

Fractional Hall Physics from Large N Interacting Fermions

Kristan Jensen^{1,*} and Amir Raz^{2,†}

¹*Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia V8W 3P6, Canada*

²*University of Texas, Austin, Physics Department, Austin, Texas 78712, USA*

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We solve models of N species of fermions in the lowest Landau level with $U(N)$ -invariant interactions in the $N \gg 1$ limit. We find saddles of the second quantized path integral at fixed chemical potential corresponding to fractional Hall states with filling (p/q) , where the integers p and q depend on the chemical potential and interactions. On a long torus there are q such states related by translation symmetry, and $SU(N)$ -invariant excitations of fractional charge. Remarkably, these saddles and their filling persist as extrema of the second-quantized action at $N = 1$. Our construction gives a first-principles derivation of fractional Hall states from strongly interacting fermions.

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Introduction—The fractional quantum Hall effect, first observed in [1], is a relatively simple arena for strong coupling physics. Noninteracting electrons in an applied magnetic field develop a series of gapped states, completely filled Landau levels, with integer filling fraction. That picture is completely spoiled by interactions, which split the degeneracy of the Landau levels and leave behind a network of fractionally filled gapped states. There is good reason to believe that the states with filling fraction $0 < \nu < 1$ arise from interacting electrons in the lowest Landau level, for which the effective Hamiltonian is entirely composed of interactions. Beginning with Laughlin [2], a mature and successful theory of the fractional Hall effect has been developed [3–11]; see [12,13] for reviews. However, this existing theory is a bit roundabout, for example, constructing trial wave functions for the many-body states ground states or positing an effective Chern-Simons-matter field theory, rather than directly diagonalizing the interacting Hamiltonian.

In contrast, the goal of this Letter is to develop a first-principles theory of the fractional Hall effect. We work with a second quantized, many-body description of interacting fermions in the lowest Landau level, which we then solve at finite interaction strength. To do so we introduce N species of fermions, mandate that the interactions are $U(N)$ invariant, and take $N \gg 1$, so that we can expand in the small parameter $1/N$. This sort of approximation has a long

history in theoretical particle physics beginning with [14], where it has been used to uncover nonperturbative phenomena like chiral symmetry breaking, dynamically generated scales, confinement, and more. Usually a large N approximation is expected to describe qualitative features of a finite N system; in this setting, due to the quantized nature of fractional Hall states, we will see that certain large N computations are applicable even at $N = 1$.

Besides solvability at finite interaction strength, there are two more advantages of a second quantized approach worth noting. The first is that, rather than constructing trial wave functions for individual fractional Hall states, we are able to uncover a hierarchy of plateaux along with the transitions between them. The second, related to the first, is that the many-body problem is naturally phrased in the grand canonical ensemble. Working in the canonical ensemble of fixed particle number raises the question of how the filling fraction stays constant as the magnetic field is adjusted. While this can be explained with disorder, see, for example, [15]; it is much simpler to work in the grand canonical ensemble of fixed chemical potential, wherein a plateau arises when the chemical potential lies in a gap of the spectrum and no disorder is required.

In this Letter we study spin-polarized nonrelativistic fermions confined to the surface of a cylinder of radius r , subject to a perpendicular magnetic field pointing out of the cylinder. The fermions are projected to the lowest Landau level, and the filling fraction is determined by varying a chemical potential μ . As we will see, our results are closely related to those of [16], and especially the thin torus limit of [17], though both these references primarily worked in the canonical ensemble with $r \rightarrow 0$.

The main takeaway of this Letter is that we can study the emergence of fractional Hall physics and hierarchies in a controlled way starting from a UV theory of interacting fermions. While the focus of this Letter is identifying the

*Contact author: kristanj@uvic.ca

†Contact author: araz@utexas.edu

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gapped states and their transitions, akin to a proof of principle, this method can be adapted to study correlation and electromagnetic response functions in these states, which require an additional order in $1/N$. We believe this viewpoint complements existing methods, and contributes a more comprehensive understanding of the fractional Hall effect.

