# Towards a Quintic Ginzburg-Landau Description of the (2,7) Minimal Model

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We discuss dimensional continuation of the massless scalar field theory with the $i\phi^5$ interaction term. It preserves the so-called $\mathcal{PT}$ symmetry, which acts by $\phi \to -\phi$ accompanied by $i \to -i$. Below its upper critical dimension $10/3$, this theory has interacting infrared fixed points. We argue that the fixed point in $d = 2$ describes the nonunitary minimal conformal model $M(2,7)$. We identify the operators $\phi$ and $\phi^2$ with the Virasoro primaries $\phi_{1,2}$ and $\phi_{1,3}$, respectively, and $i\phi^3$ with a quasiprimary operator, which is a Virasoro descendant of $\phi_{1,3}$. Our identifications appear to be consistent with the operator product expansions and with considerations based on integrability. Using constrained Padé extrapolations, we provide estimates of the critical exponents in $d = 3$. We also comment on possible lattice descriptions of $M(2,7)$ and discuss RG flows to and from this conformal field theory (CFT). Finally, we conjecture that the minimal models $M(2,2n+1)$ are described by the massless scalar field theories with the $i\phi^{2n-1}$ interaction terms.

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*Introduction*—Massless scalar field theories can describe critical phenomena in various dimensions. A textbook example is the Euclidean $\phi^4$ field theory, which describes the critical Ising model in dimensions $1 < d < 4$. In $d = 2$, this strongly interacting field theory becomes equivalent to the minimal conformal model $M(3,4)$ [1]. The $d$ dependence of the scaling dimensions and operator product expansion (OPE) coefficients can be studied using various methods, such as the $4-\epsilon$ expansion [2] and the conformal bootstrap [3], and the results are in excellent agreement with each other [4,5]. More generally, the massless Euclidean field theories with $\phi^{2n}$ interactions are equivalent in $d = 2$ with the $A$ series unitary minimal models $M(n+1,n+2)$ [6]. In particular, for the $\phi^6$ field theory, which describes the tricritical Ising model $M(4,5)$, the $d$ dependence of observables was studied in [7] using a combination of $3-\epsilon$ expansions and a conformal bootstrap.

Similar correspondences between Ginzburg-Landau (GL) scalar field theories and critical phenomena also apply in some nonunitary cases. The original such example is the massless $i\phi^3$ Euclidean field theory, which describes the Yang-Lee universality class [8]. This field theory possesses the so-called $\mathcal{PT}$ symmetry, which acts by $i \to -i$, $\phi \to -\phi$. The $6-\epsilon$ expansion of the $i\phi^3$ theory was introduced in [8] and in recent years has been carried out to much higher orders [9–11]. In $d = 2$, this theory becomes equivalent to the minimal model $M(2,5)$ [12], and $\phi$ is identified with the only nontrivial Virasoro primary operator $\phi_{1,2}$ [13]. The operator $\phi^2$ is a conformal descendant, while the higher powers of $\phi$ should be identified with operators in $M(2,5)$ that are *quasiprimary*, i.e., primary only under the conformal group $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$. In particular, $i\phi^3$ has been identified [16] with the lowest dimension quasiprimary operator $T\bar{T}$. Imposing the $d = 2$ constraints improves the Padé extrapolations of the $6-\epsilon$ expansions [16,17]. The resulting estimates are in good agreement with the high-temperature expansions [18] and with numerical calculations of quantum criticality of the Ising model in an imaginary magnetic field [16,19,20], which were pioneered in $d = 2$ by von Gehlen [21].

