

Orbital Magnetic Field Driven Metal-Insulator Transition in Strongly Correlated Electron Systems

Georg Rohringer^{1,2} and Anton Markov^{3,4}

¹*Theory and Simulation of Condensed Matter, Department of Physics, King's College London, The Strand, London WC2R 2LS, United Kingdom*

²*Institute of Theoretical Physics, University of Hamburg, 20355 Hamburg, Germany*

³*Russian Quantum Center, Skolkovo Innovation City, 121205 Moscow, Russia*

⁴*Center for Nonlinear Phenomena and Complex Systems, Université Libre de Bruxelles, CP 231, Campus Plaine, B-1050 Brussels, Belgium*



(Received 27 June 2024; revised 12 August 2025; accepted 15 October 2025; published 6 November 2025)

We study the effects of an orbital magnetic field on the Mott metal-insulator transition in the Hubbard-Hofstadter model. We demonstrate that sufficiently large magnetic fields induce a Mott insulator-to-metal phase transition supporting our claim with dynamical mean field theory (DMFT) numerical results. For both competing phases (metal and insulator) we observe a magnetic-field-induced metallization reflected in an enhancement of kinetic and potential energy. The kinetic energy of the Mott insulator increases due to the Aharonov-Bohm effect experienced by electrons virtually tunneling around an elementary plaquette which is, however, suppressed by strong correlations. The kinetic energy of the metallic phase, on the other hand, is more strongly affected by the magnetic field through a field-driven redistribution of spectral weight due to the formation of magnetic minibands. This leads to an increase of the kinetic energy which tends to stabilize the metallic state. Our theoretical results might be relevant for recent experimental studies on magnetic field driven insulator-to-metal transitions in strongly correlated materials such as VO₂, λ -type organic conductors, and moiré multilayers.

DOI: [10.1103/bw7f-p8mp](https://doi.org/10.1103/bw7f-p8mp)

Introduction—Transitions between metallic and insulating states in solids play a fundamental role in condensed matter physics. Apart from their theoretical importance they are potentially also technologically highly relevant as they allow controlling the flow of the electrical current. There exists a variety of distinct types of insulators that are driven by very different physical mechanisms: Band insulators [1] emerge due to the absence of spectral weight at the Fermi level, Slater insulators are induced by strong (antiferromagnetic) spin fluctuations [2], in Anderson insulators [3,4] disorder leads to a localization of particles, Peierls insulators [5] originate from structural changes of the underlying lattice system, and in charge transfer insulators [6] electrons or holes are transferred from the conduction to the valence band by correlation effects. A particularly interesting class are Mott insulators [7], where the insulating state is induced by correlation effects between the electrons due to their strong Coulomb repulsion [8–11].

A transition between a metallic and an insulating state can be triggered by different control parameters including temperature, strain [12], doping [13], (chemical) pressure, or electric fields [14]. Importantly for the context of this Letter, the transition between a metal and an insulator can be also controlled by the application of an external magnetic field [15,16]. A good example for such a phenomenon in strongly correlated electron systems is the colossal magnetoresistance observed in manganites [17–19], where an external magnetic field can change the resistivity by several orders of magnitude.

Recently magnetic field driven insulator-to-metal transitions were experimentally observed in several strongly correlated materials including the transition metal oxide VO₂ [20,21], λ -type organic conductors [22], and twisted bilayer graphene [23–25]. The magnitudes of the critical magnetic field at which these transitions occur appear to be inversely proportional to the size of the unit cell when comparing this metallization effect across the different compounds. Although a relevant contribution of the Zeeman term cannot be ruled out with absolute certainty, this observation rather suggests that the phase transitions are controlled by the magnetic flux through a unit cell, which has inspired us to study the *orbital effects* of the magnetic field in the presence of strong electronic correlations [26–31].

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

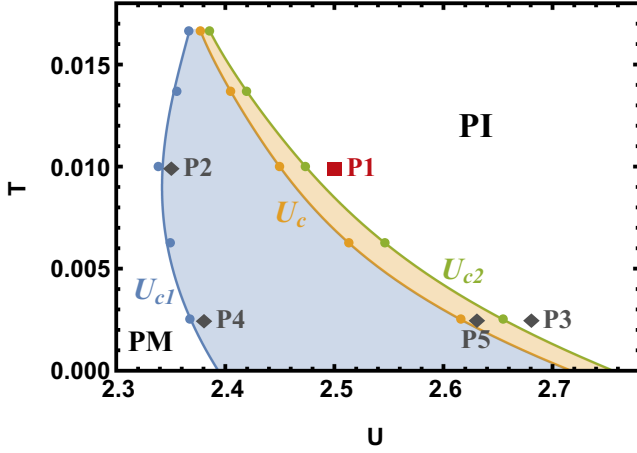


FIG. 1. DMFT phase diagram of the 2d Hubbard model on a simple (bipartite) square lattice as a function of interaction strength U and temperature T at magnetic field $B = 0$. U_{c1} and U_{c2} are the boundaries between the coexistence region (blue and orange shaded area) and the paramagnetic metallic (PM) and paramagnetic insulating (PI) phase, respectively. U_c corresponds to the line where the thermodynamic phase transition occurs. P1 to P5 indicate the points for which calculations at finite orbital magnetic fields have been performed.

