

**Dwarf Galaxies Imply Dark Matter Is Heavier than  $2.2 \times 10^{-21}$  eV**Tim Zimmermann<sup>1,\*</sup> James Alvey<sup>2,3,4,†</sup> David J. E. Marsh<sup>5,‡</sup> Malcolm Fairbairn<sup>5,§</sup> and Justin I. Read<sup>6,||</sup><sup>1</sup>*Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, Oslo, Norway*<sup>2</sup>*Kavli Institute for Cosmology Cambridge, Madingley Road, Cambridge CB3 0HA, United Kingdom*<sup>3</sup>*Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, United Kingdom*<sup>4</sup>*GRAPPA Institute, Institute for Theoretical Physics Amsterdam, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*<sup>5</sup>*Theoretical Particle Physics and Cosmology, King's College London, Strand, London WC2R 2LS, United Kingdom*<sup>6</sup>*Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom*

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It is widely established that a lower bound on the dark matter particle mass,  $m$ , can be obtained by demanding that the de Broglie wavelength in a given galaxy must be smaller than the virial radius of the galaxy, leading to  $m \gtrsim 10^{-22}$  eV when applied to typical dwarf galaxies. This lower limit has never been derived precisely or rigorously. We use stellar kinematical data for the Milky Way satellite galaxy Leo II to self-consistently reconstruct a statistical ensemble of dark matter wave functions and corresponding density profiles. By comparison to a data-driven, model-independent reconstruction, and using a variant of the maximum mean discrepancy as a statistical measure, we determine that a self-consistent description of dark matter in the local Universe requires  $m > 2.2 \times 10^{-21}$  eV (CL > 95%). This lower limit is free of any assumptions pertaining to cosmology, microphysics (including spin), or dynamics of dark matter, and only assumes that it is predominantly composed of a single bosonic particle species.

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**Introduction**—Astrophysical evidence for a large amount of gravitationally interacting matter, which cannot be explained in the context of the standard model of particle physics and general relativity, has been accumulating for more than 100 yr [1,2]. Nonetheless, the identity and precise nature of dark matter (DM) remains one of the biggest questions in our understanding of the Universe. There is a myriad of evidence—coming from, e.g., observations of the cosmic microwave background [3], galaxy clusters (e.g., [4]), large-scale structure (e.g., [5]), or stellar kinematics (e.g., [6])—for the existence of such a nonbaryonic matter component. To date, however, no definitive laboratory or nongravitational signature has been observed. As such, despite the efforts of direct detection facilities (e.g., [7,8]), large-scale cosmology surveys [3–5], and telescopes (see Ref. [9] for a review of indirect detection searches), the dark matter parameter space remains wide open.

If dark matter is thought of as dominantly composed of a single particle or composite object, it is natural to categorise DM according to the mass,  $m$ , of the constituent. At the highest end, we have macroscopic objects, such as black holes or other massive objects. Dynamical effects of these objects on star clusters, for example, lead to an upper limit on the allowed mass in the range of 10 solar masses ( $M_\odot$ ) [10]. For  $m < M_{\text{Pl}}$ , where  $M_{\text{Pl}}$  is the Planck mass, DM may be composed of a new fundamental particle. For decades, experimental programs have been developed to search for such particles, for example scattering of DM with  $m \approx 1$  GeV off atomic nuclei (e.g., [7]), or resonant production of radio waves in microwave cavities for DM with  $m \approx 1$   $\mu$ eV (e.g., [8]).

In this Letter we ask the simple question: what is the smallest value  $m$  can possibly take, consistent with kinematical considerations? Specifically, we are interested in the implications of kinematical observations of Milky Way dwarf spheroidal galaxies.

If DM is a fermion, then the resulting lower limit is known as the Tremaine-Gunn bound [11]. As a result of the Pauli exclusion principle, fermions cannot multiply occupy states in phase space, and so bounding the distribution function  $f \leq 1$ , one can estimate a lower limit on  $m$  given the mass and radius (and hence virial equilibrium velocity) of a galaxy. Modeling a dwarf galaxy as a homogeneous, spherically symmetric and degenerate Fermi gas with  $M \simeq 10^8 M_\odot$  and  $R \simeq 1.0$  kpc, this gives an estimated lower

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bound  $m_f \gtrsim 120$  eV [12]. This limit can be improved by accurately reconstructing the phase space distribution function via the observed kinematics of tracer stars hosted by the Milky way dwarfs, which leads to  $m_f > 130$  eV at the 95% confidence level (CL) [13]. However, this limit does not apply if DM is a boson.