In this Letter, we will be primarily concerned with the analogous $\mathcal{PT}$-symmetric quintic theory

$$S = \int d^d x \left( \frac{1}{2} (\partial_\mu \phi)^2 + \frac{g\phi^5}{120} \right), \qquad g \in i\mathbb{R} \qquad (1)$$

and will argue that in $d = 2$ it should be identified with the $M(2,7)$ minimal model. An early proposal for the Ginzburg-Landau description of $M(2,7)$ [22,23] also involved an $i\phi^5$ interaction term, but the derivative term was unconventional [24]. Since then, there have been a number of works on this subject including [15,25–32], but

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some disagreements between them have persisted, and in some of them the theory (1) was identified with $M(2,9)$ instead of $M(2,7)$. The lattice models proposed for $M(2,7)$ have involved the Blume-Capel model [33,34] with an added imaginary magnetic field [25,35], and we will make some comments about them.

The main feature of our new argument in favor of the identification of (1) with $M(2,7)$ is that only the operators $\phi$ and $\phi^2$ become Virasoro primaries in $d=2$. We identify them with $\phi_{1,2}$ and $\phi_{1,3}$, respectively, and their OPEs agree with the semiclassical treatment of the quintic theory (1). Importantly, we identify the operator $i\phi^3$ with the *quasiprimary* operator $(L_{-2}-(21/2)L_{-1}^2)(\bar{L}_{-2}-(21/2)\bar{L}_{-1}^2)\phi_{1,3}$.

Using the $(10/3)-\epsilon$ expansions developed for (1.1) in [28,36] and imposing the $M(2,7)$ boundary conditions in $d=2$, we will provide Padé estimates of the $d$ dependence of operator dimensions in this universality class. In $d=3$, our estimates of the critical exponents can be hopefully compared with numerical results in appropriate lattice approaches to $M(2,7)$. At the same time, we mention possible difficulties with continuing the quintic conformal field theory (CFT) all the way to $d=2$. We also discuss RG flows $M(4,5)\to M(2,7)$ and $M(2,7)\to M(2,5)$, which provide further checks of our arguments.

*Matching the GL description with $M(2,7)$*—Let us describe the $M(2,7)$ minimal model in detail. It has central charge $c(2,7)=-(68/7)$ and effective central charge [37,38] $c_{\text{eff}}(2,7)=\frac{4}{7}$. This model contains two nontrivial primary operators $\phi_{1,2}$ and $\phi_{1,3}$. Our proposal for operator identifications is summarized in Table I.

The $\mathcal{PT}$-odd operator $\phi_{1,2}\sim\phi$ has negative scaling dimension $\Delta_{1,2}=2h_{1,2}=-\frac{4}{7}$. The $\mathcal{PT}$-even operator $\phi_{1,3}\sim\phi^2$ has dimension $\Delta_{1,3}=-\frac{6}{7}$. We identify the scalar operators $i\phi^3$, $i\phi^5$, and $i\phi^6$ with quasiprimary Virasoro descendants [39],

$$i\phi^3\sim Q_2\bar{Q}_2\phi_{1,3},\quad i\phi^5\sim T\bar{T},\quad i\phi^6\sim Q_3\bar{Q}_3\phi_{1,2},\quad (2)$$

where $Q_2=L_{-2}-(21/2)L_{-1}^2$ and $Q_3=L_{-3}+7L_{-2}L_{-1}-(49/10)L_{-1}^3$. Thus, in 2D, the operator $i\phi^3$ is irrelevant, while it is relevant in $d=(10/3)-\epsilon$. We note, however, that in 2D there is a relevant spin-2 quasiprimary operator of dimension $\frac{8}{7}$,

$$T'_{zz}=Q_2\phi_{1,3},\qquad T'_{\bar{z}\bar{z}}=\bar{Q}_2\phi_{1,3}.\quad (3)$$

In the GL description, the corresponding operator is $T'_{\mu\nu}=\phi T_{\mu\nu}$.

