

Next-to-Next-to-Leading-Order QCD Corrections to Hadron Production in Deep-Inelastic Scattering at Finite Transverse Momentum

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We present the *first* calculation of hadron production in deep-inelastic scattering (DIS) at finite transverse momentum to next-to-next-to-leading order (NNLO) in perturbative QCD. To overcome the long-standing challenge of infrared divergences in semi-inclusive processes with identified final state hadrons at finite transverse momentum, we implement the recently developed q_T -subtraction framework based on the recoil-free jet definition. By utilizing the winner-take-all recombination scheme, we achieve a consistent factorization for hadron-jet associated production, enabling the inclusion of $\mathcal{O}(\alpha_s^3)$ corrections. Our NNLO results generally demonstrate an improved convergence of the perturbative expansion and a reduction in scale uncertainties compared to previous next-to-leading order ones, especially for comparisons to multiplicity data from the ZEUS Collaboration. This Letter provides a high precision theoretical foundation for the upcoming electron-ion collider era and establishes a new benchmark for the exploration of the nucleon's three-dimensional structure.

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Introduction—The semi-inclusive deep inelastic scattering (SIDIS) process, $l(l) + N(P) \rightarrow l'(l') + h(P_h) + X$, is fundamental to mapping the three-dimensional nucleon structure. While inclusive DIS has historically provided a precise mapping of collinear parton distribution functions (PDFs), the detection of a final state hadron with finite transverse momentum P_{hT} enables the exploration of the transverse momentum dependent (TMD) dynamics of partons and the hadronization process [1,2]. In particular, the high- P_{hT} region is of great theoretical interest. It

provides a unique window into the transition between the nonperturbative TMD framework and the perturbative collinear factorization regime, where the P_{hT} is generated by hard radiation.

Over the past decades, experimental programs have provided a detailed mapping of P_{hT} distributions across diverse kinematic ranges. Early measurements by the H1 [3–5] and ZEUS [6,7] Collaborations at HERA established the initial benchmarks for inclusive hadron production at high P_{hT} . These were followed by higher-precision studies from HERMES [8], COMPASS [9,10], and Jefferson Lab (JLab) [11], which revealed intricate flavor dependencies and spin-momentum correlations within the nucleon.

Despite these experimental advances, theoretical predictions for the transverse momentum spectrum have largely been restricted to next-to-leading order (NLO) accuracy [12–14], which exhibit significant scale uncertainties and struggle to provide a simultaneous global description of experimental data [15]. This status contrasts sharply with rapid progress for inclusive quantities in DIS: N³LO corrections have been computed for single jet production in DIS [16], as well as next-to-next-to-leading order (NNLO) corrections for dijet production in DIS [17].

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Furthermore, NNLO coefficient functions were recently derived for inclusive SIDIS cross sections [18–30].

The fully differential description of identified hadrons at finite P_{hT} remains a critical frontier, and achieving NNLO precision is essential to stabilize the perturbative expansion and provide robust theory inputs for the upcoming Electron-Ion Collider (EIC) [31,32]. However, extending calculations to this order for final states involving identified particles faces significant hurdles due to the complex cancellation of infrared (IR) divergences [33,34]. For the description of inclusive quantities, various strategies have emerged to resolve these singularities, broadly categorized into local subtraction [35–37], projection-to-Born methods [38], and slicing techniques [39–44]. Yet, applying these techniques to semi-inclusive processes has been limited by the difficulty of consistently treating soft and collinear radiation in the presence of identified hadrons and jets, with a few successful applications [30,45–48].