For a boson, a limit as fundamental as the fermionic Tremaine-Gunn bound can be estimated by invoking the uncertainty principle  $m\sigma_v\sigma_r \gtrsim 3/2 \times \hbar$  [14]. For a steady-state, spherically symmetric object like a dwarf galaxy composed of bosonic DM to exist, its spatial extent  $r_{\text{vir}}$  and characteristic velocity dispersion  $\sigma_v$  must satisfy  $m \gtrsim 3/2 \times \hbar(\sigma_v r_{\text{vir}})^{-1}$ . If we estimate the virial radius,  $r_{\text{vir}}$ , and velocity dispersion,  $\sigma_v$ , for the Milky Way dwarf Leo II [15] as  $r_{\text{vir}} \simeq 9$  kpc and  $\sigma_v \simeq 15$  km s $^{-1}$  [16,17] (As is customary, we set  $r_{\text{vir}} = r_{200}$ , i.e., the radius within which the mean density equals 200 times the critical density, and estimate  $\sigma_v$  via the scalar virial theorem [18]:  $\sigma_v^2 = 2\pi M_{\text{tot}}^{-1} |\int dr r^2 \rho(r) V(r)|$ ), this leads to  $m \gtrsim 2 \times 10^{-23}$  eV. This is at the same order of magnitude as the commonly assumed lower limit on the DM particle mass. In the rest of this Letter, we work to derive a more rigorous limit, which will turn out to be 2 orders of magnitude stronger. In particular, we strengthen the limit to  $m > 2.2 \times 10^{-21}$  eV (CL > 95%). Of course, there are a number of other relevant limits on ultralight dark matter candidates; see, in particular, Refs. [19–21]. Such limits, however, depend on details of cosmology, nonlinear gas physics, and/or long-time dynamics, all of which our limit is independent of. We postpone a detailed comparison and discussion to the end of this Letter.

Input to our rigorous analysis is a set of 5000 stationary and spherically symmetric density-potential pairs derived from stellar kinematical data of Leo II. Our mass limit is then derived in two steps: For each potential, we reconstruct a DM wave function as an exhaustive expansion of its energy eigenstates. We then compare the set of reconstructed wave functions and data-driven input densities at the population level and vary the boson’s mass  $m$  until both sets are statistically indistinguishable. We formalize this idea as a nonparametric, two-sample hypothesis test based on the *fused maximum mean discrepancy* [22,23] for functional data [24]. The analysis relies on JAXSP, our differentiable and scalable wave function reconstruction tool based on a semianalytical treatment of Schrödinger’s equation. We describe JAXSP in the Supplemental Material [25].

**Leo II dSph data**—To carry out the wave function fit, we require robust data-driven reconstructions of the DM density profiles within dwarf galaxies. Specifically, we need a statistical ensemble of spherically symmetric DM density profiles  $\rho(r)$  that are consistent with the photometric and kinematical measurements of the stellar tracer population in Leo II [15]. For this we use GRAVSPHERE [16,17,46,47], a Markov-chain-Monte-Carlo sampler that infers realizations of  $\rho$  by solving the spherical