Model and method—We investigate the single-band Hubbard-Hofstadter model on a square lattice with hopping amplitudes t_{ij} for electrons tunneling between sites $\mathbf{R}_i$ and $\mathbf{R}_j$, where a homogeneous magnetic field perpendicular to the two-dimensional lattice is coupled to the orbital degrees of freedom by multiplying t_{ij} with a Peierls phase (for details see Sec. S1 in Supplemental Material [32] which includes Refs. [33–39]). Correlation effects are introduced via a purely local Hubbard interaction U within or close to the coexistence region between the metallic and the insulating state of the field-free model (see Fig. 1). To investigate these correlation effects, we exploit a recently developed extension of dynamical mean field theory (DMFT) [40–43] to finite orbital magnetic fields [27–30]. Our analysis indeed predicts an orbital magnetic field driven metallization and a related metal-insulator transition that can be explained by a redistribution of spectral weight due to the formation of magnetic minibands and a delocalization effect induced by the magnetic field which drags the electrons on closed orbits around a plaquette of the underlying lattice.

We omit the Zeeman effect of the magnetic field to disentangle the relevance of an orbital magnetic field from its impact on the spin degrees of freedom. The latter has been studied for the Hubbard model within DMFT [44–46] and the Gutzwiller approach [47] predicting a crossover from a Mott to a fully polarized band insulator and an extended insulating region, respectively, rather than an enhancement of metallicity due to the magnetic field. Let us, however note, that an investigation of the interplay between the response of orbital and spin degrees of freedom to a magnetic field is a very interesting future research

direction. Moreover, paramagnetic DMFT calculations neglect the strong antiferromagnetic fluctuations emerging for the bipartite lattice with nearest neighbor hopping $t = t_{i(i+1)}$, which we consider in the Letter just for simplicity. To justify this restriction, we show that we obtain completely analogous results for frustrated lattices in Sec. S7 of Supplemental Material [32], where such fluctuations are strongly suppressed as is the case also for VO_2 or λ -type organic conductors.

In the following, we will use the half bandwidth $(W/2) = 4t$ at $B = 0$ as unit of energy while the magnetic field is given in terms of (Φ_0/a^2) , where a is the lattice constant. For convenience we set $\hbar = k_B = e = 1$ allowing us to express temperature and inverse time in terms of our energy unit $(W/2)$ and the conductivity as dimensionless quantity.

Results—We demonstrate the emergence of a magnetic-field-driven metal-insulator transition by performing DMFT calculations for the free energy F , the paramagnetic dc conductivity $\sigma_{xx} \equiv \sigma_{xx}(\omega = 0)$, and the local spectrum $A(\omega)$ as functions of the magnetic field. While the behavior of the free energy indicates the presence of a phase transition, the latter two quantities reveal its physical nature. In Fig. 2 we show our results for point P1 indicated by a red square in the phase diagram of Fig. 1, corresponding to $U = 2.5$ and $T = 0.01$, where the system is a Mott insulator for $B = 0$. Our data for points P2 to P5 as well as for systems where geometric frustration has been introduced by a next-nearest neighbor hopping $t' = t_{i(i+2)}$ are discussed in Secs. S4, S6, and S7 of Supplemental Material [32], respectively. In the left panel, the free energy F is shown as a function of the magnetic field B for two different calculation sets, denoted by $B_\uparrow$ and $B_\downarrow$. For the first set, $B_\uparrow$, we started from the insulating state S1 at $B = 0$ and gradually increased the magnetic field (dashed purple line). The free energy evolves smoothly, and no signs of a phase transition are observed; i.e., the system remains in its initial thermodynamic state. For the second set, $B_\downarrow$, we started at $B = 0.5$ and gradually decreased the magnetic field (green solid line). Remarkably, at $B = 0.5$ we can stabilize not only state S1 but also a second solution, S2, which has a substantially lower free energy. As B decreases, the difference in the free energy between the two solutions diminishes and eventually vanishes at $B_c = 0.2069$, indicating a thermodynamic phase transition from S2 to S1. We can stabilize S2 even for smaller values of $B < B_c$, where its free energy is already higher than that of S1, until S2 becomes unstable at $B_{c2} = 0.1388$ and collapses onto S1. This behavior is a distinctive hallmark of a first-order phase transition, where B_{c2} marks the lower boundary of the coexistence region between states S1 and S2.