In this Letter, we present the *first* NNLO QCD calculation of identified hadron production in DIS at finite P_{hT} . The core of our theoretical approach is the implementation of a new q_T -subtraction framework recently proposed in Refs. [45,49] (see also Refs. [50–52]), which generalizes the classical transverse momentum subtraction method [39] to final states involving both jets and hadrons. By utilizing a recoil-free jet axis, specifically the winner-take-all (WTA) recombination scheme [53,54], this method allows for the consistent treatment of radiation from the identified hadron and other jets. By incorporating $\mathcal{O}(\alpha_s^3)$ corrections through this framework, we demonstrate good convergence of theoretical predictions and reduced theoretical uncertainties. This result marks a new era of precision for SIDIS phenomenology and provides a rigorous test of the QCD factorization theorems in the high- P_{hT} domain.

q_T -subtraction formalism—In this Letter, we employ the azimuthal decorrelation $\delta\phi$, or, equivalently, the hadron momentum component p_{out} perpendicular to the scattering plane, as the slicing variable. The scattering kinematics in the Breit frame are illustrated in Fig. 1 with the scattering plane defined by the leading recoiling jet and beam directions. This partitions the cross section into contributions from an unresolved region, dominated by soft and collinear radiation, and a resolved region characterized by large-angle emissions. Explicitly, we have

$$\frac{d\sigma}{d\mathcal{O}} = \int_0^{\delta\phi^{\text{cut}}} d\delta\phi \frac{d\sigma}{d\delta\phi d\mathcal{O}} + \int_{\delta\phi^{\text{cut}}}^{\delta\phi^{\text{max}}} d\delta\phi \frac{d\sigma}{d\delta\phi d\mathcal{O}}, \quad (1)$$

where $\delta\phi^{\text{cut}}$ is a small resolution or slicing parameter introduced in the q_T -subtraction method. The first (unresolved) part is approximated by the leading-power (LP) contribution in the unresolved limit ($\delta\phi^{\text{cut}} \rightarrow 0$), subject to power corrections which vanish in this limit.

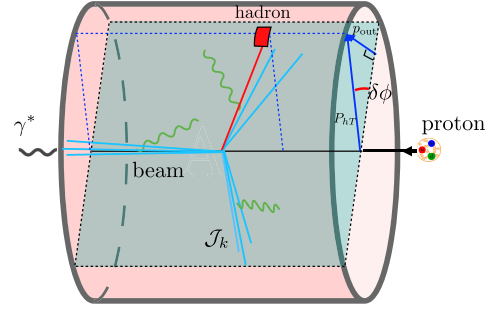


FIG. 1. Schematic definition of the kinematic variables used for the q_T -subtraction method in the Breit frame. The incoming virtual photon γ^* scatters off the proton, producing an identified hadron and a leading recoiling jet $\mathcal{J}_k$. The blue lines along the proton direction denote beam collinear radiation. The green curved lines indicate the soft radiation. The slicing variable is defined by the azimuthal decorrelation $\delta\phi$ (or, equivalently, the out-of-plane momentum p_{out} of the hadron with respect to the beam-jet plane). The limit $\delta\phi \rightarrow 0$ corresponds to the unresolved back-to-back configuration governed by the factorization formula in Eq. (2).

The cancellation of the dependence on $\delta\phi^{\text{cut}}$ in the full results serves as a rigorous check of IR safety and numerical stability of our framework.

In the resolved region ($\delta\phi > \delta\phi^{\text{cut}}$) where the hadron and jets are well separated, the differential cross sections can be calculated by requiring production of an inclusive hadron in association with a dijet in DIS up to NLO. We use the one-loop matrix elements for $ei \rightarrow ejkl$ and the tree-level matrix elements for $ei \rightarrow ejklm$ calculated in Refs. [55–58], where $ijklm$ represent all possible flavor combinations of QCD partons. The jets are reconstructed using the k_T algorithm [59,60] with WTA recombination scheme. Crucially, the jet algorithm dependence cancels between the resolved and unresolved regions, ensuring the final result is algorithm independent. The calculations are carried out in the FMNLO framework [61] for identified hadron production, with IR singularities handled using a method of dipole subtraction [62] in combination with phase space slicing.

