

Waiting Time Distribution and Multifractality Analysis of Solar Flares

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The statistical properties of solar flare occurrence are critical for understanding coronal energy release, yet the nature of their underlying dynamics remains heavily debated. Analyzing over two decades of soft x-ray observations from a comprehensive geostationary operational environmental satellite (GOES) dataset, we demonstrate that flare waiting time distributions exhibit a previously unreported scale invariance under coarse graining by flare size. This self-similarity proves the existence of long-range temporal correlations between events. This finding is supported by a complementary multifractal analysis of the x-ray flux, which confirms the presence of intermittency and turbulentlike scaling. By ruling out independent event occurrence, these results establish new statistical constraints on solar flare dynamics and link their intermittent behavior to shared multiscale avalanche phenomena in complex systems.

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Solar and stellar flares are among the most energetic phenomena that may arise from a magnetically active star, reaching, in the case of the Sun, up to 10^{26} J [1,2] in just a few hours and descending to values currently undetectable by modern instrumentation [3]. Given the broad range of energy and timescales covered by these events, studying the associated energy and waiting time (i.e., the time interval between the end of an event and the start of the following one) can provide insights into a wide range of processes, such as how stellar atmospheres maintain an energy balance and forecasting such events, as well as understanding the habitability of exoplanets. As large-scale flares are rare and cannot supply the required energy to continuously heat the corona [4–6], Parker [7] first proposed in 1983 the idea that small-scale and continuous thermal energy release processes within the magnetised atmosphere, later referred to as nanoflares, could play a key role in the energetic budget of a star. Following the

pioneering work of Parker, many authors studied the power-law relationship [8,9] between the occurrence frequency of flares and their energy, further suggesting that nanoflares could sustain the required energy output to heat the solar corona [10–15]. However, while magnetic reconnection is recognized as the fundamental mechanism for flare formation, a comprehensive understanding of the underlying physical processes has not yet been achieved, despite the many efforts [16–18], leaving unanswered the question of coronal heating. This interest extends beyond fundamental science to technological considerations. Indeed, solar flares are known to be correlated with solar energetic particle (SEP) emissions and coronal mass ejections (CMEs), all of which fall under the umbrella term “space weather events” (SWEs). These events can potentially lead to catastrophic consequences for technology, both on the ground and in space, as well as for the crew of manned space missions. Therefore, understanding the effects and causes of SWEs, as well as developing an early warning system, has become a top priority for many space agencies and companies.

As flares are part of a complex and continuous energy dissipation process spanning many scales, understanding their statistics is essential, even more so as direct modeling of these processes remains difficult given the multiscale and nonlinear nature of magnetohydrodynamic (MHD)

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turbulence. Hence, flare statistics could be considered a diagnostic of the underlying physical processes regulating energy transport and dissipation in stellar coronae. To this scope, in the last decades, the statistical properties of flare events have been investigated using several observational data and methods. In Crosby *et al.* [19], it has been observed that the hard x-ray flux peak probability distribution, as well as the total energy and duration, can be described by power laws for a wide range of scales. These features are similar to many different systems characterized by avalanchelike dynamics, which involve intermittent bursts of energy dissipation. Then, the statistical properties of the system can be characterised by three quantities: event size S (i.e., energy released), event duration t_E , and waiting time between events t_w . Usually t_E and S are correlated, while no correlation is observed between S and t_w . In Boffetta *et al.* [20], using a simplified model of MHD dynamics, event sizes are defined whenever the total energy dissipation $\epsilon(t)$ is larger than a certain threshold ϵ_0 . In this particular case, the probability distributions $P[S]$ and $P[t_w]$ display power law behavior, along with $P[t_w] \sim \tau^{-\alpha}$ and $\alpha \sim 2.2$. Similar results have been found by analyzing the solar flare events in Wheatland and Litvinenko [21] and Baiesi *et al.* [22]. The interpretation of these power-law-like distributions has been widely debated and remains a delicate topic, as different models can reproduce it. For instance, in Boffetta *et al.* [20], to explain the observed scaling law for $P[t_w]$, the authors highlight the complex dynamics of the system with long correlation times, in contrast to a self-organised criticality model (SOC) [23], which expects no correlations between the occurrence times of successive bursts. An approach based on Bayesian decomposition was proposed instead in Wheatland and Litvinenko [21] in terms of a Poisson process with a time-varying flare rate. In this Letter, we show that the waiting time distributions of flare events observed by GOES for more than 20 years are characterized not only by scale invariance but also by a new property of self-similarity, which is described in the following.