The dc conductivity [48] $\sigma_{xx} \equiv \sigma_{xx}(\omega = 0)$ and spectral function $A(\omega)$ in the middle and right panels allow us to unambiguously identify the states S1 and S2 as insulator and metal, respectively. In fact, we can see that σ_{xx} [as well

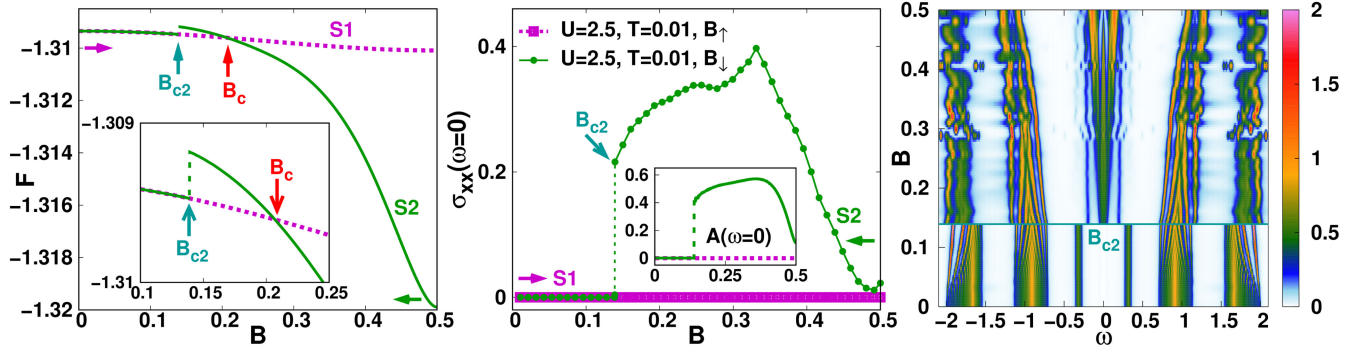


FIG. 2. Free energy F (left panel) and dc conductivity $\sigma_{xx}(\omega = 0)$ (middle panel) as a function of the orbital magnetic field B as well as local spectral function $A(\omega)$ (right panel) plotted as a heat map in the (ω, B) plane for the parameter set P1 in Fig. 1 corresponding to $U = 2.5$ and $T = 0.01$. $B_{\uparrow}$ and $B_{\downarrow}$ indicate that calculations have been started from $B = 0$ (S1) or $B = 0.5$ (S2), respectively. For $A(\omega)$ only results for S2 are shown. Inset left panel: blow up of the transition region. Inset middle panel: spectral function at the Fermi level $A(\omega = 0)$ as a function of B .

as $A(\omega = 0)$ in the inset of the middle panel] is finite for S2 (solid green circles) for $B > B_{c2}$ and it vanishes for S1 (open violet squares) on which S2 collapses at $B \leq B_{c2}$. Similarly, the spectral density $A(\omega)$ which is shown for state S2 as a heat map in the (ω, B) plane in the right panel features a charge gap around $\omega = 0$ for $B < B_{c2}$ whose size $U - W \approx 0.5$ is in excellent agreement with predictions for the gap size in the Mott insulating state in the literature [49,50]. For $B > B_{c2}$ we observe final spectral weight around $\omega = 0$ signaling a metal-insulator transition at B_{c2} . More details about the spectral function are provided in Sec. S2 A of Supplemental Material [32] (see also Refs. [51–55] therein).

Let us note that the upper boundary B_{c1} , beyond which the system always remains in the metallic state S2, is not reached for the largest magnetic field $B = 0.5$ [56] in the bipartite case considered here. Instead, B_{c1} is found at lower interaction strengths (albeit without B_{c2}), as discussed in Sec. S4 of Supplemental Material [32]. For systems with strong geometrical frustration [57,58], both boundaries B_{c1} and B_{c2} of the coexistence region are located within the interval $B \in [0, 0.5]$ (see Fig. S12 in Sec. S7 of Supplemental Material [32]).