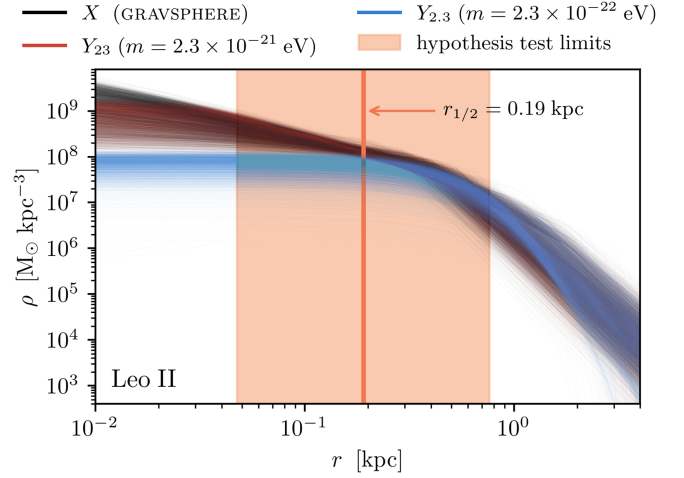


FIG. 1. Direct comparison of different density ensembles for the Milky way dwarf Leo II. Black curves depict density samples generated by the Jeans code GRAVSPHERE and act as data-driven input set  $X$  to our analysis pipeline. The pipeline output is a population  $Y_{m_{22}}$  of reconstructed DM wave functions densities  $\langle |\psi|^2 \rangle$  at boson mass  $m = m_{22} \times 10^{-22}$  eV (red and blue curves). Higher mass eigenstates enjoy a stronger spatial localization, and are therefore able to resolve more structure of a cuspy input density at small radii. Testing this small scale discrepancy in the validity region of GRAVSPHERE (orange sector) at the population level, cf. Fig. 3, summarizes the key idea behind our lower mass limit.

Jeans equation [18,48,49]. To mitigate the well-known degeneracy between the halo density and the stellar velocity anisotropy inherent to Jeans modeling [50–53], GRAVSPHERE incorporates higher order velocity moments, so-called virial shape parameters [50,54,55], into its likelihood. The results of this analysis are posterior parameter samples for the highly flexible, CORENFWTIDES-density model,  $\rho_{\text{CNFWT}}$ . This density model augments the canonical NFW profile [56] with four additional parameters. At small radii, a “coredness” parameter,  $0 \leq n \leq 1$ , and core radius  $r_c$  permit us to interpolate between a perfect core  $\rho(r \ll r_c) \sim \rho_0$  or cusp  $\rho(r \ll r_c) \sim r^{-1}$ . At large radii, the introduction of a tidal radius  $r_t$  and power law decay  $\rho(r \gg r_t) \sim r^{-\delta}$  allow for an effective model of tidal forces stripping peripheral mass away, if a larger host halo is present. This choice to allow for an inner core component accommodates the presence of unmodeled systematics such as, e.g., baryonic feedback or wavelike-DM dynamical heating [57,58]. Since a more cored profile leads to weaker limits within our method, this renders this modeling choice a conservative one. We discuss the impact of additional technical modeling choices, such as the spatial cutoff and sphericity assumptions in the Supplemental Material [25].

Our choice of Leo II was motivated by its ability to set a strong limit due to its steep inner profile, and its statistically robust description as a static [59,60], spherically symmetric distribution [60–63] with a well measured cusp. Draco is another possible candidate object [64], however, it is computationally more challenging with the current iteration

of our method [25]. We have explicitly checked in a pared-down analysis that Draco gives a consistent limit with Leo II.

Figure 1 displays the 5000 posterior samples for Leo II (black lines) that we use in our analysis. Note that Collins *et al.* [17] also validate GRAVSPHERE’s mass modeling on the spherical mock data described in Ref. [65] and find it to recover the true density within the 95% confidence interval of the density samples over the radial range  $0.25 \leq r/r_{1/2} \leq 4$  (orange sector in Fig. 1) around the half-light radius  $r_{1/2}$  even if only  $O(100)$  tracers are available. We adopt this radial range for our analysis.

*Self-consistent reconstruction of the DM wave function*—In the nonrelativistic limit, the dynamics of spin zero, bosonic DM with  $m \ll O(10)$  eV are described by a complex scalar  $\psi(\mathbf{x}, t)$  that obeys the Schrödinger-Poisson (SP) equation [66] and sources its gravitational potential  $V$  via the DM density  $\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$ . Assuming stationary conditions, i.e., no explicit time dependency in  $\rho$  and  $V$ , and restricting to spherically symmetric objects, the linear set of equations

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial r^2} - \frac{l(l+1)}{r^2} \right) u_{nl} + mV u_{nl} = E_{nl} u_{nl},$$

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) V = 4\pi G \rho, \quad (1)$$

with eigenmode  $\psi_{nlm}(\mathbf{x}, t) = r^{-1} u_{nl}(r) Y_l^m(\phi, \theta) e^{iE_{nl}t/\hbar}$  has proven to be an effective approximation for the equilibrium phenomena encapsulated in the full-fledged SP equation [67–69].