In the unresolved region ($\delta\phi < \delta\phi^{\text{cut}}$), the LP cross section is governed by a factorization formula in impact parameter (b) space, which captures the azimuthal decorrelation between the identified hadron and the recoiling jet in the back-to-back limit. A central feature of our framework is the WTA recombination scheme. Because the WTA axis is recoil-free, it remains insensitive to soft radiation, providing a stable reference for the hard scattering direction. This property eliminates nonglobal logarithms [63,64], substantially simplifying the structure of the factorization formula [65,66]. Building upon the soft-collinear effective theory (SCET) framework [67–71] and following recent developments in Refs. [45,66,72–74], we obtain the factorized cross section

$$\begin{aligned}
 \frac{d\sigma_{\text{LP}}}{dx dy dz d^2\vec{P}_{hT} dp_{\text{out}}} &= \int \frac{db}{2\pi} e^{ip_{\text{out}}b/\zeta} \\
 &\times \sum_{ijk} \int d\xi H_{ei \rightarrow ejk}(Q, \xi, \zeta) \\
 &\times \mathcal{B}_{i/p}(\xi, b) \mathcal{D}_{h/j}(\zeta, b) \mathcal{J}_k(b) S_{ijk}(b),
 \end{aligned} \quad (2)$$

where p_{out} is the hadron's transverse momentum relative to the scattering plane. The standard kinematic variables are defined as

$$\begin{aligned}
 Q^2 &= -q^2, \quad x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \\
 z &= \frac{P \cdot P_h}{P \cdot q},
 \end{aligned} \quad (3)$$

with virtual photon momentum $q = l - l'$. The factorized ingredients H , $\mathcal{B}$, $\mathcal{D}$, S , and $\mathcal{J}$ represent the hard, TMD beam, fragmentation, soft, and WTA jet functions, respectively, where dependence on the renormalization and rapidity scales (μ, ν) is implicit. ξ and $\zeta = z + xP_{hT}^2/[z(\xi - x)Q^2]$ are the momentum fractions for the beam and fragmentation functions. Note that the dependence on the rapidity scale ν cancels in the full results of the fixed-order (FO) expansion, and we have set the factorization scale equal to the renormalization scale. Finally, the sum $\sum_{ijk}$ runs over all contributing partonic channels, specifically photon-gluon fusion ($\gamma^* g \rightarrow q\bar{q}$) and QCD Compton scattering ($\gamma^* q \rightarrow qg$).

Achieving NNLO accuracy requires two-loop corrections for each factorized ingredient. The hard function $H_{ei \rightarrow ejk}$ encodes the tree-level and virtual corrections to the partonic process $ei \rightarrow ejk$. We extract them from [75,76] and find full agreement with those used in Refs. [77–79]. Crucially, linearly polarized contributions from both incoming and outgoing gluons are essential for a complete description of the cross section beyond LO [66,80,81].

We employ standard TMD beam ($\mathcal{B}_{i/p}$) and fragmentation ($\mathcal{D}_{h/j}$) functions [82,83], applying their well-established NNLO matching onto collinear distribution functions $f_{a/p}$ and $d_{h/b}$ [84–91]

$$\begin{aligned}
 \mathcal{B}_{i/p}(\xi, b) &= \sum_a \int_{\xi}^1 \frac{d\hat{\xi}}{\hat{\xi}} \mathcal{I}_{i/a}(\hat{\xi}, b) f_{a/p}\left(\frac{\xi}{\hat{\xi}}, \mu\right), \\
 \mathcal{D}_{h/j}(\zeta, b) &= \frac{1}{\zeta^2} \sum_b \int_{\zeta}^1 \frac{d\hat{\zeta}}{\hat{\zeta}} \mathcal{C}_{b/j}(\hat{\zeta}, b) d_{h/b}\left(\frac{\zeta}{\hat{\zeta}}, \mu\right),
 \end{aligned} \quad (4)$$

with $\mathcal{I}_{i/a} = \delta_{ia}\delta(1 - \hat{\xi}) + \mathcal{O}(\alpha_s)$ and $\mathcal{C}_{b/j} = \delta_{bj}\delta(1 - \hat{\zeta}) + \mathcal{O}(\alpha_s)$. The soft function S_{ijk} follows the universal NNLO TMD soft function definition [92,93], retaining a simple

analytic form because the hard scattering process is in a planar configuration [45,66,72,81,94].