First, we consider our dataset. We exploited the unprecedented statistical consistency and flexibility of the archival solar flares catalogue (ASR) flare catalog [24] to investigate the properties of the waiting times t_w . The available dataset covers from January 1, 1995, to December 31, 2024, with a temporal cadence of 1 min. In Fig. 1, we show an example of the GOES soft x-ray signal, from now on denoted by $\Phi(t)$, as well as the definition of the flare event inherited from the ASR catalog. To quickly summarize the flare event classification procedure, each local minimum of the signal is considered a potential starting point of a flare, and each subsequent local maximum is regarded as a potential flare peak. If the value of the flux at the peak is above that of the preceding minimum plus a threshold equal to $\sigma = 2 \times 10^{-8} \text{ W/m}^2$, the event is identified as a flare event. This σ value is related to the intrinsic sensitivity

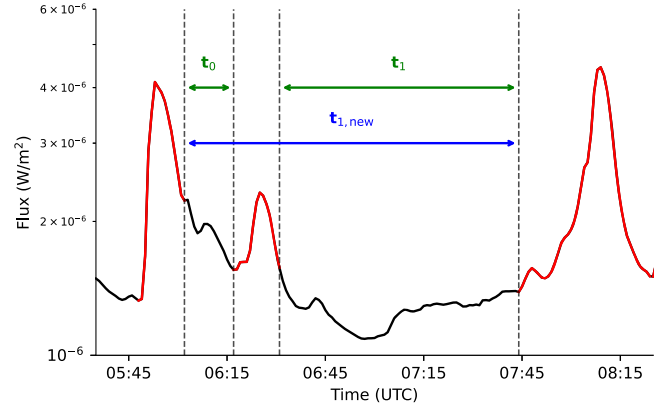


FIG. 1. The time series of GOES soft x-ray flux $\Phi(t)$ from December 14, 2014, is shown. The black line describes $\Phi(t)$ while the red line highlights the solar flare profiles. Dashed vertical lines show the starting and ending points of the flare events. Each waiting time (t_0 and t_1) in figure is highlighted by a green horizontal line. A horizontal blue line marks the new waiting time ($t_{1,new}$) once a constraint on the flare size is adopted.

and noise floor of the GOES x-rays (XRS) instrument during quiescent periods of solar activity [25]. The end of the event is defined as either when the flux returns to a value below that of the starting minimum plus the threshold or when a new minimum is encountered. Furthermore, all events that end due to the occurrence of another minimum are grouped into a single flare event. We point out that $\Phi(t)$ is not statistically stationary during the time considered, which is the reason for the definition of *ad hoc* starting and peak points. Further explanations on the event definition can be found in Berretti *et al.* [24]. Based on our definition of solar flare, we can identify the waiting times t_w between two successive events. In Fig. 1, we show a snapshot of the signal with three flare events (marked in red) and separated by two waiting times t_w (t_0 , t_1). The flare size S_i is computed as the integral flux of the flare events. In Fig. 2, we show the probability distribution of the flare size $P[S]$, the waiting time $P[t_w]$, and the peak flux $P[f_p]$, where f_p is the value of the local maximum observed in the signal. Consistent with the literature, our distributions can be represented by power laws for a large range of scales, with spectral scaling varying between ~ 1.9 and 2.3 . The spectral index of $P[S]$ steepens at $\sim 10^{-5} \text{ (W/m}^2\text{)}$ from ~ 1.3 to 1.9 . In our event definition, multiple sequential peaks are considered as a single event, which translates into a redistribution in the region of lower fluences compared to previous studies [24] and is the reason for the observed spectral transition. A similar approach is also used in the analysis of amorphous system dynamics Pinaki *et al.* [26]. $P[f_p]$ is well described by a single spectral index equal to ~ 2.3 . The results presented in Fig. 2 show that our definition of solar flares is consistent with other definitions used in the literature [25]. In particular, the waiting time probability distribution turns out to be very close to what

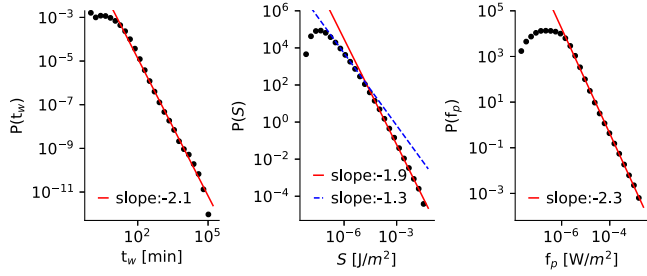


FIG. 2. Distributions of waiting time $P[t_w]$, size $P[S]$, and peak flux $P[f_p]$. Each distribution is characterized by a power-law trend on a wide range of scales. The best-fit power law indices are highlighted with straight and dashed lines, and their exact values are given in the legend.

has been previously reported; see, for instance, Baiesi *et al.* [22]. We also verify that there are no significant changes in the scaling properties of Fig. 2 when we increase the value of the threshold σ by a factor of 10.