From a practical perspective, and following Refs. [69,70], Eq. (1) breaks the nonlinear relation between wave function  $\psi$  and potential  $V$ . In addition, it approximates  $\rho$  as a stationary density that is *independent* of the wave function. In this way, Eq. (1) is equivalent to a linear order perturbative treatment of  $\psi$  with  $V$  as the zeroth-order potential [68].

The full process of this reconstruction of the wave function, [25], may be seen as an evolution of the approaches presented in Refs. [69,70]. Here we recall the main three steps: First, given a density realization, we compute its time-independent gravitational potential. We then compute an extensive library of eigenstates that contains all modes  $u_{nl}$  with a classically allowed region within  $r_{99}$ , i.e., the radius enclosing 99% of the total mass. This involves repeatedly solving Eq. (1) for increasing values of the angular momentum quantum number  $l$ —a task we solve in practice by application of a piecewise, perturbative approximation [25]. Lastly, we establish self-consistency between the density sample  $\rho_{\text{cNFWt}}$  and the steady-state contribution  $\langle |\psi|^2 \rangle = (4\pi r^2)^{-1} \sum_{nl} (2l+1) |a_{nl}|^2 u_{nl}^2(r)$ , [69], by minimizing an objective function of our choice over the mode coefficients  $|a_{nl}|$ —in this work the symmetric Jensen-Shannon divergence. (Together with the eigenstate library construction and mode coefficient optimisation, for a single representative

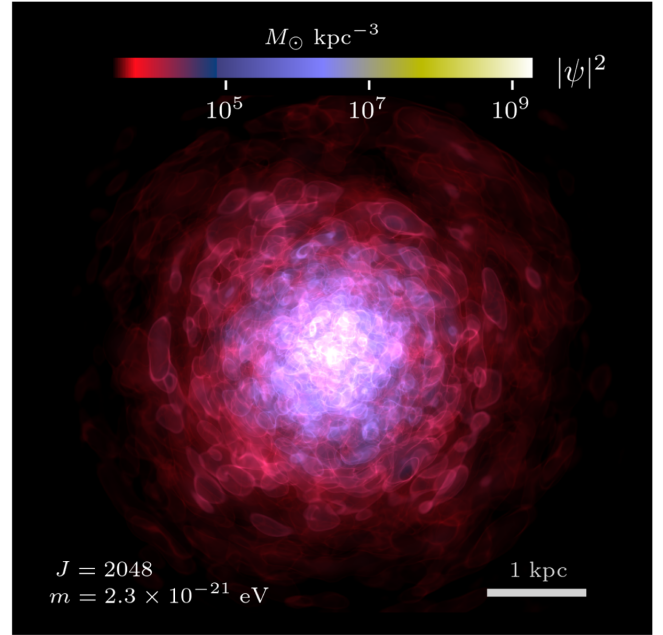


FIG. 2. Volume rendering of the total wave function density  $|\psi|^2$  reconstructed from the average profile  $\langle \rho_{\text{cNFWt}} \rangle$  of Fig. 1 for the allowed boson mass  $m = 2.3 \times 10^{-21}$  eV according to the hypothesis test in Fig. 3.  $J$  denotes the total number of radial modes  $u_{nl}$  in the eigenstate expansion.

mass  $m = 2.3 \times 10^{-21}$  eV, our pipeline takes  $O(2)$  days) to run on 100 CPU cores for 5000 profiles. Alongside this, there is typically a significant memory footprint of  $O(2)$  TB when holding the eigenstate libraries in memory during the optimization step.)