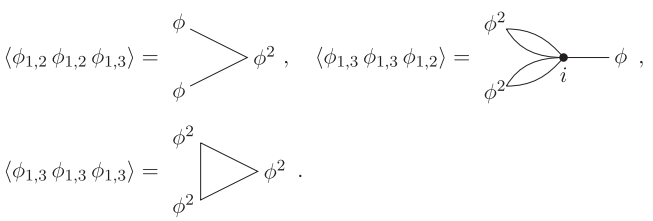
The OPEs of Virasoro primary operators are given by

$$\begin{aligned}\phi_{1,2}\times\phi_{1,2}&=1+\phi_{1,3},\\ \phi_{1,2}\times\phi_{1,3}&=\phi_{1,2}+i\phi_{1,3},\\ \phi_{1,3}\times\phi_{1,3}&=1+i\phi_{1,2}+\phi_{1,3}.\end{aligned}\quad (4)$$

TABLE I. Properties of the primary and some quasiprimary scalar operators of the $M(2,7)$ minimal model.

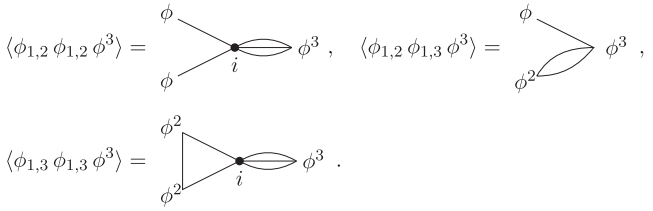
| $M(2,7)$ | $\phi_{1,1}$ | $\phi_{1,2}$ | $\phi_{1,3}$ | $Q_2\bar{Q}_2\phi_{1,3}$ | $T\bar{T}$ | $Q_3\bar{Q}_3\phi_{1,2}$ |
|---|---|---|---|---|---|---|
| $\Delta$ | 0 | $-\frac{4}{7}$ | $-\frac{6}{7}$ | (22/7) | 4 | (38/7) |
| $\mathcal{PT}$ | Even | Odd | Even | Even | Even | Odd |
| GL | 1 | $\phi$ | $\phi^2$ | $i\phi^3$ | $i\phi^5$ | $i\phi^6$ |

The nonvanishing three-point functions of primary operators, which are implied by the OPE, are $\langle\phi_{1,2}\phi_{1,2}\phi_{1,3}\rangle$, $\langle\phi_{1,3}\phi_{1,3}\phi_{1,2}\rangle$, and $\langle\phi_{1,3}\phi_{1,3}\phi_{1,3}\rangle$. Using the dictionary in Table I, we can reproduce them using the following Feynman graphs.:




(5)

The three-point functions of the lowest quasiprimary operator, $\phi^3$, with two Virasoro primaries are given by the diagrams




(6)

These perturbative explanations of the OPE are a good test of our GL interpretation of $M(2,7)$. They can be considered as generalizations of Zamolodchikov's OPE-based argument for the GL description of unitary minimal models $M(n,n+1)$ [6].

The quintic theory (1) is weakly coupled in $d=(10/3)-\epsilon$ dimensions. Its beta function was computed up to the $g^3$ order [28,36]. Rescaling $g$ as in [28], the beta function reads

$$\beta=-\frac{3}{2}g\epsilon-\frac{1377}{16}g^3+\mathcal{O}(g^5),\quad (7)$$

which admits a pair of imaginary fixed points $g_*\propto\pm i\sqrt{\epsilon}$. Using the anomalous dimensions calculated in [28,36], we obtain the scaling dimensions of $\phi,\phi^2,\phi^3$, and $\phi^5$ at the imaginary fixed points to order $\epsilon$:

$$\Delta_\phi = \frac{2}{3} - \frac{383\epsilon}{765} + \mathcal{O}(\epsilon^2), \qquad \Delta_{\phi^2} = \frac{4}{3} - \frac{262\epsilon}{255} + \mathcal{O}(\epsilon^2),$$
$$\Delta_{\phi^3} = 2 - \frac{313\epsilon}{255} + \mathcal{O}(\epsilon^2), \qquad \Delta_{\phi^5} = \frac{10}{3} + 2\epsilon + \mathcal{O}(\epsilon^2). \tag{8}$$