Now we introduce our main idea. The event size distribution is defined on the interval $[S_0, S_M]$, where S_0 and S_M are the minimum and maximum sizes, respectively. In principle, we can be interested in computing the waiting time distribution between flares of size larger than S_0 , say λS_0 , where $\lambda > 1$ is a dimensionless scaling parameter. We consider the interevent time distribution t_w for events of sizes larger than S_0 , denoted by $P[t_w|\lambda S_0]$. To clarify this concept, in Fig. 1, we show the change in the waiting time values by neglecting the smaller intermediate flare: we first have three flare events, separated by waiting time values t_0 and t_1 ; after introducing a constraint in size, we consider only the two larger flare events separated now by a single waiting time $t_{1,\text{new}}$. The relevant question is how $P[t_w|\lambda S_0]$ depends on λ . Notice that by increasing λ we are not changing the flare definition, and $P[S]$ remains unaffected. Thus, by increasing λ , we are performing a *coarse-grained* investigation of the system dynamics. The same idea has been applied in many different systems displaying avalanchelike dynamics [27–30], where it has been found that $P[t_w|\lambda S_0]$ shows scaling properties invariant with respect to λ . We find that this is also the case for solar flares. In Fig. 3(a), we show $P[t_w|\lambda S_0]$ for five different values of $\lambda = 1.0, 3.2, 10, 32$, and 100 and $S_0 = 10^{-7}$ W/m². These values are selected to obtain reasonably good statistics for each distribution and are consistent with previous works (e.g., Baiesi *et al.* [22]). The distributions shown have been rescaled to facilitate a direct comparison. We observe that the five distributions show a power-law behavior described by the same scaling for waiting times larger than 10^2 min. We remark that, to our knowledge, the combined property of size and scale invariance observed in Fig. 3 has never been reported for the statistical analysis of solar flare events. The difference between our results, in which we use the same definition of an event regardless of the value of λ , and the one discussed in Baiesi *et al.* [22], where the definition of an event

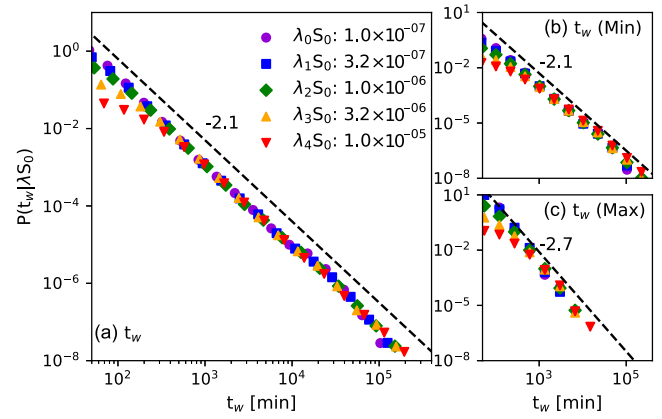


FIG. 3. (a) Distributions of the waiting time associated with five logarithmically spaced values of λ , $P[t_w|\lambda S_0]$, for the entire dataset. By gradually excluding from the distribution smaller size flare events, that is increasing λ , the distributions preserve the same scaling of ~ -2.1 for over 2 orders of magnitude. We observe that the same is also valid for datasets containing only periods of solar minima and solar maxima, panels (b) and (c).