The rest of this Letter is organized as follows. We begin by writing down the models in question and solving them at leading order in large N . We find saddles of the second quantized description corresponding to Abelian fractional Hall states at filling fractions (p/q) , which dominate at zero temperature with transitions between them. These solutions spontaneously break the magnetic translations along the cylinder by a discrete subgroup, similar to the TT solutions of [16,17], and the Laughlin states on the cylinder [18]. We then find the basic fractionally charged quasiparticle excitations, and discuss what aspects of our solution remain at $N = 1$. We also discuss a way to get “ $N = 1$ ” physics, meaning states with filling fraction (p/q) and anyons of charge $(1/q)$ (for p and q coprime), by projecting the large N theory to its $SU(N)$ singlet sector. We relegate a discussion of our numerical methods and of the decompactification limit to Supplemental Material [19].

Interacting fermions in the Lowest Landau level—We work in a second-quantized description and consider N species of fermionic fields ψ^α living on cylinder of radius r , subject to an external magnetic field B . Throughout this Letter we will work in the Landau gauge

$$\mathbf{A} = Bx\hat{y}, \quad (1)$$

where the y direction is periodic: $y \equiv y + 2\pi r$ and $x \in \mathbb{R}$. This gauge choice preserves continuous translations in y as well as a discrete magnetic translation $x \rightarrow x + (1/\text{Br})$. As our gauge choice is already periodic in y , we can simply impose periodic boundary conditions on the fermions: $\psi^\alpha(x, y + 2\pi r) = \psi^\alpha(x, y)$. (We can also study “twisted” boundary conditions, but these do not substantially affect our results.)

To project these fermions to the lowest Landau level we impose the constraint (see, e.g., [11])

$$\bar{D}\psi^\alpha = \frac{1}{2}(\partial_x + Bx - i\partial_y)\psi^\alpha = 0, \quad (2)$$

through N Lagrange multiplier fields χ_α similar to [20]. The field theory describing interacting fermions in the lowest Landau level then has the action

$$S = \int dt \, d^2x \left(i\bar{\psi}_\alpha D_t \psi^\alpha + \bar{\chi}_\alpha \bar{D}\psi^\alpha + D\bar{\psi}_\alpha \chi^\alpha \right) + S_{\text{int}}, \quad (3)$$

with $D_t\psi = (\partial_t - i\mu)\psi$. We take S_{int} to be a translationally-invariant two-body interaction labeled by a potential V ,

$$S_{\text{int}} = -\frac{1}{N} \int dt d^2x d^2x' \rho(t, \vec{x}) V(|\vec{x} - \vec{x}'|) \rho(t, \vec{x}'), \quad (4)$$

with $\rho(t, \vec{x}) = \bar{\psi}_\alpha(t, \vec{x})\psi^\alpha(t, \vec{x})$.

On the cylinder the linearly-independent solutions to the lowest Landau level constraint f_n are labeled by an integer n with

$$f_n(x, y) = \left(\frac{B}{4\pi^3 r^2} \right)^{\frac{1}{4}} e^{\frac{iny}{r} - \frac{B}{2}(x + \frac{n}{\text{Br}})^2}. \quad (5)$$

Thus upon integrating out the χ_α we have

$$\psi^\alpha(t, x, y) = \sum_{n \in \mathbb{Z}} c_n^\alpha(t) f_n(x, y), \quad (6)$$

and the path integral over $(\chi_\alpha, \psi^\beta)$ reduces to one over the c_n^α on which magnetic translations act as $n \rightarrow n + 1$. The action for the c_n^α is that of a 1D fermionic chain

$$S = \int dt \left(i\bar{c}_\alpha^n D_t c_n^\alpha - \frac{1}{N} V_{nk}^{ml} \bar{c}_\alpha^n c_m^\alpha \bar{c}_\beta^k c_l^\beta \right), \quad (7)$$

where the interaction matrix V_{nk}^{ml} is given by

$$V_{nk}^{ml} = \int d^2x d^2x' \bar{f}_n(x) f_m(x) V(|x - x'|) \bar{f}_k(x') f_l(x'). \quad (8)$$

We have in mind attractive interactions $V_{nk}^{ml} < 0$ with V_{nk}^{ml} vanishing when $m = l$ and $n = k$ as guaranteed by Fermi statistics when $N = 1$.

