

New Exceptional Effective Field Theories in de Sitter Space from Generalized Energy Conservation

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We discover a surprising relationship between exceptional effective field theories (EFTs) in de Sitter space and a notion of *generalized energy conservation* (GEC) of an S matrix defined in an extended Poincaré patch of four-dimensional de Sitter. By demanding that such an S matrix has support only when the total energies of in and out states are equal, we constrain the coupling constants in theories of self-interacting scalars living in the exceptional series of de Sitter representations. We rediscover the theories of Dirac-Born-Infeld (DBI) and special Galileon, and when increasing the conformal dimension, we find evidence for new exceptional theories where the four-point scalar self-interactions are uniquely fixed in terms of a single coupling constant. We conjecture that for each integer conformal dimension $\Delta \geq 4$, there is at least one exceptional EFT that can be entirely fixed by GEC.

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Introduction and summary—Energy conservation is usually perceived as a consequence of global time-translation symmetry by Noether’s theorem [1]. In flat space, where time translation is an exact symmetry, the probability for the incoming and outgoing states to have different total energies is exactly zero. When there is no time translation symmetry, as in de Sitter (dS) space, however, energy conservation should not be imposed *a priori* even if we have asymptotic states where well-separated wave packets can scatter.

Recently, the authors of [2] suggested that one can extend the Poincaré patch of dS space by the following (we will discuss the $\eta = 0$ elephant in the room in detail in this Letter):

$$ds^2 = \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2}, \quad \eta \in (-\infty, +\infty). \quad (1)$$

The idea is to glue together two Poincaré patches of dS in order to define an in-out correlator from which one can define an S matrix via a Lehmann-Symanzik-Zimmermann

(LSZ) reduction (see Refs. [3–6] for other definitions of scattering in dS). The $\alpha \rightarrow \beta$ element of this S matrix can be defined in the interaction picture by the Dyson series [2],

$$S_{\beta\alpha} \equiv \langle \beta | \mathcal{T} \{ e^{-i \int_{-\infty}^{+\infty} d\eta \mathcal{H}_{\text{int}}} \} | \alpha \rangle \quad (2)$$

$$\equiv 1 + i(2\pi)^4 \mathcal{A} \delta^3(\vec{k}_{\text{in}} - \vec{k}_{\text{out}}), \quad (3)$$

where $\mathcal{H}_{\text{int}}$ is the interaction Hamiltonian in the interaction picture, and β, α are out and in states in the interaction picture, respectively. The delta function enforcing momentum conservation $\delta^3(\vec{k}_{\text{in}} - \vec{k}_{\text{out}})$ arises as a consequence of the spatial translation symmetry of Eq. (1).

In [2], it was suggested that this object should be considered for theories that are *manifestly* infrared (IR) finite where there is no divergence in the interacting Lagrangian, as $\eta \rightarrow 0$. In such cases, every contribution to the S matrix contains an energy-conserving delta function, or derivatives thereof, thereby ensuring a generalized version of energy conservation:

$$\mathcal{A} = \sum_{m=0} \mathcal{A}^{(m)} \partial_{k_T}^m \delta(k_T), \quad (4)$$

where $k_T \equiv \sum_{a \in \text{in}} k_a - \sum_{b \in \text{out}} k_b$, k_a are the magnitudes of three-momenta, and the amplitudes $\mathcal{A}^{(m)}$ are functions of the external kinematics. Such energy-conserving delta functions, when scattering, e.g., conformally coupled or

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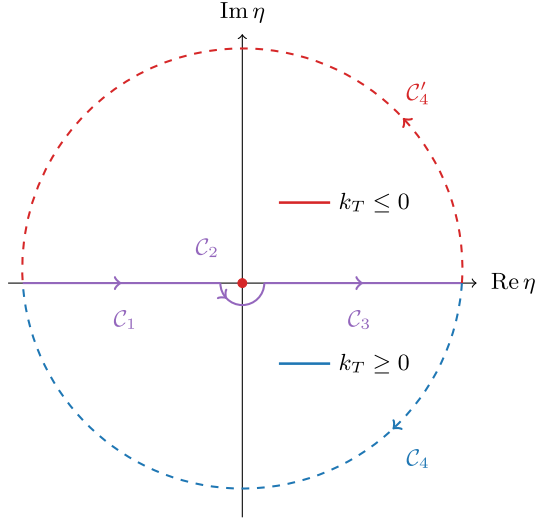