depends on the threshold and the scaling law of $P[t_w]$ changes by altering the threshold, highlights that this scale invariance property is not trivially linked to the probability distribution of $\Phi(t)$, which is substantially the same. As previously remarked, upon increasing λ , we are summing together waiting times. Thus, Fig. 3 suggests that the statistical properties of $P[t_w|\lambda S_0]$ are stable under convolution. For statistically independent t_w , the only stable distribution is Poisson, while we observe a well defined scaling behavior of $P[t_w|\lambda S_0]$ invariant over a rather wide range in λ . This suggests that the waiting times t_w are correlated. We remark that the results shown in Fig. 3 are not generically true for any avalanchelike system. For instance, in Pinaki *et al.* [26], it is shown that the invariance of $P[t_w|\lambda S_0]$ occurs in amorphous systems if the system stiffness is large enough. As a further test, we perform the same analysis on restricted time intervals corresponding to solar maximum and minimum. In particular, we use the years 2006–2009 and 2018–2020 for the solar minimum, and 2000–2003, 2014–2015, and 2022–2024 for the solar maximum. We remark that this separation is quite arbitrary and our main goal remains to describe the global properties of flares, as shown in Fig. 3(a). Nonetheless, we observe that the relative distributions conserve the same self-similarity property in both cases, as shown in Figs. 3(b) and 3(c), even though the distribution for the solar maximum is characterized by a limited range of waiting times, as expected from the higher overall flare occurrence rate, and by a steeper slope. Also, upon increasing λ , the number of events with relatively small interevent times reduces, which explains the flattening of the probability density at small t_w and large λ . Thus, this self-similarity property is not only true in the statistical average sense but appears to be a characteristic of the system’s dynamics, present during different solar activity phases.

Results similar to the one shown in Fig. 3 have been observed in Benzi *et al.* [30] with a shell model for turbulence with constant power input; see also Supplemental Material [31] for similar results for the case of stochastic forcing. Whereas the scaling exponents of $P[t_w|\lambda S_0]$ are different, as is the definition of events, the probability distribution of the waiting time is invariant upon increasing λ . At any rate, in Benzi *et al.* [30], the definition of an event is based on the instantaneous rate of energy dissipation. It is known that, for shell models, as well as for any fully developed turbulent flows, the probability distribution of energy dissipation is intermittent, and it can be successfully described by multifractal theory [32] independently of the large-scale forcing mechanism. In our cases, x-ray emission plays the role of energy dissipation; therefore, it is natural to inquire whether the signal under investigation is consistent with a multifractal characterization. We perform the multifractal analysis of $\Phi(t)$ between 2002 and 2003, an interval of time characterized by particularly high solar activity, and compare it with the shell model used in Benzi *et al.* [30]. This analysis provides crucial information on the fractal scaling properties and long-range time correlations in time series and is widely used in turbulence analysis. For shell models, we consider the quantity:

$$\epsilon(\tau) \equiv \frac{1}{\tau} \int_t^{t+\tau} \epsilon(t) dt, \quad (1)$$

where $\epsilon(t)$ is the instantaneous rate of energy dissipation and τ is a time increment in time that defines a moving average window and serves as the independent variable for our scaling analysis. We then compute the statistical moments $M_p(\tau|\epsilon) \equiv \langle \epsilon(\tau)^p \rangle$, where $\langle \dots \rangle$ is the time average. $M_p(\tau|\epsilon)$ quantifies the fluctuations of the signal across different orders p . For multifractal systems, $M_p(\tau|\epsilon) \sim \tau^{-H(p)}$, where the scaling exponent $H(p)$ is a nonlinear function of p with $H(1) = 0$. This nonlinearity is the hallmark of multifractality and indicates that, to describe the system's fluctuations, we require a continuous spectrum of exponents driven by strong, intermittent bursts. Given that $H(1) = 0$, demonstrating that $H(3) < 1.5H(2)$ is sufficient to prove this nonlinear relationship and confirm the multifractal nature of the signal. For the GOES data, we use the definition in Eq. (1) with $\Phi(t)$ instead of $\epsilon(t)$. For both ϵ and Φ , we rescale the available time between $[0:1]$ and normalize the signals such that $\langle \epsilon \rangle = \langle \Phi \rangle = 1$. The final results are shown in Fig. 4. In the inset of Fig. 4, we show $M_2(\tau|\epsilon)$ and $M_3(\tau|\epsilon)$ for the shell model discussed in Benzi *et al.* [30]. We note that $M_p(\tau|\epsilon) \sim \tau^{-H(p)}$, with $H(2) = 0.35$ and $H(3) = 0.85$, which demonstrates that $H(p)$ is nonlinear. In the main panel of Fig. 4, we show $M_2(\tau|\Phi)$ and $M_3(\tau|\Phi)$: a nice scaling behavior is observed for both quantities with scaling exponents $H(2) = 0.5$ and $H(3) = 1.25$. The same analysis for the year 2004 is presented in the Supplemental