To solve this model we take $N \gg 1$ with V finite. We also work in the imaginary time formalism and work in $B = 1$ units. We then employ large N vector model techniques, which have been used to study $O(N)$ vector models [21,22], Chern-Simons-matter theories [23,24], the SYK model [25], and dipole-symmetric models [26]. This is done by performing a “generalized Hubbard-Stratonovich transformation,” rewriting the problem in terms of bilocal collective fields $G_{mn}(t, t') \equiv \bar{c}_\alpha^m(t) c_n^\alpha(t')$, and the self energy $\Sigma_{mn}(t, t')$ which enforces this constraint. These collective fields are weakly coupled at large N , so solving the large N model amounts to solving their classical equations of motion. We ansatz that these fields are invariant under translations in imaginary time and y , implying that

$$\begin{aligned} G_{n,m}(\omega_k, \omega_{k'}) &= \beta N G_n(\omega_{k'}) \delta_{n,m} \delta_{\omega_k + \omega_{k'}}, \\ \Sigma_{n,m}(\omega_k, \omega_{k'}) &= \beta \Sigma_n(\omega_{k'}) \delta_{n,m} \delta_{\omega_k + \omega_{k'}}, \end{aligned} \quad (9)$$

where $\omega_k = [\pi/\beta](2k + 1)$ for $k \in \mathbb{Z}$ is the k th Matsubara mode. The large N Dyson equations then are

$$G_n(\omega_k) = \frac{1}{i\omega_k + \Sigma_n(\omega_k)},$$

$$\Sigma_n(\omega_k) = -\mu + \frac{2}{\beta} \sum_{k' \in \mathbb{Z}} \sum_{m \in \mathbb{Z}} V_{n,m} G_m(\omega_{k'}), \quad (10)$$

where $V_{m,n} = V_{nm}^{nm}$. Note that $V_{m,n}$ is only a function of $|n - m|$, due to the symmetry under magnetic translations in the x direction. These coefficients $V_{m,n}$ are precisely the part of V_{pq}^{mn} that survive in the thin torus limit of [16,17]. Here they govern the problem at finite radius but large N . Now, the second equation implies that Σ_n is frequency-independent. The two Dyson equations can then be combined into a single equation for the self energy

$$\Sigma_n = -\mu + \sum_{m \in \mathbb{Z}} V_{n,m} \tanh\left(\frac{\beta \Sigma_m}{2}\right). \quad (11)$$

The large N free energy evaluated on the solution to the Dyson equations is

$$\frac{F}{NL} = -\frac{1}{L} \sum_{n \in \mathbb{Z}} \left[\frac{1}{\beta} \ln \left(\cosh \left(\frac{\beta \Sigma_n}{2} \right) \right) - \frac{1}{4} (\Sigma_n + \mu) \tanh \left(\frac{\beta \Sigma_n}{2} \right) \right], \quad (12)$$

where L is the length of the cylinder in units of magnetic translations, or equivalently, L is the IR cutoff for the sum over n . The maximum possible number of filled states is NL . From the free energy we can calculate the filling fraction, ν , corresponding to the solution to be

$$\nu = \frac{1}{2} - \frac{1}{2L} \sum_{n \in \mathbb{Z}} \tanh \left(\frac{\beta \Sigma_n}{2} \right). \quad (13)$$

