

Spontaneous Quantum Turbulence in a Newborn Bose-Einstein Condensate via the Kibble-Zurek Mechanism

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(Received 20 July 2025; revised 5 December 2025; accepted 28 May 2026; published 7 July 2026)

The Kibble-Zurek mechanism (KZM) predicts the spontaneous formation of topological defects in a continuous phase transition driven at a finite rate. We propose the generation of spontaneous quantum turbulence (SQT) via the KZM during Bose-Einstein condensation induced by a thermal quench. Using numerical simulations of the stochastic projected Gross-Pitaevskii equation in two spatial dimensions, we describe the formation of a newborn Bose-Einstein condensate proliferated by quantum vortices. We establish the nonequilibrium universality of SQT through the Kibble-Zurek and Kolmogorov scaling of the incompressible kinetic energy.

DOI: [10.1103/b16v-hwp2](https://doi.org/10.1103/b16v-hwp2)

Understanding turbulence is one of the long-standing unsolved problems in physics, with broad applications in biology, medicine, industry, and weather prediction [1,2]. Feynman pioneered the idea of quantum turbulence (QT) as an analog in superfluids proliferated by a tangle of vortices [3]. With the development of technology to create, manipulate, and probe quantum fluids, the study of quantum turbulence emerged as a new research field at the frontiers of nonequilibrium physics of complex systems, arising in superfluid helium, Bose-Einstein condensates, polaritonic condensates, and neutron stars [4–12], with applications ranging from atomtronics to astrophysics.

The turbulent regime in quantum fluids describes the complex, chaotic dynamics of interacting quantized vortices, which are topological defects. This definition is loose, and as we shall see, different criteria for its diagnosis have been put forward, including the scaling of the energy spectrum on the wave number, velocity autocorrelation functions, circulation statistics, etc. The situation is reminiscent of the use of various inequivalent definitions of quantum chaos, emphasizing complementary features brought out by a diversity of diagnostic tools (spectral statistics, semiclassical orbits, out-of-time order correlators, etc.) [13].

QT was discovered in low-temperature physics studies of superfluid helium. However, the remarkable experimental progress in the mid-90s offered a new highly controllable and tunable platform for its exploration: Bose-Einstein condensates (BECs) of ultracold atomic gases [14] and the closely related quantum fluids of light [15]. The experimental measurement of quantized vortices in BEC was crucial in probing their superfluid character. A BEC is characterized by a macroscopic wave function that acts as a complex-valued order parameter and can be conveniently written in a polar decomposition $\Psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\theta(\mathbf{r})}$ [16]. A defining feature of a superfluid is that the circulation $\Gamma[C] = \oint_C \mathbf{v} \cdot d\boldsymbol{\ell}$ (that is, the contour integral of the velocity around a given contour C) is quantized as a result of the single-valued character of the macroscopic wave function and the relation between the superfluid velocity $\mathbf{v}$ (equivalently, the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$) and the condensate phase, $\mathbf{v}(\mathbf{r}) = \hbar \nabla \theta(\mathbf{r})/m$, where m is the mass of the particle. QT differs from its classical counterpart in that quantum vortices are true topological defects and thus more stable than eddies, having conserved and quantized circulation [9,17].

The phenomenology of turbulence varies significantly with the spatial dimension. In three spatial dimensions (3D), turbulence is characterized by a cascade of energy from large to small scales, with eddies breaking into ever-decreasing whirlpools. By contrast, two-dimensional (2D) turbulence is governed by the presence of an additional invariant of motion known as the enstrophy, and gives rise to an inverse energy cascade, with energy flowing from small to large scales. In the quantum case, the spatial dimensionality of the superfluid can be varied by tuning the anisotropy of the external confinement in BEC

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experiments. Notably, in 2D QT, enstrophy may not be conserved due to vortex-antivortex annihilation, and a direct energy cascade can therefore appear, as observed in numerical simulations [18,19] and BEC experiments [20]. In contrast, its conservation leads to the clustering of vortices in bigger patches, a behavior predicted by Onsager and experimentally reported in BECs [21,22] and exciton-polariton superfluids [23].