Discussion—Let us now address the question which physical mechanism is at work to drive the transition between a metal and an insulator upon tuning the orbital magnetic field B . A first naive attempt to understand this phenomenon is to investigate the magnetic field dependence of the spectral width $W(B)$ of the noninteracting system which is defined as the difference between the maximum and minimum eigenvalue E_{lk} of the dispersion matrix $\epsilon_{ll'}(\mathbf{k})$ [see Eq. (S4) in Sec. S1 of Supplemental Material [32]], i.e., $W(B) = \max_{lk} E_{lk} - \min_{lk} E_{lk}$. The corresponding results are shown in panel (c) of Fig. 3. We observe that $W(B)$ decreases with B which enhances the interaction-spectral width ratio $[U/W(B)]$ and should, hence, lead to a more insulating state rather than the observed metallization. Next, we consider the second moment of the noninteracting

density of states $M_2 = (1/q) \sum_{\mathbf{k}} \text{Tr} e_{\mathbf{k}}^2$ which is approximately related to the critical interaction $U_c \propto \sqrt{M_1}$ [59]. However, quite remarkably for the Hofstadter model M_1 does not depend on the magnetic field B and consequently no effect of the orbital magnetic field on the Mott transition can be expected from this consideration. Hence, a more thorough analysis of the observed phenomenon is required.

From a general perspective the Mott transition is governed by the balance between kinetic and potential energy. The Mott insulating phase is characterized by a small absolute value of kinetic and potential energy compared to the metallic state due to the localization of particles. An increase in the (absolute) value of these thermodynamic variables thus signals a tendency toward metallization. We indeed observe such an enhancement of E_{kin} with increasing magnetic field for both solutions S1 (solid green circles) and S2 (open purple squares) in panel (a) of Fig. 3 (analogous results for the potential energy E_{pot} are shown in Sec. S3 of Supplemental Material [32]). The effect is more pronounced in the metallic regime $B > B_{c2}$ of S2, where E_{kin} becomes substantially larger and grows rapidly with B . A similar trend was reported in determinant quantum Monte Carlo [31] and ED [26] studies, albeit without addressing the metal-insulator transition.

On the metallic side the increase of E_{kin} with B is in excellent quantitative agreement with the evolution of the noninteracting kinetic energy E_{kin}^0 [see inset of panel (a)] renormalized by the quasiparticle weight Z , corresponding to the enhancement of the effective mass of a quasiparticle due to correlations $(1/Z) = (m_{\text{eff}}/m)$ (for the rigorous definition see Sec. S3 in Supplemental Material [32]), and shifted by a field-independent constant C , which accounts for the contribution from the incoherent portion of the spectrum (yellow crosses). This behavior of E_{kin} is consistent with predictions of the Gutzwiller approach [60–62] which allows for a physical transparent understanding of the Mott transition in Hubbard-type models at low temperature. This method predicts a critical interaction

strength, at which the metal-insulator transition occurs, of $U_c = 8|E_{\text{kin}}^0|$. Hence, as $|E_{\text{kin}}^0|$ increases with B we can expect a corresponding increase of U_c or, for a fixed value of U , a transition from an insulator to a metal at the value of B , where $8|E_{\text{kin}}^0|$ reaches this value of U . To quantify these expectations and determine the energy scale that is responsible for the magnetic-field driven metal-insulator transition we show U_c (solid red squares) and U_{c2} (solid blue circles) as a function of B in Fig. 3(b) [which are obtained by inverting the monotonous functions $B_c(U)$ and $B_{c2}(U)$] and compared it to $|E_{\text{kin}}^0(B)|$ (green line). We find a good agreement for $U_c(B)$ with $6|E_{\text{kin}}^0(B)|$ where the reduction of the factor 8 in the original version of the Gutzwiller approach [61] to 6 is indeed in excellent agreement with the predictions obtained from an improved treatment of fluctuations within the Gutzwiller method in Ref. [63] and the reduced value of U_c at finite temperatures with respect to $T = 0$ [64].

We, hence, conclude that the magnetic-field-driven metal-insulator transition is governed by the increase of $|E_{\text{kin}}^0|$ with B . The increase can be explained by a redistribution of spectral weight in the noninteracting system due to the formation of magnetic minibands that generically feature van Hove singularities at finite energies, which are associated with the topological properties of the system [65]. This is illustrated exemplary in Fig. 3(d) for $B = \frac{1}{3}$. The enhanced contribution to $|E_{\text{kin}}^0|$ arises from the van Hove singularity at $\omega \approx -0.55$ for $B = \frac{1}{3}$, while the singularity at $\omega = 0$ for $B = 0$ has minimal impact on E_{kin} .

In the Mott insulating regime, the magnetic-field-induced increase of kinetic (and potential) energy can be explained in a simplified way by a strong coupling expansion [66–68] of these thermodynamic variables in terms of (t/U) which is carried out explicitly in Sec. S1 D of Supplemental Material [32]. The leading order B -dependent term for both E_{pot} and $|E_{\text{kin}}|$ is proportional to $-(t^4/U^3)\cos(2\pi B)$ and indeed leads to an increase of these energies when B increases from 0 to 0.5. This behavior can be attributed to the Aharonov-Bohm [69] effect experienced by the electrons hopping around a plaquette [Fig. 3(e)] which leads to a delocalization of these particles.