Large N solutions: FQH hierarchies and transitions—As this setup has a $\mathbb{Z}$ symmetry of magnetic translations along the cylinder, it is natural to look for solutions to the Dyson equations that preserve this symmetry. Thus we can take a translationally invariant ansatz $\Sigma_n = \Sigma$, a constant value, which when nonzero signals a gap in the single-particle spectrum. Using this ansatz we see that there is a unique translationally invariant solution to the Dyson equations (11) given by

$$\Sigma = -\mu + V \tanh \left(\frac{\beta \Sigma}{2} \right), \quad (14)$$

where $V = \sum_{n \in \mathbb{Z}} V_{m,n} < 0$ is an m -independent constant, as a result of the interaction matrix being a function of $|n - m|$. At small but nonzero temperature this solution is, for $|\mu| > |V|$, $\Sigma \approx -[\mu + \text{sgn}(\mu)V]$, while for $|\mu| < |V|$ it is $\Sigma \approx (2/\beta) \text{arctanh}(\mu/V)$. The free energy and filling fraction for this solution are simply

$$\frac{F}{NL} = -\frac{1}{\beta} \ln \left(\cosh \left(\frac{\beta \Sigma}{2} \right) \right) + \frac{1}{4} (\Sigma + \mu) \tanh \left(\frac{\beta \Sigma}{2} \right),$$

$$\nu = \frac{1}{2} - \frac{1}{2} \tanh \left(\frac{\beta \Sigma}{2} \right). \quad (15)$$

As zero temperature this solution has $\nu = 0$ for $\mu < -|V|$, $\nu = 1$ for $\mu > |V|$, and ν linearly increasing in the region $\mu \in (-|V|, |V|)$. This describes two gapped phases for $|\mu| > |V|$, completely empty and completely filled, continuously connected to each other by a gapless phase, whose existence is due to the interaction.

We also look for solutions to the Dyson equations which only preserve a $q\mathbb{Z}$ subgroup of the translations, i.e., for which magnetic translations are broken to the subgroup generated by translation by q units. Such a solution obeys $\Sigma_{n+q} = \Sigma_n$, and is characterized by q undetermined numbers $\{\Sigma_0, \Sigma_1, \dots, \Sigma_{q-1}\}$ which comprise a unit cell. On such an ansatz the Dyson equation (11) becomes

$$\Sigma_n = -\mu + \sum_{m=0}^{q-1} V_{n,m}^{(q)} \tanh \left(\frac{\beta \Sigma_m}{2} \right), \quad (16)$$

where $V_{n,m}^{(q)} = \sum_{k \in \mathbb{Z}} V_{n,m+kq}$. The free energy and filling fraction for these solutions are

$$\frac{F}{NL} = -\frac{1}{p} \sum_{n=0}^{q-1} \left[\frac{1}{\beta} \ln \left(\cosh \left(\frac{\beta \Sigma_n}{2} \right) \right) - \frac{1}{4} (\Sigma_n + \mu) \tanh \left(\frac{\beta \Sigma_n}{2} \right) \right],$$

$$\nu = \frac{1}{2} - \frac{1}{2q} \sum_{n=0}^{q-1} \tanh \left(\frac{\beta \Sigma_n}{2} \right). \quad (17)$$

At values of μ where more than one solution exists, we compare the free energy of the various solutions to find which one is dominant.

This set of equations, (16), has a trivial solution where all the Σ_n are equal, the solution invariant under magnetic translations that we discussed above. However, and crucially, we also find nontrivial solutions which at zero temperature correspond to gapped, fractionally filled states with $\nu = (p/q)$, where exactly p of the different Σ_n 's in a unit cell of length q are negative and the remaining are positive. A schematic picture of these solutions is presented in Fig. 1. We have found these solutions numerically for a number of different interactions as we describe shortly. Here let us focus on the general picture.