FIG. 1. The time integration contour for a dS scattering amplitude in an eternal energy-creating universe. The contour is chosen such that the integral on the large arc vanishes for $k_T \neq 0$, while the scattering amplitude is nonvanishing only when $k_T = k_{\text{in}} - k_{\text{out}} \leq 0$.

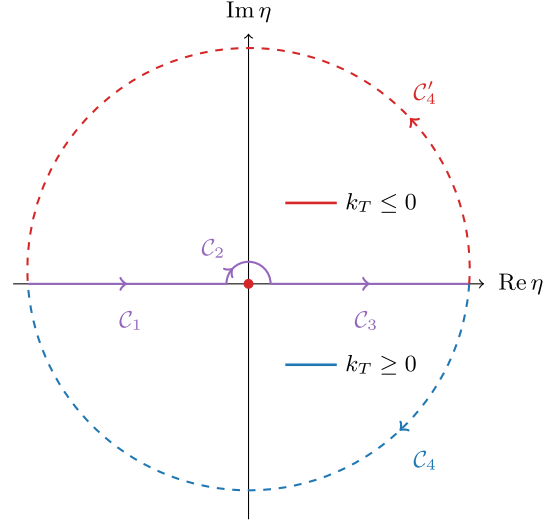


FIG. 2. The time integration contour for a dS scattering amplitude in an eternal energy-annihilating universe. The contour is chosen such that the integral on the large arc vanishes for $k_T \neq 0$, while the scattering amplitude is nonvanishing only when $k_T = k_{\text{in}} - k_{\text{out}} \geq 0$.

exceptional series scalars, arise from integrals of the form $\int_{-\infty}^{+\infty} d\eta \eta^p e^{-ik_T \eta}$ with integer $p \geq 0$ [2].

In this Letter, however, we allow ourselves to consider theories that are *not* manifestly IR finite. By imposing Bunch-Davies (BD) vacuum conditions for both the asymptotic past and future, where the free vacuum $|0\rangle$ ($\langle 0|$) is annihilated by the annihilation (creation) operator $a_{\vec{k}}$ ($a_{\vec{k}}^\dagger$), we consider scattering processes by shooting particles from the far past to the far future with a contour deformation around $\eta = 0$. In our case, an extra term $\mathcal{A}_{k_T \neq 0}$ is, in general, possible in the S matrix without any energy-conserving delta function:

$$\mathcal{A} = \mathcal{A}_{k_T \neq 0}^{(\pm)} + \sum_{m=0} \mathcal{A}^{(m)} \partial_{k_T}^m \delta(k_T). \quad (5)$$

Such an amplitude therefore has support when energy is not conserved. However, there are, perhaps, good reasons to demand $\mathcal{A}_{k_T \neq 0}^{(\pm)} = 0$ and therefore, generalized energy conservation (GEC). Indeed, a nonzero $\mathcal{A}_{k_T \neq 0}^{(\pm)}$ implies eternal energy creation or annihilation, which could lead to instabilities.

The calculation of $\mathcal{A}_{k_T \neq 0}^{(\pm)}$, and indeed the full amplitude, involves choosing between two different time integration contours ($\pm$), as shown in Figs. 1 and 2. For $k_T \leq 0$ ($k_T \geq 0$), the contour in Fig. 1 (Fig. 2) encloses the potential pole at the origin, whereas for the contour in Fig. 2 (Fig. 1), it does not. In all cases, the arc at infinity vanishes by our BD boundary conditions, and so the integral along the real line (which computes the amplitude) can be computed by residue theorem. Only the residue

at $\eta = 0$ can contribute to $\mathcal{A}_{k_T \neq 0}^{(\pm)}$, since all other non-zero contributions come from integrals of the form $\int_{-\infty}^{+\infty} d\eta \eta^p e^{-ik_T \eta}$ with integer $p \geq 0$, which yield energy-conserving contributions only, as above (we go through this in detail in Supplemental Material [7], which includes Ref. [8]). Imposing the absence of instabilities is, therefore, equivalent to imposing that the residue at $\eta = 0$ vanishes. Surprisingly, we find that such a condition can impose relations between various coupling constants appearing in general Lagrangians that are not manifestly IR finite. Imposing GEC can, therefore, be used to pick out special theories.