Relation to experimental studies—At the end let us briefly address three experimental studies for which our theoretical analysis of the orbital-magnetic-field-driven insulator-to-metal transition can be potentially relevant. In all these strongly correlated materials an insulator-to-metal transition was observed for magnetic fields corresponding to a substantial part of the flux quantum which indicates the importance of the orbital effects of the magnetic field. Moreover, the relevance of the predicted mechanism is supported by the fact that the value of the critical magnetic field strength is approximately inversely proportional to the unit cell area. Let us, however, stress that the exact magnetic field at which the transition occurs in these complicated

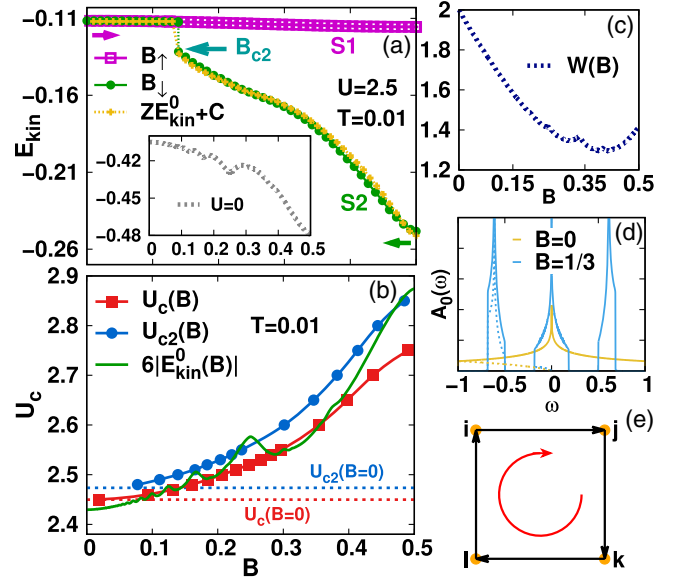


FIG. 3. (a) Kinetic energy for $U = 2.5$ and $T = 0.01$ (point P1 in Fig. 1) as a function of the orbital magnetic field B obtained by the two sets of calculations $B_{\uparrow}$ and $B_{\downarrow}$ discussed in Fig. 2. Yellow crosses correspond to the non-interacting kinetic energy E_{kin}^0 (shown in the inset) rescaled by the quasiparticle weight Z and shifted by a magnetic field independent constant C . (b) Critical interactions U_c and U_{c2} (see Fig. 1) as a function of B compared to the kinetic energy of the noninteracting system for $T = 0.01$. (c) Spectral width $W(B)$ of the noninteracting Hofstadter model as function of B . (d) Noninteracting density of states $A_0(\omega)$ for $B = 0$ and $B = \frac{1}{3}$. Dashed lines indicate the (negative) contribution $-\omega A_0(\omega)$ to the kinetic energy. (e) Illustration of an electron hopping around a plaquette due to the magnetic field.

materials can obviously not be predicted by a simple single-band Hubbard model calculation.

(i) In recent experiments [20,21] a magnetic-field-driven insulator-to-metal transition was observed in tungsten-doped vanadium dioxide $\text{V}_{1-x}\text{W}_x\text{O}_2$ at doping $x = 0.06$, and ultrahigh magnetic fields $B = 500$ T. Let us note that no metal-insulator transition has been observed in the experiment for lower W doping in the considered range of the magnetic field, which indeed fits our predictions for the role of the orbital effects. Since the lattice constant in $\text{V}_{1-x}\text{W}_x\text{O}_2$, which can be estimated by $a \approx 0.6$ nm, decreases with decreasing doping [70,71] a gradually increasing magnetic field is required to generate the same total flux through a unit cell at which the phase transition can be observed.

(ii) In λ -type organic conductors [22] a magnetic field induced transition has been found at $B \sim 50$ T, which is much lower than the corresponding field strength at which the transition occurs in VO_2 . Within the scenario of the orbital magnetic field driven Mott transition this can be attributed to the much larger unit cell in λ -type organic conductors (lattice constant $a \sim 1.2$ nm), which facilitates large fluxes also for smaller magnetic fields.

(iii) Finally, a magnetic field driven phase transition was observed in magic angle twisted bilayer graphene [23] at moderate magnetic fields of $B \approx 5$ T for the half-filled band close to the first magic angle. Within the picture of the orbital-magnetic-field-driven Mott transition this low value of B_c can be explained by the large lattice constant at this twist angle of ≈ 13.6 nm. Let us note that Zeeman effect of the magnetic field could not easily explain the appearance of the metallic phase as discussed in Refs. [24,25].