Several protocols are used to induce QT in a superfluid sample. They generally rely on continuous pumping of energy. A predominant approach involves sweeping a repulsive potential [24,25]. This procedure is often too violent, complicating the study of QT, as a result of the generation of a variety of excitations. As an alternative, one may wonder whether QT can emerge spontaneously, as topological defects emerge across a continuous phase transition. We refer to the QT that arises through finite-time spontaneous symmetry breaking as *spontaneous quantum turbulence* (SQT). In this context, the Kibble-Zurek mechanism (KZM) constitutes a powerful paradigm for describing universal critical dynamics [26–30]. It does so by exploiting equilibrium properties such as the divergence of the correlation length $\xi = \xi_0/|\varepsilon|^\nu$ and the relaxation time $\tau = \tau_0/|\varepsilon|^{z\nu}$ in the neighborhood of the critical point [30]. Here, ν and z are the correlation-length and dynamic critical exponents, $\varepsilon = (\lambda - \lambda_c)/\lambda_c$ with λ the control parameter (e.g., temperature) and λ_c its critical value, while ξ_0 and τ_0 are nonuniversal constants with dimensions of length and time, respectively. For a linearized quench $\varepsilon = t/\tau_Q$, the KZM introduces the universal scaling of the nonequilibrium correlation length $\hat{\xi}$ with the quench time τ_Q , i.e., $\hat{\xi} = \xi_0(\tau_Q/\tau_0)^{\nu/(1+z\nu)}$. For pointlike defects, such as solitons in a cigar-shaped BEC [31,32], and vortices in a pancake BEC [33,34], the KZM further predicts the formation of vortices at a density ρ_v that scales universally with the quench time as $\rho_v \propto 1/\hat{\xi}^2 = (1/\xi_0^2)(\tau_0/\tau_Q)^{2\nu/(1+z\nu)}$. In addition, it dictates that the response of the system is delayed by the freeze-out timescale $\hat{t} = (\tau_0\tau_Q^{z\nu})^{1/(1+z\nu)}$. In scenarios characterized by symmetry breaking that lead to vortex formation, the KZM has been widely tested in theoretical studies [35–41], with several experiments also supporting its role in superfluid formation and the BEC transition [33,42–45].

In this Letter, we establish and describe SQT in a newborn BEC prepared by a thermal quench in finite time. We establish the Kolmogorov scaling of the incompressible kinetic energy spectrum and demonstrate its universal scaling for different values of the quench time when rescaled by the nonequilibrium correlation length predicted by the Kibble-Zurek mechanism.

Kibble-Zurek dynamics of the BEC transition—To describe the nonequilibrium condensate dynamics and the emergence of SQT, we consider the 2D stochastic projected Gross-Pitaevskii equation [46–51]. This

formalism has been widely used to model the BEC transition, with numerous studies demonstrating its quantitative agreement with experimental observations [33,37]. Specifically, it describes the evolution of the low-energy coherent modes of a Bose gas via a complex order parameter Ψ_C which, in dimensionless units used throughout the manuscript, evolves according to

$$d\Psi_C = \mathcal{P}_C[-(i + \gamma)(H_{\text{sp}} + g|\Psi_C|^2 - \mu)\Psi_C dt + d\eta], \quad (1)$$

where μ is the chemical potential, γ is the damping rate modeling energy dissipation due to the contact with the thermal bath, g is the coupling strength of the delta-function interaction between bosons, and $d\eta$ is a complex Gaussian noise increment accounting for thermal fluctuations. $H_{\text{sp}} = -(1/2)\nabla^2 + V(\mathbf{r})$ denotes the single-particle Hamiltonian, with $V(\mathbf{r})$ the external trapping potential. In this work, we focus on a periodic homogeneous system and set $V(\mathbf{r}) = 0$. Further details of Eq. (1) and its numerical implementation are provided in [52].