These nontrivial solutions only exist for a finite range of chemical potential in the region $\mu \in (-|V|, |V|)$, where they compete with the translationally invariant gapless phase identified above. However, we find that these gapped fractionally filled states have lower free energy than the gapless one. The ensuing phase diagram at zero temperature then interpolates between the translation-invariant fully empty state for $\mu < -|V|$ and the fully filled state for

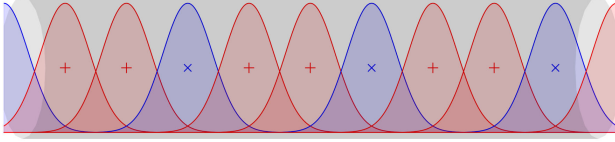


FIG. 1. A schematic of a solution to the Dyson equations with $q = 3$ and $p = 1$ corresponding to a state of filling fraction $(\nu/N) = 1/3$. The sites and their corresponding single particle wave-functions on the cylinder are presented, with the blue sites being “filled” and having $\Sigma_n < 0$ and red sites being “empty” and having $\Sigma_n > 0$.

$\mu > |V|$, by way of a series of first order transitions between fractionally filled states.

Fractional filling is not by itself sufficient to demonstrate that these are fractional Hall states. To demonstrate the latter, we first note that these nontrivial solutions in general have degeneracy q as a consequence of the spontaneously broken magnetic translation symmetry. Given a nontrivial solution with periodicity q , we obtain $q - 1$ more by mapping $\Sigma_n \rightarrow \Sigma_{n+m}$ for $m = 1, \dots, q - 1$. Additionally, there are anyonic excitations above these states, domain walls between these different solutions, which themselves can condense and so generate a hierarchy of FQH states as in [3,4].

These anyons, their charges, and their condensation can be understood in a very similar way to that of the thin torus limit as explained in [17]. Consider a unit cell $\{\Sigma_0, \dots, \Sigma_{q-1}\}$ of a nontrivial solution. Define a domain wall to be a set of m self-energies $\mathcal{D} = \{\tilde{\Sigma}_0, \dots, \tilde{\Sigma}_{m-1}\}$. Inserting this domain wall into a solution generates a new configuration, not a solution to the Dyson equations (11), that can be interpreted as a quasiparticle excitation. On either side of the domain wall one has one of the q degenerate configurations of the Σ_n 's that on their own correspond to a genuine solution to the Dyson equations. The charge of these domain walls can be calculated using the counting trick of [27]. Insert q widely separated domain walls into the chain, with an integer number of unit cells between each, and remove m unit cells from the end so as to maintain $L \gg 1$ sites. The ensuing configuration can be interpreted as a single solution, punctured by an interpolating region where the domain walls are inserted. Accounting for the fact that a single negative Σ_n contributes N to the charge and a positive one 0, the total change in the charge relative to the original configuration encodes a fractional domain wall charge $Q_{\mathcal{D}} = N(n_{\mathcal{D}} - m\nu)$ where $n_{\mathcal{D}}$ is the number of negative $\tilde{\Sigma}$'s in $\mathcal{D}$. When p and q are coprime the elementary fractional charge is (N/q) . We can also condense these domain walls to get new states by enlarging the unit cell, defining a new one formed from some number of original unit cells and one domain wall, together with a suitable adjustment of the Σ 's in the cell (without changing their sign) so that it is a solution to the Dyson equations for some chemical potential. This is similar to the procedure

used in [17] to generate the Haldane-Halperin hierarchy of Abelian FQH states.

We note that generic TQFT arguments require additional anyons in the low energy TQFT when p and q are not co-prime [28]. In fact, the only anyons accessible from the solutions of the Dyson equation are the ones generated by magnetic translations, so we cannot identify the precise TQFT without additional data from other observables, such as the total quantum dimension. In the states where p and q are co-prime and q is odd magnetic translations do generate a full TQFT, so these states seem like genuine Abelian states, although we cannot be certain without computing the total quantum dimension. When p and q are co-prime and q is even, the magnetic translations cannot generate a full TQFT due to the spin-charge constraint [28]. While in single layer Hall systems with this filling the magnetic translations cannot have periodicity q without violating spin charge, in multilayer Hall systems it is allowed, though additional anyons are required in the low energy TQFT. The large N problem can be thought of as a multilayer system with N layers, and so these states where p and q are co-prime and q is even are really a large N artifact, and must have additional anyons in the low energy TQFT.