We concentrate on massive scalars that live in the exceptional series of four-dimensional de Sitter representations with integer conformal dimension $\Delta \geq 3$, and find that by imposing GEC, there is a unique four-point amplitude for each mass where $m^2 = \Delta(3 - \Delta)H^2$ when we allow for operators with, at most, $2\Delta - 4$ derivatives. We focus on these masses, since such scalars in dS space are naturally packaged with nonlinearly realized symmetries [9]. Despite the tachyonic masses, these scalars are indeed unitary irreducible representations of dS isometries (see, e.g., Refs. [10–12] for discussions). We recover the known theories of Dirac-Born-Infeld ($\Delta = 4, m^2 = -4H^2$) and special Galileon ($\Delta = 5, m^2 = -10H^2$) for the first two tachyonic exceptional series masses and find evidence for the existence of new theories beyond these known examples as we continue to increase the conformal dimension of the field. In the cases we consider, we find that the four-point interactions are completely fixed in terms of a single coupling constant. We therefore refer to such theories as *exceptional effective field theories (EFTs)*. We are primarily

interested in four spacetime dimensions. However, in the Supplemental Material [7], we extend our analysis to other dimensions and find that our techniques are equally powerful there too.

The space of exceptional EFTs in dS space for single-scalar theories has been considered from algebraic and braneworld constructions [9,13–17] and wave function soft limits [18] (see also Ref. [19]). No evidence of new single-scalar theories beyond the special Galileon has been found. These results suggest that the new theories we have discovered in this Letter might exist only when coupled to additional degrees of freedom. There is evidence that such theories do exist in dS, given the existence of certain extensions of the dS algebra [20]. It would be very interesting to find the symmetries or algebras and spectra that define the theories we have found evidence for in this Letter. Furthermore, the actions for such special theories in dS are rather complicated and call for a more “on-shell” approach to understanding their properties. We hope that our Letter initiates such a study.

Notation—We use $k \equiv |\vec{k}|$ for momentum magnitudes, and often write $x \equiv (\eta, \vec{x})$ with the spacetime index suppressed. We use $c_0^{(\Delta)}$ for cubic couplings and $d_{2m}^{(\Delta)}$ for quartic couplings with $2m$ derivatives. After imposing momentum conservation, we can express four-point amplitudes in terms of the following six variables:

$$k_1, \quad k_2, \quad k_3, \quad k_4, \quad \vec{k}_1 \cdot \vec{k}_2, \quad \vec{k}_1 \cdot \vec{k}_3. \quad (6)$$

There may be further redundancies forced by energy-conserving delta functions.

Exceptional EFTs from generalized energy conservation—We focus on imposing GEC on the four-point amplitudes arising from the self-interactions of exceptional series scalars to avoid potential instabilities. We could, in principle, impose a $\mathbb{Z}_2$ symmetry such that only contact diagrams contribute; however, in each case, we find that cubic vertices are forbidden by GEC, and therefore, no exchange contributions to four-point amplitudes are allowed. We show this explicitly only for the $\Delta = 3$ scalar with the generalization to other conformal dimensions simple. For four-point interactions, we include operators with, at most, $2\Delta - 4$ derivatives. A benefit of working with this amplitude over, e.g., boundary correlators is that it is field-redefinition invariant. This follows from its derivation via an LSZ reduction, as we show explicitly in the Supplemental Material [7], where we also work through two field-redefinition examples.