Based on the operator identification in Table I, we know the exact scaling dimensions of these operators in $d = 2$ dimensions. This information allows us to apply two-sided Padé resummations to the $\epsilon$ expansions in (8). We note that $\phi^3$ is relevant near the upper critical dimension, while it is irrelevant in $d = 2$; therefore, there is a value of $d$ where it crosses marginality. In Fig. 1, we show the Padé extrapolations for $2 \le d \le (10/3)$, including the rough estimates of the scaling dimensions in $d = 3$. The corresponding rough estimates of the critical exponents are $\eta = 2\gamma_\phi \approx -0.044$, $\nu = [1/(d - \Delta_{\phi^2})] \approx 0.49$, $\zeta = [(d - \Delta_{\phi^3})/(d - \Delta_\phi)] \approx 0.49$, $\sigma = [\Delta_\phi/(d - \Delta_\phi)] \approx 0.19$, $\omega = \Delta_{\phi^5} - d \approx 0.71$. For comparison, the exact critical exponents in $d = 2$ are $\eta = -\frac{8}{7}$, $\nu = (7/20)$, $\zeta = -\frac{4}{9}$, $\sigma = -\frac{2}{9}$, $\omega = 2$.

As $d$ is reduced from $10/3$, the scaling dimensions of $\mathcal{PT}$-odd operator $\phi$ and $\mathcal{PT}$-even operator $\phi^2$ appear to cross at $2 < d_* < 3$. There may be a subtlety at $d = d_*$, where the conformal and $\mathcal{PT}$ symmetries allow a non-vanishing two-point function $\langle \phi(x)\phi^2(y)\rangle$, which has an imaginary coefficient. The theory at $d = d_*$ may, thus, have unusual properties, but we hope that one can analytically continue from $d > d_*$ to $d < d_*$. Another possible pitfall is that $\Delta_{\phi^3}(d)$ crosses the marginal value $d$ somewhere between $d = 2$ and $d = 3$. In view of the general results on RG flows [40], it is not clear if such a marginality crossing is allowed [41]. In unitary examples, such as [42], a marginal operator typically appears at the limiting value of $d$ where a fixed point merges and annihilates with another fixed point. However, it is possible that nonunitary theories behave differently.

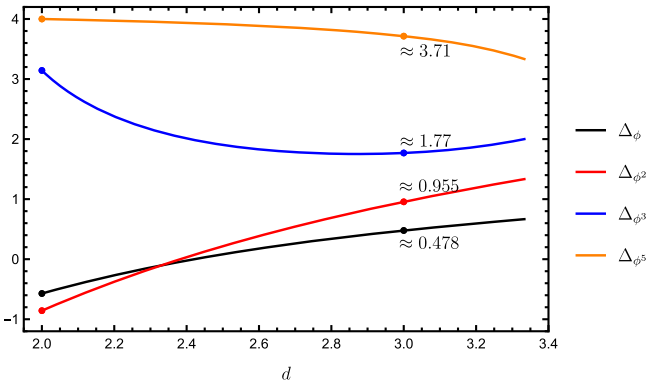



FIG. 1. Two-sided Padé extrapolations for the dimensions of operators $\phi, \phi^2, \phi^3$, and $\phi^5$. We use the [1,1] Padé approximant, except for $\phi^3$ where we use the [0,2] Padé approximant because [1,1] has a pole.