Conclusions and outlook—We have demonstrated the emergence of a metal-insulator transition in the Hubbard-Hofstadter model which is driven by the orbital effect of an external magnetic field on the electrons. As for the standard interaction-driven MIT we observe a first order behavior with a coexistence region extending to a magnetic field B_{c2} while the actual thermodynamic transition occurs at $B_c > B_{c2}$. We have argued that this magnetic-field-driven change of state can be explained by a metallization effect due to the orbital magnetic field, which is indicated by the increase of the kinetic and potential energy with increasing B . We have identified the redistribution of spectral weight due to the formation of magnetic mini-bands and the magnetic field induced virtual motion of an electron around a plaquette as theoretical explanations for this effect from a metallic and insulating perspective, respectively. In the final part of the Letter, we have discussed the relevance of our results for recent experiments on VO_2 , λ -type organic conductors and twisted bilayer graphene and argue that orbital effects of an external magnetic field might play an important role for the magnetic field induced insulator-to-metal transition in these materials.

In this Letter we have focused on the orbital effect of the magnetic field and neglected its impact on the spin degrees of freedom via a Zeeman term. In general, the importance of the orbital effects of a magnetic field with respect to the spin Zeeman term can be investigated experimentally by changing the direction of the magnetic field. While the Zeeman term should be unaffected, the orbital effects should change quite significantly. Another possibility to disentangle the spin and orbital effects of magnetic fields are synthetic gauge fields [72] generated by lasers with periodic driving which couple only to orbital degrees of freedom. Hence, a comprehensive investigation of the interplay between the effects of a magnetic field on orbital and spin degrees of freedom (Zeeman term), which will allow us to work out a unified picture of the response of correlated electron systems to magnetic fields, is a highly interesting novel research direction.

Our prediction could be potentially of technological use, as it provides a new switching mechanism between an insulator and a metal. In particular, the discussion is relevant for the moiré materials with large unit cells, such that the orbital effects of the magnetic become significant at moderate magnetic fields of a few Tesla. In these systems [73], the orbital effects of the magnetic field are under a

current focus of both experimental and theoretical effort [23,29,73,74]. Let us also note that the mechanism we discussed is not specific to the Mott insulators and could be extended to other types of insulators.

Acknowledgments—We thank T. Schäfer, A. Toschi, and G. Sangiovanni for useful discussions. G. R. acknowledges financial support from the Deutsche Forschungsgemeinschaft (DFG) through Projects No. 407372336 and No. 449872909. A. A. M. acknowledges support from the Russian Quantum Center in the framework of the Russian Quantum Technologies Roadmap and from the European Research Council (LATIS project). The authors gratefully acknowledge the computing time granted by the Resource Allocation Board and provided on the supercomputers Lise and Emmy/Grete at NHR@ZIB and NHR@Göttingen as part of the NHR infrastructure. These centers are jointly supported by the Federal Ministry of Education and Research and the state governments participating in the NHR (www.nhr-verein.de). The calculations for this research were conducted with computing resources under the project hhp00048.

-
- [1] N. Ashcroft and N. Mermin, *Solid State Physics* (Harcourt College Publishers, New York, 1976).
 - [2] T. Schäfer, F. Geles, D. Rost, G. Rohringer, E. Arrigoni, K. Held, N. Blümer, M. Aichhorn, and A. Toschi, *Phys. Rev. B* **91**, 125109 (2015).
 - [3] P. W. Anderson, *Phys. Rev.* **109**, 1492 (1958).
 - [4] F. Evers and A. D. Mirlin, *Rev. Mod. Phys.* **80**, 1355 (2008).
 - [5] R. E. Peierls, *Quantum Theory of Solids* (Clarendon Press, New York, 1996).
 - [6] J. Zaanen, G. A. Sawatzky, and J. W. Allen, *Phys. Rev. Lett.* **55**, 418 (1985).
 - [7] N. F. Mott, *Rev. Mod. Phys.* **40**, 677 (1968).
 - [8] S. Manzeli, D. Ovchinnikov, D. Pasquier, O. V. Yazyev, and A. Kis, *Nat. Rev. Mater.* **2**, 17033 (2017).
 - [9] B. H. Moon, *Emergent Mater.* **4**, 989 (2021).
 - [10] K. Haule and G. L. Pascut, *Sci. Rep.* **7**, 10375 (2017).
 - [11] Y. Fei, Z. Wu, W. Zhang, and Y. Yin, *AAPPS Bull.* **32**, 20 (2022).
 - [12] K. L. Gurunatha, S. Sathasivam, J. Li, M. Portnoi, I. P. Parkin, and I. Papakonstantinou, *Adv. Funct. Mater.* **30**, 2005311 (2020).
 - [13] C. Ling, Z. Zhao, X. Hu, J. Li, X. Zhao, Z. Wang, Y. Zhao, and H. Jin, *ACS Appl. Nano Mater.* **2**, 6738 (2019).
 - [14] G. Mazza, A. Amaricci, M. Capone, and M. Fabrizio, *Phys. Rev. Lett.* **117**, 176401 (2016).
 - [15] F. Kagawa, T. Itou, K. Miyagawa, and K. Kanoda, *Phys. Rev. Lett.* **93**, 127001 (2004).
 - [16] Z. L. Sun, A. F. Wang, H. M. Mu, H. H. Wang, Z. F. Wang, T. Wu, Z. Y. Wang, X. Y. Zhou, and X. H. Chen, *npj Quantum Mater.* **6**, 94 (2021).
 - [17] A. P. Ramirez, *J. Phys. Condens. Matter* **9**, 8171 (1997).
 - [18] E. Dagotto, T. Hotta, and A. Moreo, *Phys. Rep.* **344**, 1 (2001).