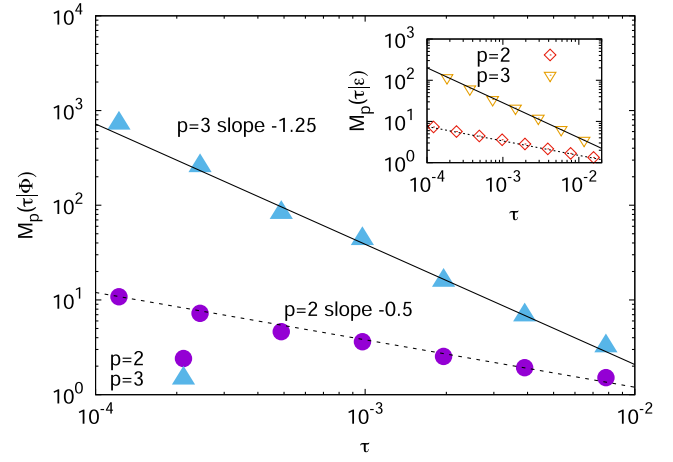


FIG. 4. The main panel shows log-log behavior of $M_p(\tau|\Phi)$ ($p = 2, 3$) as defined in the text using Eq. (1) with Φ for GOES data. The two straight lines show the slopes $H(2) = 0.5$ and $H(3) = 1.25$ using the definition $M_p(\tau|\Phi) \sim \tau^{-H(p)}$. In the inset, we show $M_p(\tau|\epsilon)$ for the shell model used in Benzi *et al.* [30]. The scaling exponents are $H(2) = 0.35$ and $H(3) = 0.85$. For both ϵ and Φ , we rescale the available time between $[0:1]$, and we normalize the signals such that $\langle \epsilon \rangle = \langle \Phi \rangle = 1$.

Material and shows results consistent with the one presented in the paper [31]. These results provide evidence of multifractal behavior of our system.

Our Letter shows that the waiting time distributions of solar flare events are not only scale invariant but also exhibit properties of self-similarity under coarse graining with respect to flare size. The invariance of $P[t_w|\lambda S_0]$ for different $\lambda > 1$ suggests a nontrivial correlation between flare events, which is hard to explain with Poissonian dynamics. Furthermore, the multifractal analysis of $\Phi(t)$ reveals long-range correlations and intermittency similar to those observed in turbulent energy dissipation, strengthening the analogy between solar flare statistics and avalanche-like turbulent systems. While identifying the underlying mechanisms and physical origins of solar flares is outside the scope of this Letter, our results challenge some of the existing theories in the literature. Classic SOC models predict power-law distributions by viewing solar flares as discrete, memoryless avalanches on a critical system, that is, a system close to a critical threshold. Alternatively, a nonstationary Poisson process can mimic power-law-like distributions through the superposition of exponential processes with varying rates. However, such models typically treat the flaring rate as a slowly varying external parameter. In contrast, our Letter demonstrates that $P[t_w]$ is invariant under coarse graining, a property that suggests the intermittent nature of flares is an emergent feature of internal, multiscale dynamics. In particular, the observed self-similarity in waiting time distributions and the multifractal nature of the x-ray flux point to the underlying turbulentlike dynamics of the physical processes occurring between the energy storage and the onset of flares, with no

reference to any critical threshold. This is consistent with the physical picture emerging in a turbulent flow [20,30], as well as in the broader context of avalanche dynamics, such as those observed in amorphous solids and earthquakes [27–30]. In these systems, energy dissipation occurs via intermittent bursts. Similar to the temporal correlations observed between plastic yielding events in soft-glass systems or successive acoustic emissions in rock fracture, our analysis shows that solar flares exhibit distinct clustering patterns and nonstationary intermittency. These dynamics fundamentally deviate from memoryless exponential distributions, revealing a shared statistical framework for avalanche phenomena across widely different physical domains. In summary, we find that the behavior of solar flares conforms to a self-similar multifractal process, where the observed invariance of $P[t_w|\lambda S_0]$ under coarse graining reveals the existence of long-range temporal correlations.

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Data availability—The data that support the findings of this article are openly available [34]. The code used to analyze the data is available from [35].

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- [31] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/b7tv-8mbk> provides two additional results referenced in the main text but not shown in the main paper: a repetition of the shell-model analysis under stochastic, rather than constant, power-input forcing, to test the robustness of the multifractal scaling exponents; and a repetition of the multifractal analysis of the GOES soft x-ray flux for a different year of solar activity, confirming that the observed scaling and multifractal signatures are consistent across different phases of the solar cycle rather than specific to the interval analyzed in the main paper.
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