*Integrable deformation by $\phi_{1,3}$*—Deforming the $M(2,7)$ model by $\phi_{1,3} \sim \phi^2$ yields a massive integrable theory, whose spectrum consists of two breathers, $A_a$, $a \in \{1,2\}$, with the golden ratio between masses [43,44]:

$$\frac{m_2}{m_1} = 2\cos\frac{\pi}{5} \approx 1.68103. \tag{9}$$

There are no kinks in this theory. The two-particle $S$ matrix of the corresponding reduced sine-Gordon model with $\beta^2 = (16\pi/7)$ is [43,44]

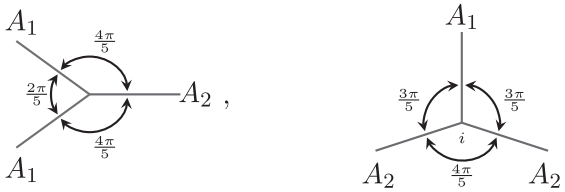
$$S_{11}(\theta) = F_{\frac{2}{5}}(\theta), \qquad S_{12}(\theta) = F_{\frac{3}{5}}(\theta)F_{\frac{4}{5}}(\theta),$$
$$S_{22}(\theta) = F^2_{\frac{3}{5}}(\theta)F_{\frac{4}{5}}(\theta), \tag{10}$$

where $\theta$ is the rapidity difference of the particles and the building block $F_\alpha(\theta)$ is a meromorphic function of $\theta$,

$$F_\alpha(\theta) = \frac{\tanh[\frac{1}{2}(\theta + i\pi\alpha)]}{\tanh[\frac{1}{2}(\theta - i\pi\alpha)]}. \tag{11}$$

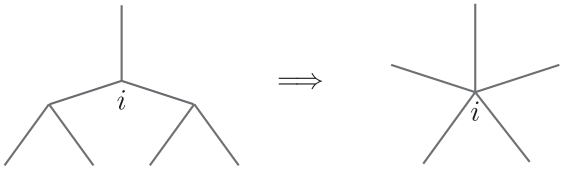
When $0 < \alpha < \frac{1}{2}$, $F_\alpha(\theta)$ has a simple pole of residue $2i \tan\alpha \in i\mathbb{R}_+$ at $i\alpha\pi$ and a simple pole of residue $-2i \tan\alpha \in i\mathbb{R}_-$ at $i(1-\alpha)\pi$. $F_{\frac{1}{2}}(\theta)$ has only a double pole at $(i\pi/2)$. The $S$ matrix satisfies the spatial reflection symmetry $S_{ab}(\theta) = S_{ba}(\theta)$, the crossing symmetry, and unitarity, respectively [45].

In the deformed theory $M(2,7) + \phi_{1,3}$, the breather $A_1$ exhibits a "$\phi^5$ property," with the scattering processes given by $A_1A_1 \to A_2$, $A_1A_2 \to A_1$, $A_1A_2 \to A_2$, $A_2A_2 \to A_1$ [46]. These scattering processes can be represented diagrammatically as follows:




(12)

where the angles are fixed by the poles of the $S$ matrix, and the label "$i$" indicates that the corresponding scattering amplitude is purely imaginary. Combining the two building blocks in (12), we obtain an effective five-particle scattering of $A_1$:




(13)

which is captured by the following identity of the $S$ matrix [49],

$$S_{11}(\theta) = S_{11}\left(\theta + \frac{i\pi}{5}\right) S_{11}\left(\theta - \frac{i\pi}{5}\right) S_{11}\left(\theta + \frac{3i\pi}{5}\right) \times S_{11}\left(\theta - \frac{3i\pi}{5}\right). \quad (14)$$

This relation can be derived from the bootstrap equations.

Comparing the amplitudes with the OPE structure of $M(2,7)$, we can identify $A_a \leftrightarrow \phi_{1,1+a}$ [44]. Using our field identification, $A_a$ can be further identified as $\phi^a$. The "$\phi^5$ property" of the fundamental particle $A_1 \leftrightarrow \phi$ supports our conjecture (1).

*RG flows between minimal models—*

Semiclassical argument for the RG flow $M(4,5) \to M(2,7)$: All of the known RG flows involving $M(2,2n+1)$ are not short, so standard conformal perturbation theory cannot be applied. To describe the flow $M(3,4) \to M(2,5)$ qualitatively, Fisher used a semiclassical argument [8].