As a concrete example, in Fig. 1 we show the set of reconstructed  $\langle |\psi|^2 \rangle$  for the input densities  $\rho_{\text{cNFWt}}$  from GRAVSPHERE at two different boson masses— $m = 2.3 \times 10^{-22}$  eV (excluded by our analysis) and  $m = 2.3 \times 10^{-21}$  eV (not excluded). In Fig. 2, we show the full wave function  $|\psi(r, \phi, \theta)|^2 = \langle |\psi|^2 \rangle(r) + \chi(r, \phi, \theta)$ , including the interference cross term  $\chi$ . The presence of  $\chi$  induces intriguing physics in its own right and may be used to put constraints on the boson mass. Our limit is derived exclusively from the time independent background contribution  $\langle |\psi|^2 \rangle$ .

*How to exclude boson masses*—It is perhaps tempting to incorporate an eigenstate expansion approach into a textbook Jeans analysis, i.e., define a parametrized potential, construct the corresponding eigenstate library, and incorporate  $\langle |\psi|^2 \rangle$  into a likelihood to sample the full posterior distribution of the mode coefficients. Unfortunately, this is computationally intractable. Depending on the boson mass and depth of the potential well, eigenstate library sizes of  $J = O(10^4)$  are common, rendering standard sampling techniques ineffective.

The question then is how to determine if a given boson mass self-consistently reproduces the true distribution of

density profiles as defined by the GRAVSPHERE Jeans analysis. In order to answer this question, we construct a nonparametric, two-sample hypothesis test that ascertains whether the set of GRAVSPHERE functions,  $X = \{\rho_{\text{cNFWt},j}\}_{j=1}^{5000}$ , and the set of wave function densities for boson mass  $m$ ,  $Y_m = \{|\psi|^2\}_j\}_{j=1}^{5000}$ , are drawn from the same underlying distribution. Our null hypothesis is therefore  $H_0: p = q_m$ , where we denote  $p$  as the “posterior distribution over density profiles consistent with Leo II stellar kinematic data” and  $q_m$  as “the corresponding distribution of reconstructed density profiles for a fixed boson mass  $m$ .” Our (lower) limit on  $m$  at confidence level (CL)  $(1 - \alpha)$  is then obtained once the null hypothesis cannot be rejected at the  $\alpha$  significance level. Intuitively, this happens at a mass with eigenstate libraries flexible enough to recover all of the GRAVSPHERE densities equally well, irrespective of whether they are cored or cusped.

There are two technical hurdles that we need to overcome to apply this test in practice. First, we need to define a test statistic  $\tau(X_1, X_2)$  over two samples of density profiles,  $X_{1,2}$ . Second, at a given boson mass  $m$ , we need to derive the distribution of  $\tau$  under the null hypothesis,  $p(\tau|H_0)$ , where  $X_{1,2}$  are both drawn from the same distribution. With this, we can then evaluate the test statistic on the “observed” samples  $\tau_o = \tau(X, Y_m)$  and compare it to the critical values of the distribution  $p(\tau|H_0)$  at the desired significance level.

In this work, we use an estimator for the *fused maximum mean discrepancy* (MMD) as a test statistic [22–24,71]. (In the context of two-sample, distribution equality testing, [23] benchmark the fused maximum mean discrepancy against a selection of state-of-the-art test statistics, and find it to be on par or outperform them in terms correctly rejecting  $H_0$ .) We refer to the Supplemental Material [25] for full technical details, however, it can be shown that MMD constitutes a metric on the space of probability distributions and that  $\tau(X, Y_m) \xrightarrow{N \rightarrow \infty} \text{MMD}(p, q_m)$ , thus providing an intuitive “distance-based” interpretation for  $\tau$ . As such, we can reject  $H_0$  under the observation of sufficiently *large* values of  $\tau_o$ .

With the test statistic chosen, we follow Ref. [23] and compute  $p(\tau|H_0)$  by using a permutation resampling, i.e., we randomly shuffle the combined set  $Z = (X, Y_m)$  (mixing GRAVSPHERE and wave function densities), split into two equal parts,  $X_1 \cup X_2 = Z$ , and compute  $\tau(X_1, X_2)$ . We repeat this process  $\mathcal{O}(10^5)$  times to obtain an empirical estimate of the cumulative distribution (CDF) of  $p(\tau|H_0)$  and reject  $H_0$  if the observed value of  $\tau_o$  exceeds the  $(1 - \alpha)$  quantile of the CDF (One may show, [72], that this resampling strategy is actually conservative, i.e. constructing the distribution of  $\tau$  in this fashion can only weaken the test. We have checked explicitly, however, that using e.g. only the GRAVSPHERE functions in the permutation approach we obtain almost identical results.). A more