For an example consider a Coulomb-like interaction between the sites, and take $V_{m,n} \propto -|m - n|^{-1}$ together with $V_{m,m} = 0$ and a cutoff on the maximum possible $|m - n|$. We solve the Dyson equations numerically to find the filling fraction as a function of chemical potential. A plot of these solutions at zero temperature as well as the free energy is presented in Fig. 2. In this figure we can see the series of transitions between the translationally invariant states for $|\mu| > |V|$ through a sequence of fractionally filled states, where states with a larger denominator in the filling have larger plateaux. For this long range interaction the plateaux transitions seem to form a “devil’s staircase” with a self-similar structure as the cutoff is increased, however this behavior is not observed for short range interactions where solutions with a periodicity that is much larger than the interaction length are subdominant. The solutions we find all have $\gcd(p, q) = 1$. In general, we do get both plateaux where p and q can be even or odd, although this may be a large N artifact as we discuss later. We elaborate more on how we obtain these numerical solutions in the Appendix, and showcase the phase diagram for a short-range interaction as well.

Extrema at $N = 1$ —While we arrived at the Dyson equations from standard large N arguments, some of our analysis holds at finite N , and even at $N = 1$. We can exactly rewrite the original problem in terms of bilocal degrees of freedom G and Σ without introducing any Jacobian, so the large N action and its equations of motion for $G_{n,m}$ and $\Sigma_{n,m}$ hold even at finite N .

On our ansatz (9) the parameter N completely drops out of those equations of motion, and therefore the solutions we identified above at large N are in fact extrema of the many-

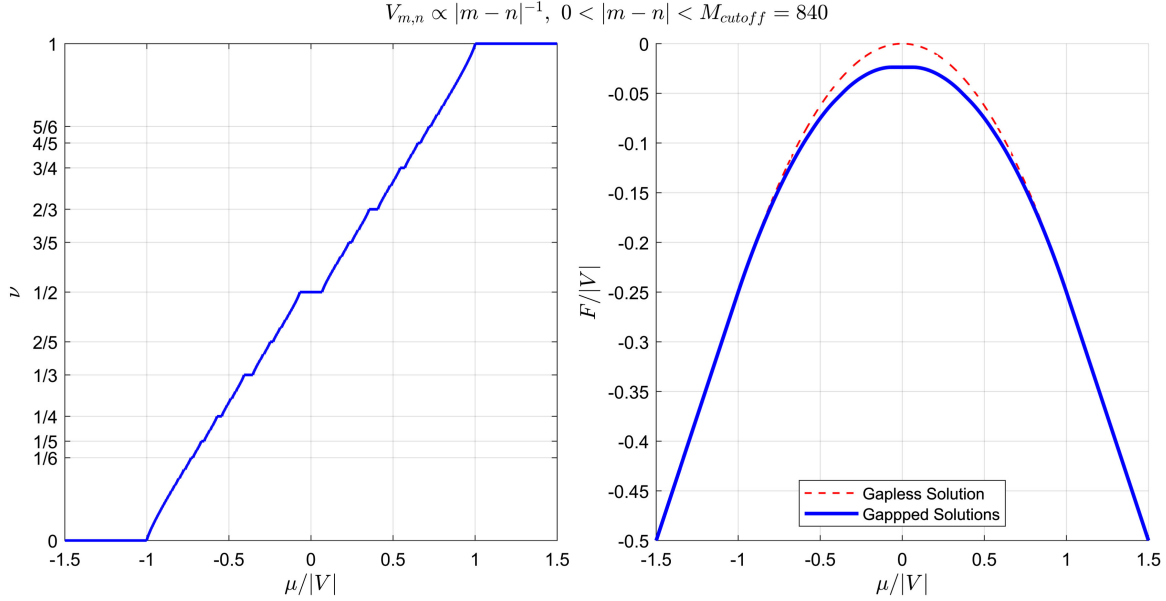


FIG. 2. A plot of the filling fraction and free energy as a function of chemical potential at zero temperature for N species of electrons in the lowest Landau level with an interaction matrix $V_{m,n} \propto -|n - m|^{-1}$. We highlight the fillings of the largest plateaus, though many more can be seen when zooming in on the curve.