Let us show how to generalize Fisher's argument to the flow $M(4,5) \to M(2,7)$, following a suggestion in [25]. The tricritical Ising model $M(4,5)$ perturbed by relevant primary operators is described by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + i g_1 \phi + g_2 \phi^2 + i g_3 \phi^3 + g_4 \phi^4 + g_6 \phi^6. \quad (15)$$

After shifting the field, $\phi \to \phi + i\phi_0$, and choosing $\phi_0 = \pm\sqrt{(g_4/15g_6)}$, the quartic term is removed. Then we find

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}(\partial_\mu \phi)^2 + i(g_1 + 2g_2\phi_0 - 3g_3\phi_0^2 - 4g_4\phi_0^3 + 6g_6\phi_0^5)\phi \\ & + (g_2 - 3g_3\phi_0 - 6g_4\phi_0^2 + 15g_6\phi_0^4)\phi^2 \\ & + i(g_3 + 4g_4\phi_0 - 20g_6\phi_0^3)\phi^3 \\ & + 6ig_6\phi_0\phi^5 + g_6\phi^6 + C. \end{aligned} \quad (16)$$

Now, by tuning three parameters $g_1 = \pm\sqrt{(64g_4^5/84375g_6^3)}$, $g_2 = -(g_4^2/5g_6)$, and $g_3 = \mp\sqrt{(64g_4^3/135g_6)}$, we obtain the quintic theory (1), which is conjectured to describe $M(2,7)$, while the operator $\phi^6$ becomes irrelevant in the IR theory.

While this semiclassical argument is expected to be reliable slightly below $d = 3$, where both the UV and IR fixed points are quite weakly coupled, it cannot be readily applied in 2D. In this strongly coupled case, one likely needs to resort to numerical methods. The RG flow $M(4,5) \to M(2,7)$ was studied in [30] using the truncated conformal space approach (TCSA). In [25], the RG flow $M(4,5) \to M(2,7)$ was discussed in the context of the quantum Blume-Capel (BC) spin chain [50] with a non-Hermitian Hamiltonian:

$$H_{\mathrm{BC}} = \sum_{i=1}^{N} (\alpha (S_i^z)^2 - S_i^z S_{i+1}^z + h_x S_i^x - i h_z S_i^z), \quad (17)$$

where $S_i^x$ and $S_i^z$ are spin-1 matrices. The spectrum of (17) is invariant under $h_x \to -h_x$. The last term breaks $\mathbb{Z}_2$ symmetry acting as $S_i^z \to -S_i^z$ but preserves $\mathcal{PT}$ symmetry $S_i^z \to -S_i^z$, $i \to -i$. In the space of $\{\alpha, h_x, h_z\}$, $M(4,5)$ is a point, $M(3,4)$ is a line, and $M(2,5)$ is a 2D surface.

In a $1+1$ dimensional lattice Hamiltonian approach, the operator $(Q_2 + \bar{Q}_2)\phi_{1,3}$ (3) is allowed in the IR $M(2,7)$. Therefore, we expect that there are three tunings required to reach the $M(2,7)$ critical point corresponding to the relevant operators $\phi_{1,2}$, $\phi_{1,3}$, and $(Q_2 + \bar{Q}_2)\phi_{1,3}$ [51]. However, in the classical Blume-Capel model with an imaginary magnetic field on a 2D square lattice [35],

$$E_{\mathrm{BC}} = -\sum_{\langle ij \rangle} \sigma_i \sigma_j + \sum_i (\alpha \sigma_i^2 - i h_z \sigma_i), \quad (18)$$

where $\sigma_i \in \{-1, 0, 1\}$, the discrete rotation symmetry forbids $T'_{\mu\nu}$ because it changes sign under a rotation by $\pi/2$. Therefore, in this case, only two tunings should be needed to reach $M(2,7)$. This is one more tuning than what was required to reach $M(2,5)$ from $M(3,4)$, so $M(2,7)$ is a candidate to be the tricritical Yang-Lee model [30].