To drive the phase transition, we consider a linear quench of the chemical potential, $\mu(t) = \mu_i + (\mu_f - \mu_i)t/\tau_Q$, with μ_i and μ_f denoting its initial and final values, respectively. This protocol drives the system through the critical point μ_c at a finite rate τ_Q , resulting in the formation of a BEC populated with vortices. A vortex is characterized by a quantized circulation of the superfluid velocity, $\oint \mathbf{v}(\mathbf{r}, t) \cdot d\boldsymbol{\ell} = 2\pi w$, where $w \in \mathbb{Z}$ is the winding number. The associated singularity of the velocity field enforces a vanishing condensate density at the vortex center, with the healing length determining the core size. An example of a single realization of the newborn BEC is shown in Fig. 1, where vortices can be recognized by the depletion of the BEC density at their core. The associated phase map indicates that only the values $w = \pm 1$ are generated in the course of the BEC transition. This is consistent with the fact that higher winding numbers are energetically costly and unstable against decay [60].

Using the mean-field values of the critical exponents $\nu = 1/2$ and $z = 2$, the KZM predicts the scaling of the freeze-out time, which sets the timescale in which the growth of the order parameter lags behind the crossing of the critical point set by the quench of $\mu(t)$. The equilibration time, when the growth of the number of bosons in BEC changes from exponential to linear [36,39,52], is proportional to the freeze-out time and scales universally as $t_{\text{eq}} \propto \hat{t} \propto \tau_Q^{1/2}$. This scaling is confirmed by the numerical results shown in the inset of Fig. 1(c), where $t_{\text{eq}} = (3.779 \pm 0.693)\tau_Q^{0.541 \pm 0.069}$. At the equilibration time, the KZM further predicts that the vortex number scales as $n_v \propto \tau_Q^{-1/2}$, in agreement with the fit shown in Fig. 1(c), where $n_v = (249.036 \pm 44.999)\tau_Q^{-0.534 \pm 0.047}$.

Since we consider a 2D system, the role of the Berezinskii-Kosterlitz-Thouless (BKT) transition and the associated thermal vortices deserves discussion. Following

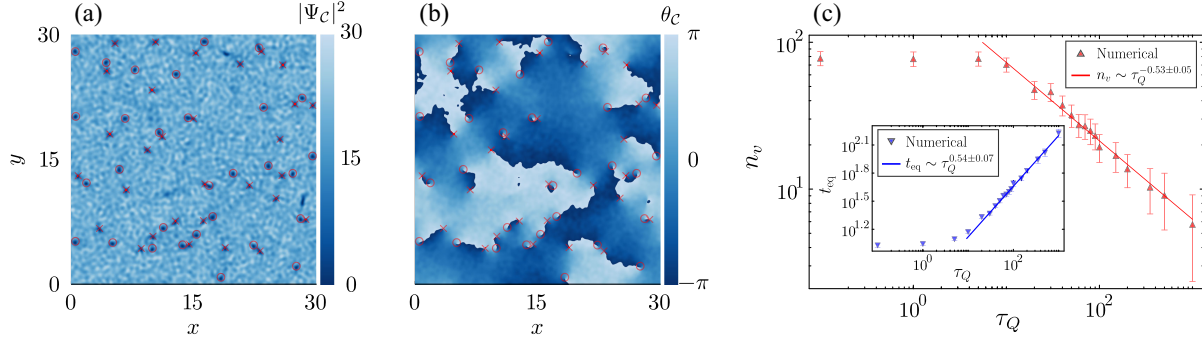


FIG. 1. Typical condensate density and phase profiles at equilibration time, together with the vortex-number scaling. Panel (a) shows the condensate density $|\Psi_C(\mathbf{r}, t_{\text{eq}})|^2$ at equilibration time t_{eq} following a quench of duration $\tau_Q = 10$, while panel (b) displays the corresponding phase of Ψ_C . Vortices with topological charge $w = +1$ are marked by red crosses, and those with $w = -1$ by circles; no vortices with $|w| > 1$ were observed. The main plot in panel (c) shows the vortex count n_v at t_{eq} versus quench time τ_Q , demonstrating the Kibble-Zurek power-law scaling for $10 \leq \tau_Q \leq 1000$. The inset plots t_{eq} against τ_Q , where t_{eq} is the equilibration time measured from the crossing of the critical point. Data are averaged over $\mathcal{R} = 1000$ realizations, with error bars indicating 1 standard deviation.