(1) $\Delta = 3$. We begin our analysis with the first allowed mass of exceptional series scalars, which is $m^2 = 0$, corresponding to $\Delta = 3$. The mode function is given by

$$f_k(\eta) = \frac{H e^{-ik\eta}}{\sqrt{2k^3}} (1 + ik\eta). \quad (7)$$

Let us first consider cubic interactions, where the only dS-invariant interaction that cannot be removed by a field redefinition is the potential $V(\phi) = c_0^{(3)} \phi^3$. The corresponding three-point amplitude is computed by

$$\mathcal{A}_3 = \frac{1}{2\pi} \frac{6c_0^{(3)}}{H^4} \sqrt{8k_1 k_2 k_3} \int_{-\infty}^{+\infty} \frac{d\eta}{\eta^4} f_{k_1}(\eta) f_{k_2}(\eta) f_{k_3}(\eta). \quad (8)$$

In order to remove any potential instabilities, we demand that the residue of the integrand vanishes, which leads to $\mathcal{A}_{k_T \neq 0}^{(\pm)} = 0$. Given that the Taylor expansion of $f_{k_1}(\eta) f_{k_2}(\eta) f_{k_3}(\eta)$ contains a nonzero η^3 part, $\mathcal{A}_{k_T \neq 0}^{(\pm)} = 0$ only if we set $c_0^{(3)} = 0$. GEC therefore forbids any non-trivial cubic vertex for this massless scalar.

Moving to quartic interactions, we consider the general Lagrangian up to two derivatives:

$$\frac{\mathcal{L}^{\Delta=3}}{\sqrt{-g}} = -\frac{1}{2}(\partial\phi)^2 + d_0^{(3)} H^2 \phi^4 + d_2^{(3)} \phi^2 (\partial\phi)^2 + \dots \quad (9)$$

We include a factor of H^2 in front of the potential such that the $d_{2m}^{(\Delta)}$ have the same nonzero mass dimension for each Δ . The potential contributes to the residue, just like for the three-point amplitude; however, the two-derivative term does not. This follows from the fact that the Taylor expansion of the mode function [Eq. (7)] does not contain a linear term in η (which is the origin of the *manifestly local test* of [21]). We show this explicitly in the Supplemental Material [7], where we discuss field redefinitions. To realize GEC, we therefore need to fix $d_0^{(3)} = 0$.

We are therefore left with only the two-derivative interaction, but since this term can be removed by a field redefinition, as we discuss in the Supplemental Material [7], the amplitude in this case becomes trivial. To yield a nonzero result, we can alter our starting point and allow for colored massless scalars. In this case, the potential $d_0^{(3)} \phi^4$, which can now be colored, is still forbidden by GEC, but the two-derivative interaction is now nontrivial and does indeed yield nonvanishing amplitudes. In this sense, GEC picks out the four-point two-derivative interaction. If we wrote the interactions as $g^{IJ}(\phi) \partial^\mu \phi_I \partial_\mu \phi_J$, for some internal manifold metric $g^{IJ}(\phi)$, then nontrivial amplitudes would arise when this symmetric metric had a nonvanishing Riemann tensor [22]. This can be the case for two or more scalars.

(2) $\Delta = 4$. We now turn our attention to a scalar with $m^2 = -4H^2$ and $\Delta = 4$. Here, the mode function is given by

$$f_k(\eta) = \frac{H e^{-ik\eta}}{\sqrt{2k^5 \eta}} [-3i + k\eta(3 + ik\eta)]. \quad (10)$$

In this case, we immediately use our freedom to choose a field basis where there is no $\phi^2(\partial\phi)^2$ operator (in the Supplemental Material [7], we verify that this interaction can indeed be removed by a field redefinition) and now work with the most general quartic interactions up to four derivatives:

$$\frac{\mathcal{L}^{\Delta=4}}{\sqrt{-g}} = -\frac{1}{2}(\partial\phi)^2 + 2H^2\phi^2 + d_0^{(4)}H^4\phi^4 + d_4^{(4)}(\partial\phi)^4 \dots \quad (11)$$