- [19] M. B. Salamon and M. Jaime, *Rev. Mod. Phys.* **73**, 583 (2001).
- [20] Y. H. Matsuda, D. Nakamura, A. Ikeda, S. Takeyama, Y. Suga, H. Nakahara, and Y. Muraoka, *Nat. Commun.* **11**, 3591 (2020).
- [21] Y. H. Matsuda, Y. Muraoka, D. Nakamura, A. Ikeda, Y. Ishii, X.-G. Zhou, H. Sawabe, and S. Takeyama, *J. Phys. Soc. Jpn.* **91**, 101008 (2022).
- [22] S. Fukuoka, T. Oka, Y. Ihara, A. Kawamoto, S. Imajo, and K. Kindo, *Phys. Rev. B* **109**, 195142 (2024).
- [23] Y. Cao, V. Fatemi, A. Demir, S. Fang, S. L. Tomarken, J. Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, R. C. Ashoori, and P. Jarillo-Herrero, *Nature (London)* **556**, 80 (2018).
- [24] J. M. Pizarro, M. J. Calderón, and E. Bascones, *J. Phys. Comm.* **3**, 035024 (2019).
- [25] Y. H. Kwan, S. A. Parameswaran, and S. L. Sondhi, *Phys. Rev. B* **101**, 205116 (2020).
- [26] K. Czajka, A. Gorczyca, M. M. Maśka, and M. Mierzejewski, *Phys. Rev. B* **74**, 125116 (2006).
- [27] S. Acheche, L.-F. Arsenault, and A.-M. S. Tremblay, *Phys. Rev. B* **96**, 235135 (2017).
- [28] A. A. Markov, G. Rohringer, and A. N. Rubtsov, *Phys. Rev. B* **100**, 115102 (2019).
- [29] J. Vučičević and R. Žitko, *Phys. Rev. Lett.* **127**, 196601 (2021).
- [30] J. Vučičević and R. Žitko, *Phys. Rev. B* **104**, 205101 (2021).
- [31] J. K. Ding, W. O. Wang, B. Moritz, Y. Schattner, E. W. Huang, and T. P. Devereaux, *Commun. Phys.* **5**, 204 (2022).
- [32] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/bw7f-p8mp> for technical details of the numerical calculations and additional numerical results.
- [33] M. Wallerberger, A. Hausoel, P. Gunacker, A. Kowalski, N. Parragh, F. Goth, K. Held, and G. Sangiovanni, *Comput. Phys. Commun.* **235**, 388 (2019).
- [34] F. Wang and D. P. Landau, *Phys. Rev. Lett.* **86**, 2050 (2001).
- [35] J. Sherman and W. J. Morrison, *Ann. Math. Stat.* **21**, 124 (1950).
- [36] W. W. Hager, *SIAM Rev.* **31**, 221 (1989).
- [37] D. A. Harville, *Matrix Algebra From a Statistician's Perspective* (Springer-Verlag, New York, 1997), <https://doi.org/10.1007/b98818>.
- [38] I. M. Niven, H. S. Zuckerman, and H. L. Montgomery, *An Introduction to the Theory of Numbers*, 5th ed. (John Wiley and Sons, New York, 1991).
- [39] W. R. Inc., *Mathematica*, version 14.1 (1988–2025), Champaign, IL, 2024.
- [40] D. Vollhardt and P. Wölfe, *Electronic Phase Transitions* (Elsevier Science Publishers B.V., New York, 1992), Chap. 1, pp. 1–78.
- [41] A. Georges and W. Krauth, *Phys. Rev. Lett.* **69**, 1240 (1992).
- [42] A. Georges and G. Kotliar, *Phys. Rev. B* **45**, 6479 (1992).
- [43] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, *Rev. Mod. Phys.* **68**, 13 (1996).
- [44] L. Laloux, A. Georges, and W. Krauth, *Phys. Rev. B* **50**, 3092 (1994).
- [45] J. Bauer and A. C. Hewson, *Phys. Rev. B* **76**, 035118 (2007).
- [46] P. G. van Dongen and C. Leinung, *Ann. Phys. (Berlin)* **509**, 45 (1997).