the quasi-2D Bose gas experiment of Ref. [34], which demonstrated negligible BKT effects and verified KZM power laws, we work in the low-temperature limit (of order 1 nK or less) to isolate the SQT arising from the symmetry-breaking phase transition. In this regime, the expected number of thermal vortices is negligible [61]. This is further supported by the time evolution of the vortex count shown in the Supplemental Material [52]: n_v decays monotonically to zero, indicating that all observed vortices are of KZM origin rather than thermally activated, consistent with the experimental findings of Ref. [34].

Compressible and incompressible energy spectra—By introducing the density-weighted velocity field $\mathbf{u}(\mathbf{r}) = |\Psi_C(\mathbf{r})|\mathbf{v}(\mathbf{r})$ [62–65], the kinetic energy of the newborn BEC takes the form $E = (1/2) \int |\mathbf{u}(\mathbf{r})|^2 d^2\mathbf{r}$. Using the canonical decomposition [62–65], $\mathbf{u}$ can be separated into a compressible (irrotational) component $\mathbf{u}_c$, satisfying $\nabla \times \mathbf{u}_c = 0$, and an incompressible (solenoidal) component $\mathbf{u}_i$, for which $\nabla \cdot \mathbf{u}_i = 0$. The total kinetic energy is accordingly decomposed into its compressible and incompressible contributions,

$$E_{i,c} = \frac{1}{2} \int |\mathbf{u}_{i,c}(\mathbf{r})|^2 d^2\mathbf{r}. \quad (2)$$

It is particularly useful to express Eq. (2) in Fourier space, resulting in $E_{i,c} = \int E_{i,c}(k) dk$, with

$$E_{i,c}(k) := \frac{k}{2} \int |\tilde{\mathbf{u}}_{i,c}(\mathbf{k})|^2 d\varphi_k, \quad (3)$$

where $\mathbf{k} = k(\cos \varphi_k, \sin \varphi_k)$ and $\tilde{\mathbf{u}}_{i,c}(\mathbf{k})$ is the Fourier transform of $\mathbf{u}_{i,c}(\mathbf{r})$ [66].

Kolmogorov scaling of the energy spectrum in SQT—The occurrence of SQT in the newborn BEC is diagnosed by the Kolmogorov scaling of the incompressible kinetic energy spectrum $E_i(k)$ [17,67], which has been

experimentally measured in 2D BECs [20]. This law was originally derived for homogeneous isotropic turbulence where large eddies (with high Reynolds number) are continuously broken into smaller eddies (with small Reynolds number), and there exists a range of length scales (inertial range) where the details of energy injection and dissipation can be neglected and the eddies follow universal statistical properties [2,68]. With the additional assumptions that, within this regime, the change of the kinetic energy per unit time is independent of k and the viscosity does not affect $E_i(k)$, dimensional arguments imply a $k^{-5/3}$ scaling of the incompressible kinetic energy spectrum, independent of spatial dimensionality [68,69]. This universal law is a direct reflection of the velocity-velocity autocorrelation function [2,70]. At higher momenta, the inertial range over which the Kolmogorov scaling extends is interrupted by a second power law, $E_i(k) \propto k^{-3}$, that reflects the short-range properties of the vortex core and is thus unrelated to turbulence [66,71]. As the radius of the vortex core is of the order of the condensate healing length $\xi_h := \sqrt{1/(2\mu_f)}$ [16,71,72], the inertial range lies below $k < 2\pi/\xi_h$. At large scales, it is further bounded by the mean intervortex distance l_v [66]. The width of this range is therefore determined by the quench speed, which sets the energy-injection scale through the universal Kibble-Zurek (KZ) correlation length $\hat{\xi} \sim l_v$. As a result, the extent of the Kolmogorov scaling shrinks for faster quenches and broadens for moderate ones. This effect is illustrated in Fig. 2, showing the incompressible energy spectra at equilibration for quenches of duration $\tau_Q = 100$ in (a) and $\tau_Q = 350$ in (b). The corresponding fits yield $E_i(k) = (176.555 \pm 14.316)k^{-1.710 \pm 0.079}$, $E_i(k) = (1028.257 \pm 123.069) \times k^{-3.025 \pm 0.030}$ in panel (a), and $E_i(k) = (37.722 \pm 1.723) \times k^{-1.654 \pm 0.031}$, $E_i(k) = (226.968 \pm 42.383)k^{-3.036 \pm 0.047}$ in (b), where in each panel the first and second expressions refer to the red dashed and black dotted lines, respectively. The power-law fitted exponents in the inertial range are thus in agreement with the Kolmogorov scaling value $5/3 \simeq 1.67$