Furthermore, we adopt the k_T algorithm and the WTA recombination scheme for the TMD jet functions $\mathcal{J}_k$, to be consistent with the numerical evaluation in the resolved region. The NNLO quark jet functions are readily available from recent calculations [74,95–98]. While the NNLO WTA gluon jet functions, comprising unpolarized and linearly polarized components, are currently unknown, renormalization invariance uniquely determines the two-loop logarithmic terms. We approximate the remaining two constants of the gluon jet functions with rescaled energy-energy correlation jet functions [90,91]. Details on components of the gluon jet functions and a validation on our default choices are also given (see Supplemental Material [99]).

Numerical results—To validate our calculation framework, we explicitly verify that the physical cross section remains independent of the arbitrary cutoff $\delta\phi^{\text{cut}}$ in the limit $\delta\phi^{\text{cut}} \rightarrow 0$. Figure 2 displays the numerical stability of the $\mathcal{O}(\alpha_s^3)$ NNLO and $\mathcal{O}(\alpha_s^2)$ NLO corrections to the differential cross section of charged pion production at the EIC. The kinematics are fixed at $\sqrt{s} = 141$ GeV, $x = 0.2$, $y = 0.3$, with integration over $0.2 < z < 0.8$ and $P_{hT} > zQ/10$. The results are shown for six representative partonic channels, including uu , ug , gu , gg , $u\bar{u}$, and ud , labeled according to the flavors of the incoming and fragmenting partons. Among these, the uu channel is always dominant due to both favored PDFs and FFs. In all our calculations we use the CT18 NNLO PDFs [101] and the NPC23 NNLO FFs of pions [102] through all orders, with a nominal choice of $\mu = Q$ for both the renormalization and factorization scales. The only exception is for calculations of unidentified charged hadrons, where we use the NPC23 NLO FFs [103,104] through all orders since the NNLO FFs are not available. Note that each of the data points in Fig. 2 is from independent Monte Carlo (MC) integrations and the shown MC uncertainties are uncorrelated.

The NLO results (red points) exhibit excellent flatness over the range $10^{-3} < \delta\phi^{\text{cut}} < 10^{-1}$, confirming a precise cancellation of IR divergences, and converge to the reference values calculated with FMNLO [61]. At NNLO (blue points), the results similarly converge to a constant plateau, despite the larger MC integration uncertainties from the resolved cross section. The solid lines represent a fit to the genuine NNLO corrections together with residual power corrections [105,106]. From the fit, we estimate a systematic uncertainty (discrepancy *with respect to* the genuine result) of less than 5% (2%) when a finite value of $\delta\phi^{\text{cut}} = 0.04(0.01)$ is used in the NNLO calculations for all partonic channels (see Supplemental Material [99] for details). The stability observed across both quark- and gluon-initiated channels establishes the robustness of our method for high precision phenomenology. In addition, we