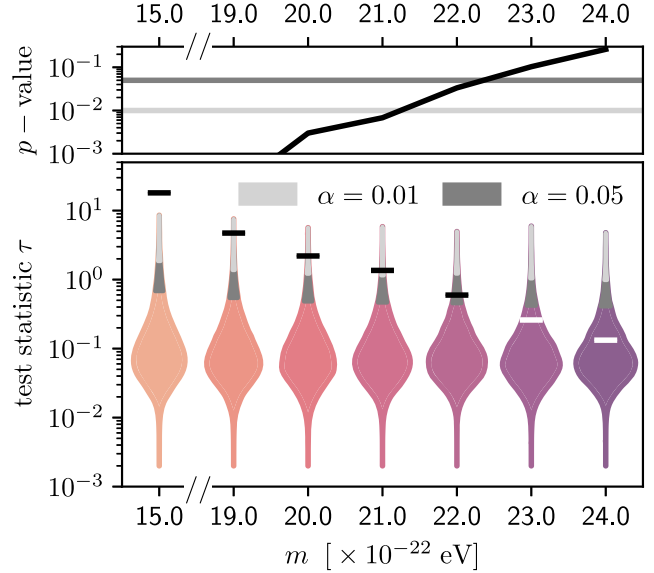


FIG. 3. Nonparametric, two-sample hypothesis test for distribution equality based on the fused maximum mean discrepancy  $\tau(Z)$  as test statistic. Shown is the distribution of  $\tau(Z)$  under the null hypothesis obtained by a permutation resampling. Areas in shades of gray illustrate the rejection regions at significance  $\alpha$ . Black and white bars depict the realized value of the test statistic. We determine  $m > 2.2 \times 10^{-21}$  eV as the first mass compatible with the null hypothesis at CL  $> 95\%$ .

intuitive example of this test applied to the coredness parameter in the CORENFWTIDES profile is provided in the Supplemental Material [25].

**Results**—Figure 3 depicts the distribution of the test statistic  $\tau$  under the null hypothesis  $H_0$ , and the critical values of this distribution at the  $\alpha = 1\%, 5\%$  levels. For a set of boson masses  $m \in [15, 24] \times 10^{-22}$  eV, we evaluate the test statistic  $\tau_o$  (shown by the black and white horizontal lines in the figure) and compare them to the critical values of the distribution. As expected, we see that at larger masses,  $\tau_o$  is fully consistent with the distribution under  $H_0$ . In this regime, the reconstructed wave functions are statistically indistinguishable from the GRAVSPHERE density profiles. On the other hand, at low masses, the eigenstate superposition is unable to reproduce key features (in particular, the cuspiest profiles) across the samples, translating into a statistical discrepancy and rejection of  $H_0$ .

The key result of this work is that we find  $m > 2.2 \times 10^{-21}$  eV (CL  $> 95\%$ ). This result is largely unaffected by moderate changes of the radial range on which the hypothesis test is conducted. Also, evaluating the two-body relaxation rate at this mass gives a timescale much longer than the age of the Universe [73], making it fully consistent with our modeling of Leo II as stationary.

Our robust lower limit on the DM mass is 2 orders of magnitude stronger compared to a naive estimate based on the uncertainty principle. The reported improvement may

be intuitively understood as a consequence of applying the uncertainty condition at the level of individual eigenstates rather than at the level of the overall wave function: Each eigenstate,  $\psi_j$ , has a spatial extent set by its uncertainty  $\sigma_r^2 = \langle \psi_j | r^2 | \psi_j \rangle - \langle \psi_j | r | \psi_j \rangle^2$  and thus contributes to the dwarf density at that particular length scale. The inferred Leo II DM density is cuspy, i.e., well modeled by a steep power law, on certain length scales. If the length scale of this cusp is comparable to the position uncertainty  $\sigma_r$  of the wave function, then the eigenstate expansion will not be able to adequately reproduce the density profile. To compensate, the spatial extent of all modes must shrink, requiring an increased boson mass  $m$ , and tightening the lower limit. If the dark matter distribution would be cored, a lower mass would suffice to reproduce the density ensemble at small radii. We checked this explicitly by applying our analysis to Fornax [74] yielding a weakened limit of  $m > 4 \times 10^{-22}$  eV (broadly consistent with related studies [53,75]).