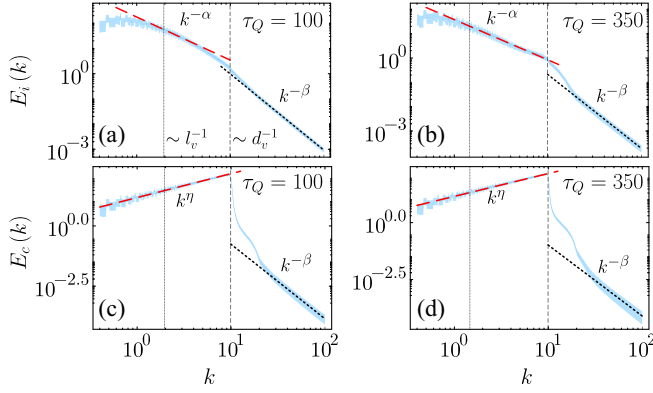


FIG. 2. Incompressible and compressible kinetic energy spectra. Panels (a) and (b) show the incompressible kinetic energy spectrum $E_i(k)$ at the equilibration time following quenches with $\tau_Q = 100$ and $\tau_Q = 350$, respectively, each averaged over $\mathcal{R} = 1000$ independent noise realizations. The corresponding fitted power-law exponents are $\alpha = 1.710 \pm 0.079$, $\beta = 3.025 \pm 0.030$ for (a), and $\alpha = 1.654 \pm 0.031$, $\beta = 3.036 \pm 0.047$ for (b). Panels (c) and (d) display the compressible spectrum $E_c(k)$ for the same quench protocols, with fit parameters $(\eta, \beta) = (0.992 \pm 0.008, 3.090 \pm 0.110)$ and $(1.002 \pm 0.008, 3.111 \pm 0.223)$ respectively. The vertical dashed lines mark $k = 2\pi/d_v$, where $d_v = 4\xi_h$ gives a good estimate of the vortex diameter in our system, while the vertical dotted lines indicate $k = 2\pi/l_v$, with l_v being the mean nearest intervortex distance. Shaded error bands correspond to 1 standard deviation.

within the numerical uncertainty. In BECs the inertial range is typically narrow, spanning no more than one decade due to the lack of a wide separation between characteristic length scales [17,65,68]. This limitation can be mitigated by analyzing the velocity structure functions, which, due to their self-similar nature, allow the extension of the Kolmogorov scaling beyond the inertial range. In particular, in [52] we compute the longitudinal velocity increments $S_p(r) = \langle |[\mathbf{u}_i(\mathbf{R} + \mathbf{r}) - \mathbf{u}_i(\mathbf{R})] \cdot \mathbf{r}/r|^p \rangle$ and find clear power-law relations $S_p(r) \propto [S_3(r)]^{\zeta(p)}$, with exponents that deviate from the K41 prediction $\zeta(p) = p/3$ [1,2,20,68,70,73–76], and are more accurately described by the refined K62 model [77]. While the KZM accurately describes the universal critical dynamics across the BEC transition, it does not account for other effects such as the annihilation of vortex-antivortex pairs, the losses of vortices at the edges of the trap, and coarsening. For slow quenches, the system evolves nearly adiabatically, resulting in suppressed vortex formation and diminished turbulence. As a consequence, the inertial range shrinks and the Kolmogorov scaling becomes less pronounced.