- [47] D. Vollhardt, *Rev. Mod. Phys.* **56**, 99 (1984).
- [48] R. Kubo, *J. Phys. Soc. Jpn.* **12**, 570 (1957).
- [49] G. Sangiovanni, A. Toschi, E. Koch, K. Held, M. Capone, C. Castellani, O. Gunnarsson, S.-K. Mo, J. W. Allen, H.-D. Kim, A. Sekiyama, A. Yamasaki, S. Suga, and P. Metcalf, *Phys. Rev. B* **73**, 205121 (2006).
- [50] X. Wang, E. Gull, L. de' Medici, M. Capone, and A. J. Millis, *Phys. Rev. B* **80**, 045101 (2009).
- [51] J. Fei, C.-N. Yeh, and E. Gull, *Phys. Rev. Lett.* **126**, 056402 (2021).
- [52] K. Nogaki, J. Fei, E. Gull, and H. Shinaoka, *SciPost Phys. Codebases* **19** (2023).
- [53] K. Nogaki, J. Fei, E. Gull, and H. Shinaoka, *SciPost Phys. Codebases* **19-r1.0** (2023).
- [54] M. Jarrell and J. E. Gubernatis, *Phys. Rep.* **269**, 133 (1996).
- [55] J. Kaufmann and K. Held, *Comput. Phys. Commun.* **282**, 108519 (2023).
- [56] Larger magnetic fields ($B > 0.5$) are not meaningful in our model, as it is symmetric under the transformation $B \rightarrow 1 - B$.
- [57] Y. Hatsugai and M. Kohmoto, *Phys. Rev. B* **42**, 8282 (1990).
- [58] N. Hyttrek, Topological quantization of the Hall conductivity in superconducting cuprates—A Hubbard-Hofstadter model study, Master's thesis, University of Hamburg, 2024.
- [59] R. Bulla and M. Potthoff, *Eur. Phys. J. B* **13**, 257 (2000).
- [60] M. C. Gutzwiller, *Phys. Rev.* **137**, A1726 (1965).
- [61] W. F. Brinkman and T. M. Rice, *Phys. Rev. B* **2**, 4302 (1970).
- [62] D. Vollhardt, *Rev. Mod. Phys.* **56**, 99 (1984).
- [63] M. M. Wysockiński and M. Fabrizio, *Phys. Rev. B* **95**, 161106(R) (2017).
- [64] The improved treatment of fluctuations in Ref. [63] reduces U_c by a factor of ~ 0.822 . Moreover, according to our phase diagram the critical $U_c = 2.71$ at $T = 0$ is reduced to $U_c = 2.45$ at $T = 0.01$. Hence, we have $U_c = (2.45/2.71) \times 0.822 \times 8|E_{\text{kin}}^0| = 5.945|E_{\text{kin}}^0| \sim 6|E_{\text{kin}}^0|$.
- [65] G. G. Naumis, *Phys. Lett. A* **380**, 1772 (2016).
- [66] S. Pairault, D. Sénéchal, and A.-M. S. Tremblay, *Phys. Rev. Lett.* **80**, 5389 (1998).
- [67] S. Pairault, D. Sénéchal, and A.-M. S. Tremblay, *Eur. Phys. J. B* **16**, 85 (2000).
- [68] D. Sen and R. Chitra, *Phys. Rev. B* **51**, 1922 (1995).
- [69] Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
- [70] S. Choi, G. Ahn, S. J. Moon, and S. Lee, *Sci. Rep.* **10**, 9721 (2020).
- [71] H. Lu and J. Robertson, *Phys. Status Solidi (b)* **256**, 1900210 (2019).
- [72] J. Dalibard, in *Proceedings of the International School of Physics "Enrico Fermi" 2014* (2015), arXiv:1504.05520.
- [73] J. Kang, B. A. Bernevig, and O. Vafek, *Phys. Rev. Lett.* **127**, 266402 (2021).
- [74] C. R. Kometter, J. Yu, T. Devakul, A. P. Reddy, Y. Zhang, B. A. Foutty, K. Watanabe, T. Taniguchi, L. Fu, and B. E. Feldman, *Nat. Phys.* **19**, 1861 (2023).