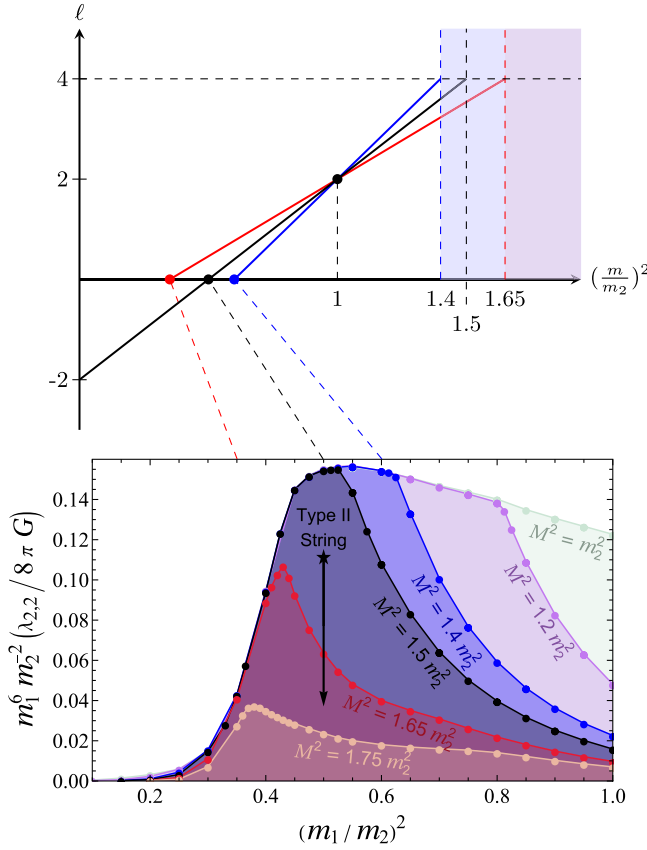


FIG. 4. Top: a cartoon of linear spectra, for different choices of M^2 . Blue, black, and red lines represent lower-than-critical, critical, and larger-than-critical slopes, respectively. Bottom: maximization of $\lambda_{2,2} m_1^6 / 8\pi G m_2^2$ in a system with two resonances below threshold, for different values of m_1^2 and M^2 . The vertical black arrow correspond to the amplitude in Eq. (15), with the star indicating the $\epsilon = 0$ case.

Three-state system: Further considering three resonances below threshold, we fix (M^2, m_3^2, m_1^2) and maximize $\lambda_{3,4}$ with respect to m_2^2 . The result for $m_3^2 = 3m_1^2$ is shown in Fig. 5. A narrow peak emerges at the critical value $m_2^2 = (m_1^2 + M^2)/2$ which satisfies Eq. (13), and also lies on the graviton trajectory. When M^2 falls below $4m_1^2$, the graviton trajectory is no longer viable, and the peak drastically decreases.

Conclusion—In this Letter we have shown that, in the narrow-width approximation, maximizing low-lying higher-spin couplings selects spectra organized near linear Regge trajectories. This occurs in both 0-subtracted and -2 -subtracted dispersion relations, and in the gravitational case the optimal trajectory is fixed by the graviton pole. Thus linearity emerges here not from assuming an infinite Regge tower or extraordinary UV softness, but from an IR extremization principle.

In the traditional Regge theory program, linear Regge trajectories emerged as a global feature, arising from the requirement that an infinite tower of resonances

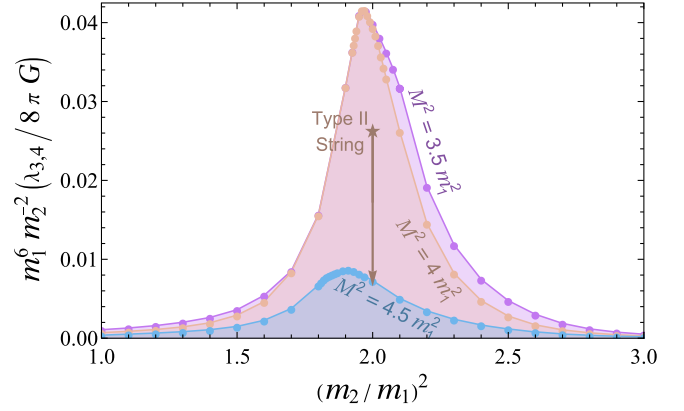


FIG. 5. Maximum of $\lambda_{3,4} m_1^6 / 8\pi G m_2^2$, as a function of $(m_2/m_1)^2$, for $m_3^2 = 3m_1^2$ and different values of M^2/m_1^2 .

collectively produce finite high-energy growth of scattering amplitudes, i.e., Reggeization. By contrast, in our analysis, we do not assume the existence of an entire Regge tower *a priori*, nor do we assume any extraordinary UV softness; yet, linear trajectories emerge.

We further believe that these results generalize qualitatively to twice subtracted dispersion relations without SUSY, where the role of the spin-0 resonance is taken by the first massive spin-4 state.

Finally, we have focused on the leading trajectory, with the understanding that daughter trajectories must be present, given the inconsistency of a single linear trajectory. It will be interesting to understand which IR principles organize the daughter trajectories as well. Furthermore, the narrow-width approximation also implies that the massive higher-spin states are stable to leading order in perturbation theory, such that the amplitudes of their scattering are amenable to the standard bootstrap constraints. The study of such mixed systems can potentially sharpen the picture of which low energy spectra maximize higher-spin couplings.

Acknowledgments—We are very grateful to Justin Berman, Li-Yuan Chiang, Lukas Lindwasser, Francesco Sciotti, and He-Chen Weng for important discussions, and Brando Bellazzini, Justin Berman, and Leonardo Rastelli for enriching comments on the manuscript. S. R. and F. R. have been supported by the SNSF under Grant No. 200021-205016. Y.-t.H. and J.-D.T. are supported by Taiwan National Science and Technology Council Grants No. 112-2628-M-002-003-MY3 and No. 114-2923-M-002-011-MY5.

Data availability—The data that support the findings of this article are openly available [60].

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