Erratum: Stabilizing Mechanism for Bose-Einstein Condensation of Interacting Magnons in Ferrimagnets and Ferromagnets [Phys. Rev. Lett. 121, 187202 (2018)]

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We correct an error and a typo in the analyses of the stability of magnon Bose-Einstein condensation (BEC) in the three cases considered in our Letter. The equations of the Bogoliubov transformation in the case of the two-sublattice ferromagnet are incorrect. After correcting this error, we find that the stability of the BEC of interacting magnons for the two-sublattice ferromagnet changes from stable to unstable. As a result, our conclusion should be altered as follows: not the sublattice structure, but the ferrimagnetic spin structure is the key ingredient for stabilizing the magnon BEC in the ferrimagnet. Namely, the difference between the spin structures of the ferrimagnet and the ferromagnet is the origin of the difference that the BEC of interacting magnons is stable in the case of the ferrimagnet, whereas it is unstable in the case of the one-sublattice ferromagnet. The typo is about the Hamiltonian of the magnon-magnon interaction in the case of the one-sublattice ferromagnet, and it does not affect any results and conclusions.

The error appears in the equations of the Bogoliubov transformation in the case of the two-sublattice ferromagnet, the third case of our model. These equations should be replaced by

$$a_{\boldsymbol{q}} = c_{\boldsymbol{q}}\alpha_{\boldsymbol{q}} - s_{\boldsymbol{q}}\beta_{\boldsymbol{q}}, \tag{1}$$

$$b_{\boldsymbol{q}} = s_{\boldsymbol{q}}\alpha_{\boldsymbol{q}} + c_{\boldsymbol{q}}\beta_{\boldsymbol{q}}, \tag{2}$$

where $c_{\boldsymbol{q}} = \cos\theta_{\boldsymbol{q}}$ and $s_{\boldsymbol{q}} = \sin\theta_{\boldsymbol{q}}$, which satisfy $\tan 2\theta_{\boldsymbol{q}} = [2\sqrt{S_A S_B}J(\boldsymbol{q})/(S_A - S_B)J(\boldsymbol{0})]$. This correction changes $\epsilon_\alpha(\boldsymbol{q})$ and $\epsilon_\beta(\boldsymbol{q})$ in the third case as follows:

$$\epsilon_\alpha(\boldsymbol{q}) = J(\boldsymbol{0})(S_A + S_B) - \Delta\epsilon(\boldsymbol{q}), \tag{3}$$

$$\epsilon_\beta(\boldsymbol{q}) = J(\boldsymbol{0})(S_A + S_B) + \Delta\epsilon(\boldsymbol{q}), \tag{4}$$

where $\Delta\epsilon(\boldsymbol{q}) = \sqrt{J(\boldsymbol{0})^2(S_A - S_B)^2 + 4S_A S_B J(\boldsymbol{q})^2}$. Since these equations show that $\epsilon_\alpha(\boldsymbol{0}) = 0$ and $\epsilon_\beta(\boldsymbol{0}) = 2J(\boldsymbol{0})(S_A + S_B)$, the stability of the BEC of interacting magnons in this case can be analyzed in a way similar to that used in the first case considered in our Letter. By doing the similar calculations, we obtain the effective Hamiltonian to describe the BEC of interacting magnons in the third case, which is of the same form as that shown in our Letter: $H_{\mathrm{eff}} = \sum_{\boldsymbol{q}} \epsilon_\alpha^*(\boldsymbol{q})\alpha_{\boldsymbol{q}}^\dagger\alpha_{\boldsymbol{q}}$, where $\epsilon_\alpha^*(\boldsymbol{q}) = \epsilon_\alpha(\boldsymbol{q}) - (4/N)\sum_{\boldsymbol{q}'}\tilde{\Gamma}_{\alpha\alpha}(\boldsymbol{q},\boldsymbol{q}')n_{\boldsymbol{q}'\alpha}$ and $n_{\boldsymbol{q}'\alpha} = \langle\alpha_{\boldsymbol{q}'}^\dagger\alpha_{\boldsymbol{q}'}\rangle$. The changes due to the above correction appear only in $\tilde{\Gamma}_{\alpha\alpha}(\boldsymbol{q},\boldsymbol{q}')$:

$$\tilde{\Gamma}_{\alpha\alpha}(\boldsymbol{q},\boldsymbol{q}') = J(\boldsymbol{0})(c_{\boldsymbol{q}}^2 s_{\boldsymbol{q}'}^2 + c_{\boldsymbol{q}'}^2 s_{\boldsymbol{q}}^2) + 2J(\boldsymbol{q}-\boldsymbol{q}')c_{\boldsymbol{q}}s_{\boldsymbol{q}}c_{\boldsymbol{q}'}s_{\boldsymbol{q}'} - \frac{J(\boldsymbol{q})}{\sqrt{S_A S_B}}c_{\boldsymbol{q}}s_{\boldsymbol{q}}(S_A s_{\boldsymbol{q}'}^2 + S_B c_{\boldsymbol{q}'}^2) - \frac{J(\boldsymbol{q}')}{\sqrt{S_A S_B}}c_{\boldsymbol{q}'}s_{\boldsymbol{q}'}(S_A s_{\boldsymbol{q}}^2 + S_B c_{\boldsymbol{q}}^2). \tag{5}$$

Namely, the signs of the third and fourth terms in the right-hand side have changed from plus to minus, as well as the changes in $c_{\boldsymbol{q}}$, $s_{\boldsymbol{q}}$, $c_{\boldsymbol{q}'}$, and $s_{\boldsymbol{q}'}$; in the correct Bogoliubov transformation, $c_{\boldsymbol{q}} = \cos\theta_{\boldsymbol{q}}$ and $s_{\boldsymbol{q}} = \sin\theta_{\boldsymbol{q}}$, as described above. Estimating the above $\tilde{\Gamma}_{\alpha\alpha}(\boldsymbol{q},\boldsymbol{q}')$ in the long-wavelength limits, we obtain $\tilde{\Gamma}_{\alpha\alpha}(\boldsymbol{q},\boldsymbol{q}') \sim \frac{2}{9}J(\boldsymbol{0})q^2q'^2[(S_A S_B)^2/(S_A + S_B)^4]$, which means that the leading term of the magnon-magnon interaction is attractive. Thus, for the two-sublattice ferromagnet, the magnon BEC becomes unstable in the presence of the magnon-magnon interaction.

Combining the above result with the results in the first and second cases considered in our Letter, we should change our conclusion as follows: the ferrimagnetic spin structure is the key to stabilizing the magnon BEC in the ferrimagnet. Namely, the sublattice structure is not important. (Note that the BEC of interacting magnons is stable in the first case, the case of the two-sublattice ferrimagnet, whereas it is unstable in the second case, the case of the one-sublattice ferromagnet.)

This conclusion suggests that the ferrimagnetic spin structure is needed to be taken into account in describing the magnon BEC of yttrium iron garnet, a three-dimensional ferrimagnet.

The typo appears in the equation of the Hamiltonian of the magnon-magnon interaction H_{int} in the second case of our model, the case of the one-sublattice ferromagnet: the third term in the square bracket of H_{int}, which was $-2J(\boldsymbol{q})c^{\dagger}_{\boldsymbol{q}'}c_{\boldsymbol{q}}c^{\dagger}_{\boldsymbol{q}'}c_{\boldsymbol{q}}$, should be replaced by $-2J(\boldsymbol{q})c^{\dagger}_{\boldsymbol{q}'}c_{\boldsymbol{q}}c_{\boldsymbol{q}'}c^{\dagger}_{\boldsymbol{q}}$. This typo does not affect any results and conclusions.

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