Although our main interest lies in the Kolmogorov scaling of the incompressible kinetic energy spectrum, we also plotted $E_c(k)$ at the equilibration time in Figs. 2(c) and 2(d). For a 2D system at thermodynamic equilibrium, it is reported that $E_c(k) \propto k$ for small k [18]. In addition, since $E(k)$

follows the k^{-3} law at large k [63,78], $E_c(k)$ is expected to follow the same power law. Our numerical results in Figs. 2(c) and 2(d) are in agreement with these expectations.

Kibble-Zurek universality of SQT—Having established that the Kolmogorov scaling describes thermal quenches for two different values of τ_Q , we now demonstrate the nonequilibrium universality dictated by KZM in SQT. To this end, let us consider the total incompressible kinetic energy E_i . Since the density-weighted velocity field $\mathbf{u}_i = \sqrt{\rho} \mathbf{v}_i$ is concentrated around the vortex cores, E_i can be approximated as a sum of isolated vortex contributions, $E_i \simeq (1/2)n_v \int_{\mathcal{D}_{\xi}} |\mathbf{u}_v(\mathbf{r})|^2 d^2\mathbf{r}$, where $|\mathbf{u}_v(\mathbf{r})| = \sqrt{\rho(\mathbf{r})}/r$, and the integration domain $\mathcal{D}_{\xi}$ is set by the KZ correlation length $\hat{\xi}$. At equilibration, the background density $\rho \simeq \mu(t_{\text{eq}})/g \propto \tau_Q^{-1/(1+z\nu)}$ and $\hat{\xi} \propto \tau_Q^{\nu/(1+z\nu)}$, so each vortex contributes $\int_{\mathcal{D}_{\xi}} |\mathbf{u}_v|^2 d^2\mathbf{r} \propto \tau_Q^{-1/(1+z\nu)}$ up to logarithmic corrections [52]. Combining this with the KZ vortex number scaling $n_v \propto \tau_Q^{-2\nu/(1+z\nu)}$ yields $E_i \propto \tau_Q^{-(1+2\nu)/(1+z\nu)}$ as confirmed numerically in the End Matter. Next, we consider the spectral density $E_i(k)$. Since the Kibble-Zurek length $\hat{\xi}$ sets the characteristic momentum scale at equilibration, it is natural to assume the scaling ansatz $E_i(k, \tau_Q) = A(\tau_Q)F(k\hat{\xi})$, where F encodes the spectral shape and $A(\tau_Q)$ is a k -independent amplitude. Using $E_i \propto \tau_Q^{-(1+2\nu)/(1+z\nu)}$ together with $\hat{\xi} \propto \tau_Q^{\nu/(1+z\nu)}$ gives $A(\tau_Q) \propto \tau_Q^{-(1+\nu)/(1+z\nu)}$, implying a collapse of the spectra across different quench rates under the rescaling

$$k \rightarrow k\tau_Q^{\nu/(1+z\nu)}, \quad E_i(k) \rightarrow E_i(k)\tau_Q^{(1+\nu)/(1+z\nu)}, \quad (4)$$

as shown in Fig. 3 for the mean-field values of the critical exponents. A universal collapse also emerges for the compressible spectrum $E_c(k)$, under a distinct rescaling, at low temperature where phonon emission from vortex-antivortex annihilation [79] dominates over thermal density fluctuations (see Ref. [52] for details). In addition, for k exceeding d_v^{-1} , the lack of collapse for $E_i(k)$ when rescaled by the KZ variables in the k^{-3} scaling regime is expected as it is governed by an equilibrium quantity, the vortex core radius, that is independent of the quench rate.