Flow $M(2,7) \to M(2,5)$: Let us study the flow from the quintic theory to the cubic theory $M(2,7) \to M(2,5)$, clarifying the discussion of [31]. It is necessary to perturb the UV CFT by two operators, $i\phi_{1,2}$ and $\phi_{1,3}$. Therefore, the GL description of such a scaling $M(2,7)$ minimal model is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + i g_1 \phi + g_2 \phi^2 + i g_5 \phi^5. \quad (19)$$

After the shift $\phi \to \phi + i\phi_0$ and choosing $\phi_0 = -\sqrt[3]{(g_2/10g_5)}$, the quadratic term is removed. Then we find

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}(\partial_\mu \phi)^2 + i(g_1 - 15g_5\phi_0^4)\phi - 10ig_5\phi_0^2\phi^3 - 5g_5\phi_0\phi^4 \\ & + ig_5\phi^5 + C. \end{aligned} \quad (20)$$

By tuning only one parameter $g_1 = \sqrt[3]{(27g_2^4/80g_5)}$, we obtain cubic Yang-Lee theory. The operators $\phi^4$ and $i\phi^5$ become irrelevant in the IR theory.

While this semiclassical argument is suggestive, it is not parametrically reliable. Even in 3D, where the UV CFT is weakly coupled due to the proximity to the upper critical dimension $10/3$, the IR Yang-Lee CFT is not. In 2D, both the $i\phi^5$ and $i\phi^3$ CFTs are strongly coupled, but we can use the TCSA to study the RG flow. This approach was introduced in [52], where it was applied to SLYM up to the level $L = 5$ (17 spin-0 states) [53]. Here we are using TCSA with the basis of all descendants excluding null vectors up to the level $L = 25$ (523,953 spin-0 states). We use the structure constants $C_{223} \approx 2.569$, $C_{233} \approx 4.592i$, $C_{333} \approx -6.019$.

The Hamiltonian of perturbed $M(2,7)$ on an infinite cylinder with circumference $R$ is

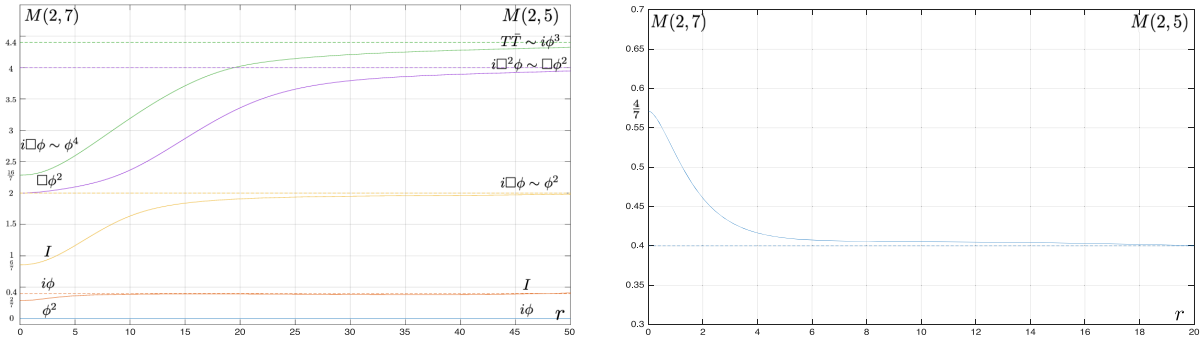



FIG. 2. (a) Operator flow and (b) effective central charge for the $M(2,7) \to M(2,5)$ flow. (a) $-(C_n(r) - C_0(r))/12$. (b) $C_0(r)$ between $c_{eff}^{UV} = 4/7$ and $c_{eff}^{IR} = 2/5$.

$$H = \frac{2\pi}{R}\left(L_0 + \bar{L}_0 - \frac{c(2,7)}{12}\right) + i\lambda_{12}\int_0^R dx\phi_{1,2}(x,0) - \lambda_{13}\int_0^R dx\phi_{1,3}(x,0). \tag{21}$$

This scaling theory has a single scaling parameter $\xi = (\lambda_{12}/|\lambda_{13}|^{9/10})\mathrm{sgn}(\lambda_{13})$, which should be tuned to its critical value to obtain the flow $M(2,7) \to M(2,5)$.