We now compute the residue of the corresponding four-point amplitude and find

$$\mathcal{A}_4^{(\pm)}|_{k_T \neq 0} = -(d_0^{(4)} + 8d_4^{(4)})H^4 i \frac{36}{35} \theta(\pm k_T) \frac{(3(k_1^7 + k_2^7 + k_3^7 + k_4^7) - 7[k_1^2 k_2^2 (k_1^3 + k_2^3) + 5\text{perms}])}{k_1^2 k_2^2 k_3^2 k_4^2}, \quad (12)$$

where the sign in front of k_T depends on the choice of contour shown in Figs. 1 and 2. We find that GEC requires us to fix $d_0^{(4)} = -8d_4^{(4)}$, which therefore fixes the interactions up to an overall coupling constant. It might seem surprising that operators with different numbers of derivatives can contribute to the amplitude in identical ways, but this is simply a consequence of scale invariance, which yields a fixed momentum scaling. We know that there is indeed a special theory with $\Delta = 4$ and interactions with this power counting, which is the Dirac-Born-Infeld (DBI) scalar (see, e.g., Refs. [9,14,16]) with leading-order Lagrangian:

$$\frac{\mathcal{L}^{\text{DBI}}}{\sqrt{-g}} = \frac{H^2 \Lambda^2}{(1 - \phi^2/\Lambda^2)^2} \sqrt{1 - \frac{(\partial\phi)^2/(H^2 \Lambda^2)}{1 - \phi^2/\Lambda^2}}, \quad (13)$$

where Λ is a dimensionful scale. Upon expanding this Lagrangian up to quartic order in ϕ and performing the field redefinition $\phi \rightarrow \phi - \phi^3/(2\Lambda^2)$, we recover our action [Eq. (11)] with $d_0^{(4)} = -8d_4^{(4)}$. GEC has, therefore, picked out the DBI scalar as being special among $\Delta = 4$ self-interacting scalars with, at most, one derivative per field.

The full corresponding amplitude is quite long and not particularly illuminating, but let us point out that the coefficient of the part of the amplitude with the highest number of derivatives acting $\delta(k_T)$ is precisely the flat-space amplitude, which is also the case for all Δ . This is analogous to the statement that cosmological correlators computed at the boundary $\eta \rightarrow 0$ contain flat-space amplitudes as the coefficient of the leading total-energy pole [23–26].

(3) $\Delta = 5$. Next, we consider a scalar with $m^2 = -10H^2$ or, equivalently, $\Delta = 5$, where the mode function is given by

$$f_\eta(k) = \frac{H e^{-ik\eta}}{\sqrt{2k^7\eta^2}} \{-15 - ik\eta[15 + k\eta(6i - k\eta)]\}. \quad (14)$$

In keeping with the previous pattern, in this case we allow for general interactions up to six derivatives, with

the two-derivative operator removed by a field redefinition. There is only a single six-derivative operator that cannot be removed, since the only Lorentz-invariant four-point amplitude in flat space is $\mathcal{A} \sim st(s+t)$. We therefore work with

$$\frac{\mathcal{L}^{\Delta=5}}{\sqrt{-g}} = -\frac{1}{2}(\partial\phi)^2 + 5H^2\phi^2 + d_0^{(5)}H^6\phi^4 + d_4^{(5)}H^2(\partial\phi)^4 + d_6^{(5)}(\partial\phi)^2(\nabla_\mu\nabla_\nu\phi)^2 + \dots \quad (15)$$

As before, we now impose GEC by demanding that $\mathcal{A}_{k_T \neq 0}^{(\pm)} = 0$ and find the conditions

$$d_4^{(5)} = -\frac{6}{125}d_0^{(5)}, \quad d_6^{(5)} = \frac{3}{500}d_0^{(5)}. \quad (16)$$

Note that in this case, there are more independent kinematical structures compared to $\Delta = 3, 4$, which is why we find more conditions.

Up to integration by parts and field redefinitions (which do not alter the amplitude), which are worked through in [18], these GEC conditions on the couplings uniquely pick out the special Galileon (see Refs. [9,14] for details on this theory).