Deviations from the universal collapse can also be expected for fast quenches (e.g., $\tau_Q \leq 10$), for which the KZM breaks down [38,80] due to the saturation of the defect density shown in Fig. 1(c). However, such deviations may be taken into account by using the associated correlation length and energy scale, which scale universally with the quench depth rather than the quench time. Naturally, other nonuniversal processes beyond KZM and its extensions, governing the evolution after the formation of the BEC, such as atom losses,

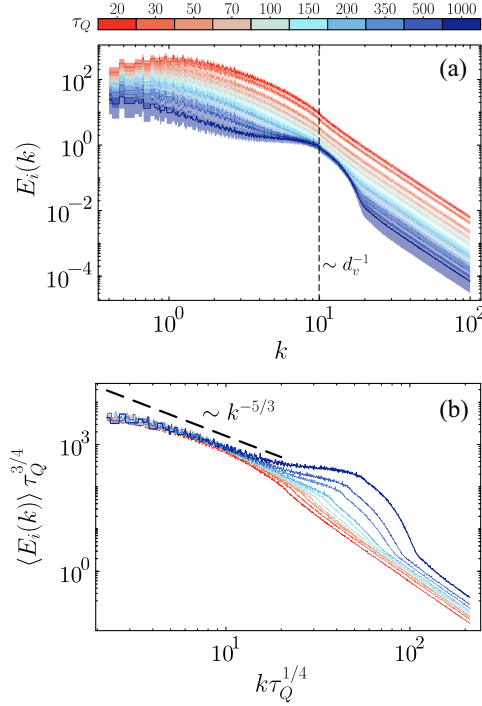


FIG. 3. Kibble-Zurek universality of the incompressible kinetic energy spectrum. Panel (a) shows $E_i(k)$ at equilibration time for various τ_Q values, each averaged over $\mathcal{R} = 1000$ stochastic realizations. The dashed vertical line marks $k = 2\pi/d_v$ with the vortex diameter estimated to be $d_v = 4\xi_h$. Panel (b) displays $\langle E_i(k) \rangle \tau_Q^{3/4}$ as a function of the scaled momentum $k\tau_Q^{1/4} \propto k\hat{\xi}$. Shaded error bands indicate 1 standard deviation in (a) and 95% confidence interval in (b).

vortex-antivortex annihilation, and coarsening are expected to suppress the Kolmogorov scaling and SQT.

Discussion and conclusion—In summary, we have shown that superfluid formation triggered by a finite-time quench gives rise to SQT. To this end, we have established that the incompressible kinetic energy of the emergent nonequilibrium BEC exhibits Kolmogorov scaling with respect to the wave vector. Moreover, using the nonequilibrium correlation length predicted by KZM and the associated energy scale, we have shown that the Kolmogorov scaling admits a universal collapse for different values of the quench time. Our findings highlight the rich interplay between distinct yet complementary notions of universality: the universal scaling of energy spectra in turbulent flows and the universal defect dynamics across continuous phase transitions.

Note added—During the completion of this work, Ref. [81] reported the study of a 4D superfluid with Kolmogorov scaling for the total energy, rather than the incompressible energy in the 2D case we discuss.

Acknowledgments—M.M. is grateful to Andr s Grabarits, Simon Fisher, Martin Gazo, and Zoran

Hadzibabic for valuable discussions. S.S. would like to further thank Sol Kim and Yong-il Shin for insightful comments, and the Institut d’ tudes Scientifiques de Carg se, for support and hospitality during the program Bridges over turbulent matters—Navigating across observations, concepts and models. The authors acknowledge financial support from the Luxembourg National Research Fund under Grant No. C22/MS/17132060/BeyondKZM. M. T. would like to thank Anusandhan National Research Foundation (ANRF), Government of India, for the financial support through the Prime Minister Early Career Research Grant with Grant No. ANRF/ECRG/2024/003150/PMS, and Christ University for funding the research through the seed grant, Sanction No. CU-ORS-SM-24/94. The numerical simulations presented in this work were carried out using the HPC resources of the University of Luxembourg (ULHPC).

Data availability—The data that support the findings of this article are openly available [82].