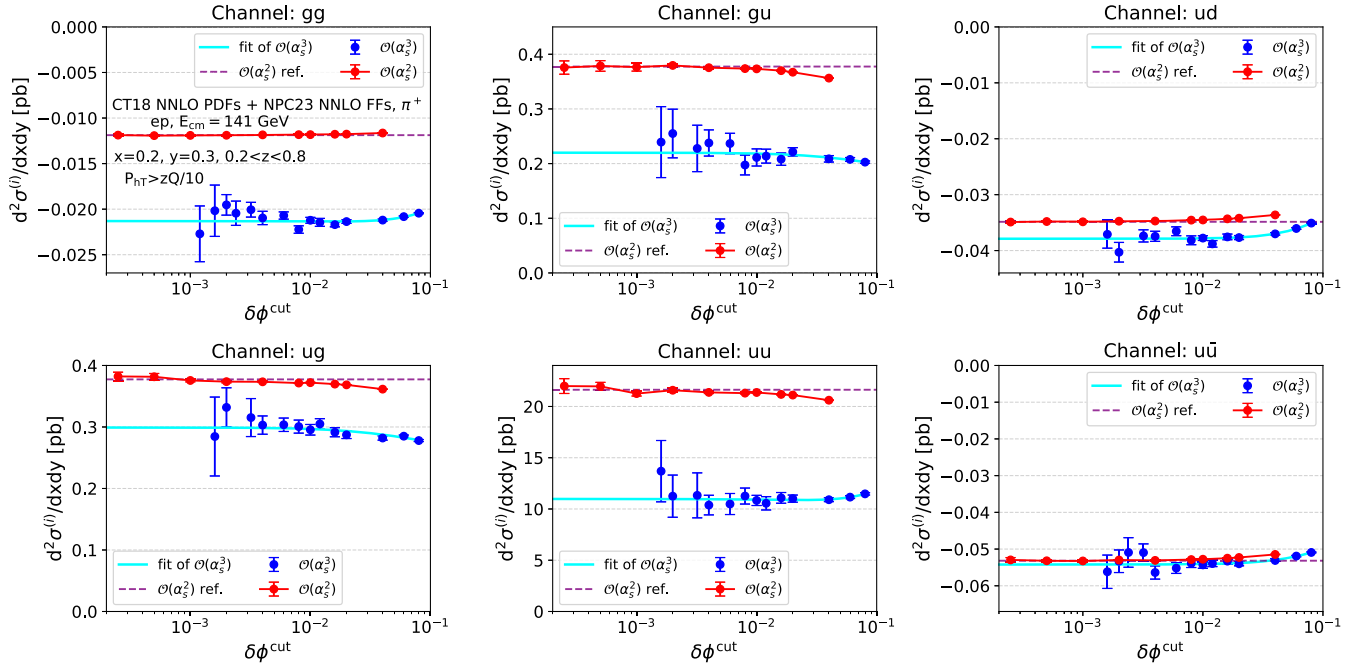


FIG. 2. Numerical stability of the total cross section with respect to the slicing parameter $\delta\phi^{\text{cut}}$. The panels display the $\mathcal{O}(\alpha_s^2)$ (red) and $\mathcal{O}(\alpha_s^3)$ (blue) contributions for six representative partonic channels. Error bars indicate Monte Carlo statistical uncertainties. The solid cyan lines represent the fit to the asymptotic limit $\delta\phi^{\text{cut}} \rightarrow 0$. The dashed purple lines denote the reference values of $\mathcal{O}(\alpha_s^2)$.

have checked that, when increasing the above unpolarized (polarized) constants by a factor of three, the NNLO corrections to the uu channel change by less than 1.5% (0.02%) of the correction itself, which is negligible.

Figures 3 and 4 present our NNLO predictions for the charged pion differential cross sections, tailored to the same kinematics of the future EIC used in Fig. 2, except for replacing the transverse momentum cut with $P_{hT} > 2$ GeV. We set $\delta\phi^{\text{cut}} = 0.01$ in the NNLO calculations. Figure 3 shows differential cross section as a function of the momentum fraction z . The upper panel displays the perturbative progression from LO (blue dotted) to NLO (red dot-dashed) and NNLO (black solid). We observe a substantial positive correction at NNLO across the entire z range, highlighting the importance of the $\mathcal{O}(\alpha_s^3)$ contributions. The lower panel, which normalizes results to the central LO prediction, reveals the improved convergence of the perturbative expansion. The scale uncertainty bands (hatched regions), calculated by varying μ by a factor of two from the nominal value Q , undergo a significant reduction when moving from NLO to NNLO for moderate z values. Note that the uncertainty bands represent only perturbative scale variations. We have checked that the systematic uncertainties from the extrapolation $\delta\phi^{\text{cut}} \rightarrow 0$ are subdominant compared to the residual scale variations across the shown kinematic ranges, and thus are not included in the bands. Furthermore, the K -factor exhibits a distinct kinematic dependence, rising noticeably at large z . This behavior indicates that in the threshold region the

soft-gluon dynamics begin to dominate [107,108]. Higher-order corrections beyond $\mathcal{O}(\alpha_s^3)$, for instance from soft-gluon resummation, can further reduce the perturbative uncertainties at large z .