(4) $\Delta = 6$. We have, so far, used GEC to recover known theories of special scalars in dS space. We now initiate the search for new theories by increasing the conformal dimension. For $m^2 = -18H^2$, or $\Delta = 6$, the mode function is given by

$$f_k(\eta) = \frac{iH e^{-ik\eta}}{\sqrt{2k^9\eta^3}} \{105 + k\eta[105i + k\eta(-45 + k\eta(-10i + k\eta))]\}. \quad (17)$$

We now consider a general Lagrangian including operators up to eight derivatives. Again, there is a single new operator that cannot be removed by a field redefinition, given that the only flat-space amplitude is $\mathcal{A} \sim (s^2 + t^2 + st)^2$, so with a choice of field basis, we can write

$$\begin{aligned} \frac{\mathcal{L}^{\Delta=6}}{\sqrt{-g}} = & -\frac{1}{2}(\partial\phi)^2 + 9H^2\phi^2 + d_0^{(6)}H^8\phi^4 + d_4^{(6)}H^4(\partial\phi)^4 \\ & + d_6^{(6)}H^2(\partial\phi)^2(\nabla_\mu\nabla_\nu\phi)^2 + d_8^{(6)}(\nabla_\mu\nabla_\nu\phi)^4 + \dots \end{aligned} \quad (18)$$

We now compute the four-point amplitude, and by imposing GEC, we find the conditions

$$d_4^{(6)} = -\frac{13}{2646}d_0^{(6)}, \quad d_6^{(6)} = -\frac{5}{1323}d_0^{(6)}, \quad d_8^{(6)} = \frac{1}{5292}d_0^{(6)}. \quad (19)$$

To the best of our knowledge, this theory has not been discussed before, but given our results, it is clearly a special subset of all possible $\Delta = 6$ theories with, at most, $2\Delta - 4$ derivatives. Following the pattern for lower conformal dimensions, we might expect that this theory has a nonlinearly realized symmetry.

(5) $\Delta = 7$. We end our analysis by considering a scalar with $m^2 = -28H^2$ or $\Delta = 7$, where the mode function is

$$f_k(\eta) = \frac{He^{-ik\eta}}{\sqrt{2k^{11}\eta^4}} \{945 + ik\eta[945 + k\eta(420i + k\eta(-105 + k\eta(-15i + k\eta)))]\}. \quad (20)$$

Continuing the previous pattern, we now allow for operators with, at most, ten derivatives. As before, there is a single new operator we can write down, given that the only possible flat-space amplitude is $\mathcal{A} \sim st(s+t)(s^2+t^2+st)$. A convenient basis for the Lagrangian is

$$\begin{aligned} \frac{\mathcal{L}^{\Delta=7}}{\sqrt{-g}} = & -\frac{1}{2}(\partial\phi)^2 + 14H^2\phi^2 + d_0^{(7)}H^{10}\phi^4 + d_4^{(7)}H^6(\partial\phi)^4 \\ & + d_6^{(7)}H^4(\partial\phi)^2(\nabla_\mu\nabla_\nu\phi)^2 + d_8^{(7)}H^2(\nabla_\mu\nabla_\nu\phi)^4 \\ & + d_{10}^{(7)}\partial^\alpha[(\nabla_\mu\nabla_\nu\phi)^2]\partial_\alpha[(\nabla_\mu\nabla_\nu\phi)^2] + \dots \end{aligned} \quad (21)$$

As with all our previous cases, the GEC condition uniquely fixes all couplings up to some overall coefficient:

$$\begin{aligned} d_4^{(7)} = & -\frac{129}{13034}d_0^{(7)}, \quad d_6^{(7)} = -\frac{141}{1981168}d_0^{(7)}, \\ d_8^{(7)} = & \frac{3}{247646}d_0^{(7)}, \quad d_{10}^{(7)} = \frac{3}{7924672}d_0^{(7)}. \end{aligned} \quad (22)$$

At this point, we conjecture that solutions to the GEC condition do not truncate as we increase the conformal dimension. We therefore expect to have infinitely many exceptional EFTs living in the exceptional series of de Sitter representations, each with unique quartic self-interactions with, at most, $2\Delta - 4$ derivatives (but quite possibly requiring additional degrees of freedom for $\Delta \geq 6$).