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End Matter

Universality of the incompressible kinetic energy—In the main text, we argued that the incompressible kinetic energy at equilibration time $E_i := (1/2) \int |\mathbf{u}_i(\mathbf{r}, t_{\text{eq}})|^2 d^2\mathbf{r} = \int_0^\infty E_i(k) dk$, scales with the quench duration as

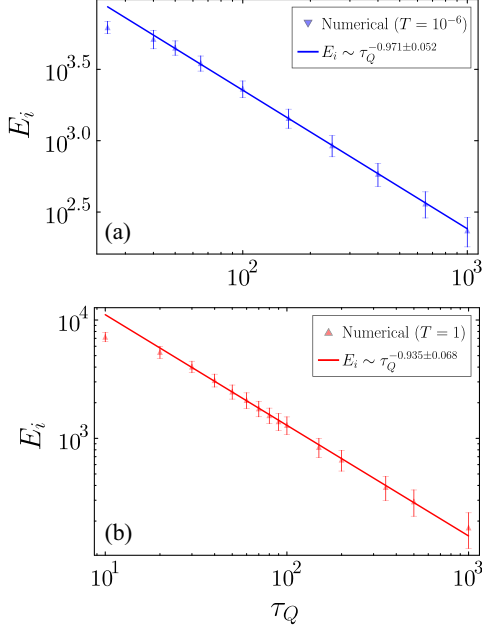


FIG. 4. Kibble-Zurek universality of the incompressible kinetic energy. Panels (a) and (b) show the incompressible kinetic energy E_i at the equilibration time for various quench times τ_Q at temperature $T = 10^{-6}$ and $T = 1$, respectively. Each data point is averaged over (a) $\mathcal{R} = 2000$ and (b) $\mathcal{R} = 1000$ stochastic realizations. In both panels, error bars denote 1 standard deviation.

$$E_i \propto \tau_Q^{-(1+2\nu)/(1+z\nu)}. \quad (\text{A1})$$

This follows from the observation that E_i can be approximated by a sum over the n_v vortex contributions, $E_i \simeq (1/2)n_v \int_{\mathcal{D}_{\hat{\xi}}} |\mathbf{u}_v(\mathbf{r})|^2 d^2\mathbf{r}$, where the integration domain $\mathcal{D}_{\hat{\xi}}$ is set by the KZ length $\hat{\xi}$, and the weighted velocity field of a single vortex is $\mathbf{u}_v(\mathbf{r}) = \sqrt{\rho(\mathbf{r})} \mathbf{e}_\phi / r$ with $\mathbf{e}_\phi$ being the unit vector along the azimuthal angle. We evaluate the single-vortex integral by assuming a vanishing condensate density within the core of radius r_v , and a uniform background value $\rho = \mu/g$ outside. This yields

$$\int_{\mathcal{D}_{\hat{\xi}}} |\mathbf{u}_v(\mathbf{r})|^2 d^2\mathbf{r} \simeq 2\pi\rho \int_{r_v}^{\hat{\xi}} \frac{1}{r} dr = 2\pi\rho \ln\left(\frac{\hat{\xi}}{r_v}\right). \quad (\text{A2})$$

The factor $\ln(\hat{\xi}/r_v)$ grows only logarithmically with $\hat{\xi}$, making it a subleading correction to the power law (A1), which is governed by the scaling of n_v and ρ at equilibration time, as discussed in the main text. We tested the prediction in Eq. (A1) numerically for two different temperature values in Fig. 4, and the results support this scaling within numerical uncertainty. The fitted power laws in Fig. 4 are $E_i = (19.774 \pm 5.010) \times 10^4 \tau_Q^{-0.971 \pm 0.052}$ for panel (a) and $E_i = (9.543 \pm 2.951) \times 10^4 \tau_Q^{-0.935 \pm 0.068}$ in panel (b), confirming the predicted τ_Q^{-1} behavior based on the mean-field values of the critical exponents $\nu = 1/2$ and $z = 2$. For all fits with exact independent variables, we use weighted least squares with weights given by the inverse variances.