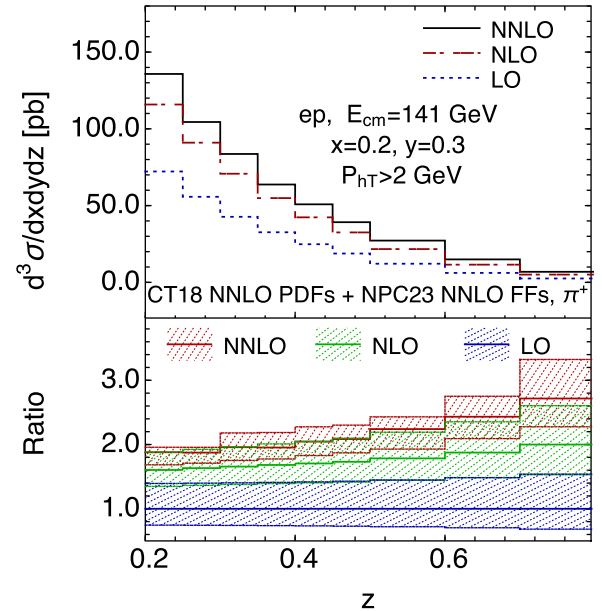


FIG. 3. Differential cross section as a function of the hadron momentum fraction z for charged pion production at the EIC at various orders in QCD together with scale variations, where we set $\delta\phi^{\text{cut}} = 0.01$. Kinematic cuts, including a lower limit of 2 GeV on the hadron transverse momentum, are applied.

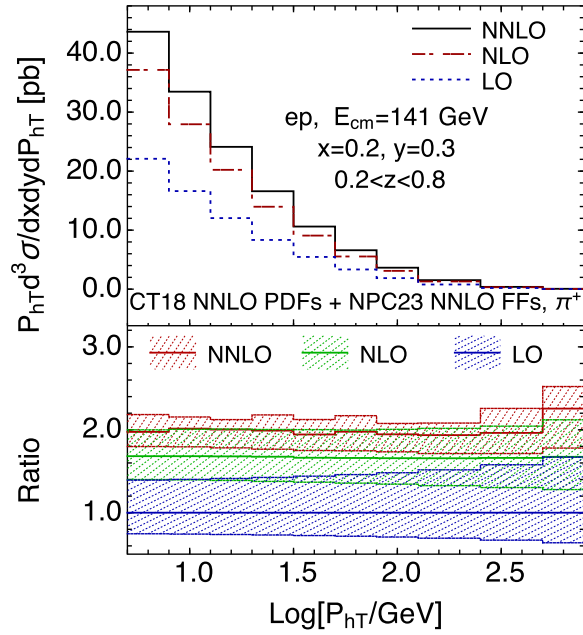


FIG. 4. Differential cross section as a function of the hadron transverse momentum P_{hT} for charged pion production at the EIC at various orders in QCD together with scale variations, where we set $\delta\phi^{\text{cut}} = 0.01$.

Figure 4 displays the differential cross section as a function of the hadron transverse momentum P_{hT} , which is of particular significance as it directly probes the transition from the non-perturbative TMD regime to the perturbative collinear domain. The upper panel reveals a clear hierarchy in the perturbative series, with the NNLO prediction (black solid line) providing a refined description of the spectrum. The ratio plot in the lower panel demonstrates improved convergence of the expansion in general. While the jump from LO to NLO is substantial—reflecting the opening of new partonic channels—the correction from NLO to NNLO is moderate and stable. Crucially, the scale uncertainty (hatched bands) is largely reduced at NNLO compared to the lower orders. This improved theoretical precision is essential for reliably identifying the matching window between FO calculations and TMD resummation, thereby facilitating a robust extraction of the nucleon’s three-dimensional structure at the EIC.