Discussion—In this Letter, we have considered the dS S matrix proposed in [2], and by demanding the absence of any instabilities, we have constrained the self-interactions of exceptional series scalars. In such theories, the corresponding amplitudes have support only when energy is conserved. We rediscovered the DBI and special Galileon theories, and found evidence for new exceptional theories, each with uniquely fixed quartic self-interactions with, at most, $2\Delta - 4$ derivatives. The solutions we have found are, therefore, the leading-order ones in the derivative expansion. Our results lead us to make two conjectures:

Conjecture 1—For each integer conformal dimension $\Delta \geq 4$, there is a unique four-point amplitude (allowing for operators with, at most, $2\Delta - 4$ derivatives) that has support only when energy is conserved.

Conjecture 2—There is at least one exceptional EFT for each $\Delta \geq 4$ exceptional series representation of dS space. For $\Delta \geq 6$, such EFTs contain additional degrees of freedom.

Our second conjecture follows from the results of [9], which suggests that *single-scalar* exceptional EFTs do not exist for $\Delta \geq 6$.

Interestingly, the exceptional EFTs are uniquely picked out by GEC or stability constraints in dS space, but not in anti-de Sitter (AdS) space, since energy is automatically conserved in AdS by exact time-translation symmetry. In that sense, no tuning of the interactions is required to yield healthy theories in AdS. This conclusion coincides with representation theory, where in dS the exceptional EFTs live in the exceptional series of dS representations, whereas in AdS, they simply correspond to conventional scalar representations [9]. We further point out that the uniqueness of the theories we have fixed using GEC, especially DBI and special Galileon, which involve only a single scalar degree of freedom, resonate with the uniqueness of Yang-Mills and general relativity in flat space as theories of massless spin-1 and spin-2 particles, respectively. Indeed, there we start with a particular representation of the Poincaré group (massless spin-1 or massless spin-2) and demand the absence of instability. To some order in derivatives, the condition yields unique theories (see, e.g., Refs. [27,28]). Here, we start with particular representations of the dS group, and by demanding the absence of instabilities, we arrive at unique theories, again up to some order in derivatives. Therefore, the exceptional theories in dS space appear more fundamental than their flat-space counterparts, since additional constraints such as soft limits [29–32] or nonlinear symmetries [33–35] are required to uniquely fix the latter beyond representation theory and stability alone. In any case, it is interesting to study the connections between GEC and nonlinearly realized symmetries in de Sitter space, and recently it was shown in [36] that solutions to the soft theorems that follow from the nonlinear symmetries require the imposition of GEC.

We hope that our Letter initiates a systematic search for new theories in dS space and a more on-shell study of special

theories. We envisage a number of possible future research directions: (1) Moving to six point might be an avenue for realizing that new degrees of freedom are necessary for $\Delta \geq 6$. Indeed, it could be that exchange and contact diagrams due to scalar self-interactions cannot combine into six-point amplitudes that adhere to GEC and that new degrees of freedom, and therefore new exchange processes, are required. (2) It would be interesting to systematically search for extensions of the dS algebra whose nonlinear realization contains the scalars we have discovered in this Letter. (3) The infinite towers of theories we have found could correspond to the longitudinal modes of the infinite higher spin fields in (A)dS in the decoupling limit [37–39]. It would be interesting to investigate this, and other connections to partially massless spinning fields, in more detail. In this regard, the spinor helicity formalism of [40] could be very useful. (4) Demanding a vanishing residue for the integrands we have studied in this Letter is equivalent to demanding the absence of logarithmic divergences in boundary correlators. The amplitude has the benefit of being field-redefinition invariant; nevertheless, it would be interesting to study the properties of boundary correlators for these exceptional theories in more detail [41].

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Data availability—Mathematica files computing the residues and resulting constraints from GEC are available from the authors upon request.

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