body action at any N including $N = 1$. However, at $N = 1$ the second-quantized path integral no longer has a saddle-point approximation. Fluctuations of the bilocal degrees of freedom around the extrema have $O(1)$ interactions at finite N . Even so, to the extent that it makes sense to organize the interactions into a perturbative expansion, the filling fractions corresponding to these extrema are exact to all orders in perturbation theory by the following argument. Loop effects will generate frequency-dependent radiative corrections to the self-energies Σ_n , but these effects are perturbatively small and therefore do not change the sign of the Σ_n . However, at zero temperature, the filling fraction is solely due to the signs of the Σ_n .

While these solutions persist at finite N and their filling fraction is exact in perturbation theory, their free energy receives $O(1)$ corrections at finite N . This means the precise phase structure and transitions between the plateaus at finite N will differ from our large N results, as different states become energetically favored as a function of N and μ . In general, we expect the gapped even denominator states to become less energetically favorable, as the observed states at even denominator filling are gapless. This is similar to the TT picture of [17], where the gapped even denominator states become energetically suppressed away from the thin torus limit.

Discussion—In this Letter we have provided a first-principles derivation of fractional Hall states and transitions from a microscopic description of strongly interacting fermions in the lowest Landau level using large N techniques. One big advantage of our approach is the ability to study many fractional quantum Hall states simultaneously, from a single Hamiltonian, and to observe Hall plateaus

without any need for disorder. This framework can also be used to study other aspects of the fractional quantum Hall system, including finite temperature effects, the nature of plateau transitions, and the conductivity of each state. Another advantage of a large N approach is the ability to systematically compute the corrections to the leading large N results.

Unfortunately our approach has one big drawback: the existence of even denominator plateaus. These even denominator gapped states may be large N artifacts, and we expect them to be subdominant to a gapless phase at $N = 1$. More generally, our solutions exist as extrema of the many-body problem even at finite N , though they receive energetic corrections of $O(1/N)$, thereby invalidating our phase diagram. Nevertheless their filling or degeneracy persist even at $N = 1$.

Our methods can, in principle, be adapted to study the problem of $N \gg 1$ species of interacting fermions in the LLL on other symmetric spatial geometries, namely, the sphere, the plane, and the torus. We are doing so presently and hope to report our findings soon.

To conclude we note that there is a controlled way to use the large N theory to obtain Abelian FQH states that resemble those expected for the physical case of a single species of interacting fermion. We simply project the large N theory onto its $SU(N)$ singlet sector. In the second quantized description this can be accomplished by coupling the matter to a $SU(N)_k$ Chern-Simons theory in the $k \rightarrow \infty$ limit. Crucially our solution of the unprojected theory is also the solution of the projected one, which only has a $U(1)$ global symmetry. After this projection the elementary fermions c_n^α are unphysical, and the basic operators that

carry U(1) charge are “baryons,” like $\epsilon_{\alpha_1 \dots \alpha_N} c_n^{\alpha_1} \dots c_n^{\alpha_N}$, whose U(1) charge is quantized in multiples of N . Rescaling the U(1) charge so that baryons have charge 1, i.e., charge is quantized in multiples of 1, the states we find have all of the expected data for FQH states built from the LLL. Namely, a filling fraction (p/q) with degeneracy q on the torus, and fractionally charged quasiparticles of charge $[\gcd(p, q)/q]$.

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