Erratum: Quantum Phase Transitions in the Sub-Ohmic Spin-Boson Model: Failure of the Quantum-Classical Mapping [Phys. Rev. Lett. 94, 070604 (2005)]

Matthias Vojta, Ning-Hua Tong, and Ralf Bulla

(Received 5 June 2009; published 19 June 2009)

DOI: 10.1103/PhysRevLett.102.249904 PACS numbers: 05.10.Cc, 05.30.Jp, 99.10.Cd

Reference [1] used results obtained by the Numerical Renormalization Group (NRG) method to argue that the critical properties of the spin-boson model with bath exponent $s < 1/2$ are not those of mean-field theory. The difference is in the order-parameter exponents β, δ, and x.

Here we point out that the interpretation of the numerical data was incorrect: By now, we have convinced ourselves that the exponents in question could not be reliably extracted from the NRG calculations if the ordered phase is controlled by an operator which is dangerously irrelevant at criticality. The two relevant issues will be discussed in turn.

(A) *Hilbert-space truncation*: The bosonic NRG is presently unable to describe the physics of the localized fixed point for $s < 1$, due to the truncation of the bosonic Hilbert space [2]. From the absence of convergence problems at the critical fixed point, we nevertheless concluded [1] that all critical exponents can be extracted from NRG. While correct for an interacting fixed point, this assertion is erroneous for exponents β and δ of a Gaussian fixed point because, in the presence of a dangerously irrelevant variable, those exponents are *not* properties of the critical fixed point and its vicinity, but instead properties of the flow towards the fixed point controlling the ordered phase. (This is what happens above the upper-critical dimension of a standard ϕ^4 theory.)

(B) *Mass flow*: The NRG algorithm integrates out the bath iteratively. In the order-parameter language, this leads to a mass renormalization along the flow, arising from the bath (not from nonlinear interactions); i.e., at any NRG step, only a partial mass renormalization has been taken into account. As a result, the critical flow trajectory in NRG is that of a system with an additional mass set by the current NRG scale (i.e., by the temperature T), and, effectively, the system is not located at the critical coupling for any finite T. For an interacting fixed point, the missing mass correction from the bath and the interaction-generated mass have the same scaling; hence, no qualitative error is introduced. In contrast, above the upper-critical dimension, the interaction-generated mass vanishes faster as $T \rightarrow 0$, and the bath mass correction dominates all observables calculated along the critical flow trajectory, like the exponent x. Note that this conceptual "mass-flow" problem of NRG arises for all baths without low-energy particle-hole symmetry.

Consequently, the NRG-extracted exponents β, δ, and x quoted in Ref. [1] were unreliable. The same applies to the analytical argument given in Ref. [1]: The RG equations (9–11) were correct, but the following analysis relied on the absence of dangerously irrelevant variables as well.

Analyzing (A) in detail, we have found that careful NRG calculations performed with a large number N_b of bosonic states in principle allow to extract the correct exponents β and δ. While the truncation always dominates asymptotically close to criticality, the physical power laws can be seen on intermediate scales. As an example, we show in Fig. 1 the

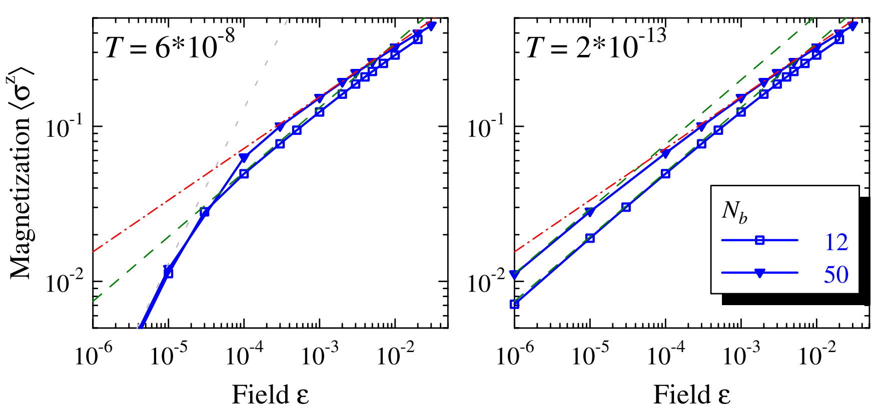

FIG. 1 (color online). Magnetization curves of the critical spin-boson model for $s = 0.4$. NRG parameters are $\Lambda = 4$, $(N_s, N_b) = (40, 12)$ and $(60, 50)$ [2]. The lines are power laws with exponents 1 (dotted line), $1/2.4$ (dashed line), and $1/3$ (dash-dotted line).

nonlinear field response at criticality for $s = 0.4$. For $N_b = 50$, a crossover from $\delta \approx 2.4$ to the mean-field value $\delta = 3$ upon increasing the field is visible before the quantum critical regime is left.

Taking into account recent numerical work based on quantum Monte Carlo and exact diagonalization techniques [3,4], which confirmed mean-field behavior in the sub-Ohmic spin-boson model for $s < 1/2$, we conclude that the quantum-to-classical mapping is valid here.

We acknowledge helpful discussions with K. Ingersent, S. Kirchner, R. Narayanan, Q. Si, M. Troyer, P. Werner, and in particular, with S. Florens, H. Rieger, and T. Vojta. This research was supported by the DFG via SFB 608, SFB TR/12, and FG 960.

[1] M. Vojta, N. Tong, and R. Bulla, Phys. Rev. Lett. **94**, 070604 (2005).
[2] R. Bulla *et al.*, Phys. Rev. B **71**, 045122 (2005).
[3] A. Winter *et al.*, Phys. Rev. Lett. **102**, 030601 (2009).
[4] A. Alvermann and H. Fehske, Phys. Rev. Lett. **102**, 150601 (2009).