*Significance and generality of the result*—There are, of course, many other lower limits on the DM mass that rely on similar physics to that which has been used in this work. We mention a few here for comparison. If the DM relic density arises from coherent motion in a harmonic potential, then the cosmic microwave background anisotropies constrain  $m > 10^{-24}$  eV [76], which is improved to  $m > 10^{-23}$  eV with weak gravitational lensing [77] and to  $m > 2 \times 10^{-20}$  eV including modeling of intergalactic gas in the Lyman-alpha forest [19]. The same cosmological models can be used to make predictions about galaxy number counts, e.g., Ref. [78] finds  $m > 2.9 \times 10^{-21}$  eV in order to produce the abundance of satellite galaxies of the Milky Way, while Ref. [79] finds  $m > 2.5 \times 10^{-22}$  eV to explain high redshift galaxies observed by Hubble and James Webb space telescopes. All of these lower limits are less generic than ours since they rely on assumptions about cosmology and/or complex nonlinear dynamics.

In addition, the physics of the SP system gives rise to a rich phenomenology of nonlinear wave effects [80]. Dynamical heating of the stellar tracer population [73] in Eridanus-II and Segue-I and II leads to the lower limit  $m > 3 \times 10^{-19}$  eV [20,21]. This limit is less fundamental than ours, since it relies on dynamics over billions of years (it is dynamical, rather than kinematical), and could be strongly affected by unmodeled events in the history of the galaxy.

We computed our lower limit for a boson with spin  $s = 0$ . Higher spin bosonic DM is also of interest (see, e.g., Ref. [81]). In such a case, there are copies of the DM wave function for each polarization state. Taking the superposition of such terms to form the total density reduces the amplitude of the interference term  $\chi(r, \phi, \theta)$ , but does not affect the spherically averaged density  $\langle |\psi^2| \rangle$  [82–84]. Thus our lower limit applies equally to  $s = 0$  and to higher spin DM.

There are a number of studies of the Milky Way satellites and other galaxies, which attempt to fit a model inspired by the behavior of ultralight bosons, namely, an outer power-law piece, with an inner solitonic core [80]. The amplitude of the ground state wave function in our analysis effectively accounts for the presence of a soliton with an arbitrary “core-halo” relation [85,86] (displacement of the soliton from the halo center in simulations is small [87] and should not affect our spherical halo assumption). For example, Refs. [53,80,88–93] also use Jeans analysis or simplifications thereof, while Ref. [94] fits rotation curves. All of these works find *preferred* values of  $m$  that fit the cored density profiles of certain galaxies. When taken globally, the smallest cores lead to results that are in tension with the observed masses of larger galaxies, leading to lower limits or very large preferred values for  $m$  as in Refs. [91–93]. It is well known, however, that cores can be explained by other physics, such as stellar heating [95]. For the first time in this context, we used a fully self-consistent density profile comprised of energy eigenstates instead of the heuristic composite, and thus our limit is unaffected by uncertainties on the core-halo relation [86]. In essence, our robust limit comes down to fitting the inner slope of a noncored object, which is therefore also unaffected by any baryonic physics that may lead to cores in other galaxies.

*Conclusions*—We have developed a robust and advanced methodology, both computationally and statistically, to set the strongest *fundamental* lower limit to date on the DM particle mass,  $m > 2.2 \times 10^{-21}$  eV. Our methodology could be extended in future to constrain the fraction of DM with  $m < 2.2 \times 10^{-21}$  eV by including an additional cold component in the gravitational potential. Such constraints on mixed DM may be competitive with cosmological bounds, and if accurate to  $\mathcal{O}(10\%)$  may probe interesting high energy physics models [75,96]. This would also relax the only assumption about the composition of DM in our analysis.

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*Data availability*—The JAXSP library is available at [97]. In addition, the scripts to generate the results in this work can be found at [98].

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