In Fig. 5, we compare our predictions with high-precision ZEUS data [6], collected at an ep center-of-mass energy of $E_{\text{cm}} = 296$ GeV. We set $\delta\phi^{\text{cut}} = 0.04$ in the NNLO calculations. The plot displays the unidentified charged hadron multiplicity as a function of transverse momentum P_{hT} in the kinematic region with $10 < Q^2 < 160$ GeV², $75 < W < 175$ GeV, and $x_F > 0.05$, where the Feynman x variable is defined as $x_F \equiv 2P_{hL}^*/W$. The longitudinal momentum P_{hL}^* is the projection of hadron momentum onto the virtual photon direction in the center of mass frame of γ^*p , and W is the invariant mass of the

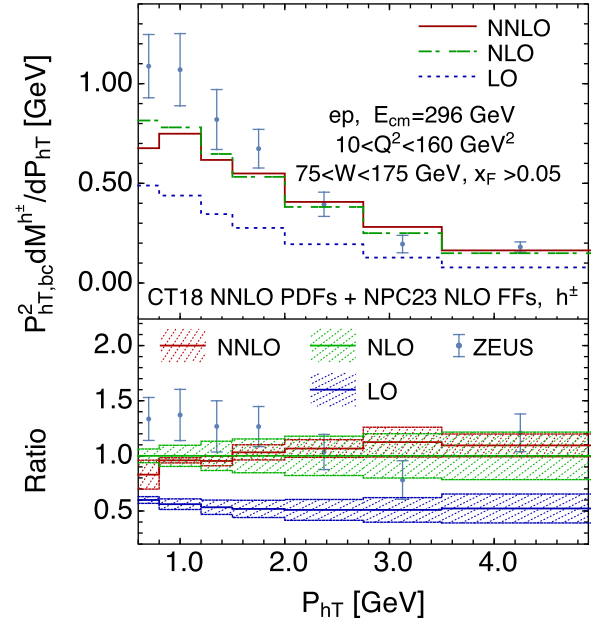


FIG. 5. Multiplicity distribution (multiplied by the square of bin center) as functions of the hadron transverse momentum P_{hT} for unidentified charged hadron production at the HERA at various orders in QCD together with scale variations, compared to the ZEUS data [6], where we set $\delta\phi^{\text{cut}} = 0.04$ to ensure adequate control over MC uncertainties.

hadronic final state. The LO result (blue dotted) grossly underestimates the data, accounting for only $\sim 50\%$ of the observed yield. This large discrepancy highlights the limitation of LO kinematics in generating high- P_{hT} recoil. While NLO predictions (green) significantly reduce this gap, they continue to systematically undershoot the experimental central values and show scale variations as large as the experimental errors. The inclusion of $\mathcal{O}(\alpha_s^3)$ corrections leads to a moderate enhancement of the normalization and reduced scale variations at large transverse momentum. On the other hand, it is clear that for $P_{hT} \sim 1$ GeV or smaller the FO calculations are insufficient and TMD resummation [30,109] is required to describe the data.

Summary and outlook—In this Letter, we have presented the first NNLO QCD calculation for single inclusive hadron production in DIS at finite transverse momentum. By implementing the q_T -subtraction method with a recoil-free WTA axis, we have successfully overcome the technical bottleneck of consistently treating IR divergences in the presence of both fragmenting hadrons and jets. Our numerical results demonstrate that NNLO corrections are substantial and significantly reduce the scale uncertainties compared to NLO. While residual higher-order effects remain relevant in certain extreme kinematic regions, this work provides a precision theoretical input for the hard scattering cross sections, enabling future global analyses to precisely delineate the transition region between collinear

and TMD factorization and to extract nonperturbative PDFs and FFs with higher precision. Future extensions of this work to polarized scattering will further unlock the potential of the EIC to probe the spin-momentum correlations of partons within hadrons. Finally, our results also serve as a fundamental step toward fully differential N³LO predictions for SIDIS [30].

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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