Acting on descendants of scaling dimension $\Delta = h + \bar{h}$, the TCSA perturbed Hamiltonian [52] takes the following form:

$$H = \frac{2\pi}{R}\left(\Delta + \frac{17}{21}\right)\mathbf{1} + i\frac{2\pi G_{12}}{R}B_{12} - \frac{2\pi G_{13}}{R}B_{13}. \tag{22}$$

In this Hamiltonian, $G_{1n} = \lambda_{1n}(2\pi)^{\Delta_{1,n}-1}R^{2-\Delta_{1,n}} = \lambda_{1n,0}(2\pi)^{\Delta_{1,n}-1}r^{2-\Delta_{1,n}}$ are dimensionless effective coupling constants, and $B_{1n}$ are dimensionless matrix elements of $\phi_{1,n}$. $\lambda_{1n,0}$ and $r$ are defined by introducing an arbitrary mass scale $m$, namely $\lambda_{1n,0} = \lambda_{1n}m^{\Delta_{1,n}-2}$ and $r = mR$, where $r$ is understood as the dimensionless scaling length. Both perturbations $M(2,7) + i\phi_{1,2}$ and $M(2,7) + \phi_{1,3}$ are integrable massive deformations with the spectra consisting of two breathers. In both cases, we denote the mass of the lighter breather by $m_1$. The couplings $\lambda_{12}$ and $\lambda_{13}$ satisfy the following relations [55–57]:

$$\lambda_{12} \approx 0.07856 m_1^{\frac{18}{7}}, \qquad \lambda_{13} \approx -0.04054 m_1^{\frac{20}{7}}. \tag{23}$$

Therefore, it is convenient to choose $m = m_1$ as the reference mass scale in the TCSA Hamiltonian. The same choice was also used in [31].

The $n$th energy level on a cylinder is [37,58,59]

$$E_n(R) = FR - \frac{C_n(R)}{12}\frac{2\pi}{R}, \tag{24}$$

where $F$ is the bulk vacuum energy density and $C_n(R)$ interpolates between $c^{\mathrm{UV}} - 12\Delta_n^{\mathrm{UV}}$ at $R \to 0$ and $c^{\mathrm{IR}} - 12\Delta_n^{\mathrm{IR}}$ at $R \to \infty$. Applying TCSA, we plot the first five energy levels $-\{[C_n(R) - C_0(R)]/12\} = (R/2\pi)[E_n(R) - E_0(R)]$ interpolating between $\Delta_n^{\mathrm{UV}} - \Delta_0^{\mathrm{UV}}$ and $\Delta_n^{\mathrm{IR}} - \Delta_0^{\mathrm{IR}}$ for tuned $\xi$ [Fig. 2(a)]. We also plot $C_0(R) = -(6R/\pi)[E_0(R) - FR)]$, which interpolates between $c_{\mathrm{eff}}^{\mathrm{UV}} = c_{\mathrm{eff}}(2,7) = \frac{4}{7}$ and $c_{\mathrm{eff}}^{\mathrm{IR}} = c_{\mathrm{eff}}(2,5) = \frac{2}{5}$ [Fig. 2(b)] as a function of the dimensionless parameter $r = m_1 R$, for appropriately tuned $\xi$ and $F$. For the flow $M(2,7) \to M(2,5)$, we obtained

$$\xi = 1.28855, \qquad F = -0.093535. \tag{25}$$

Our $\xi$ differs from [31] by 0.05%. For the flow with the same IR Yang-Lee model $M(3,4) \to M(2,5)$, [60] obtained free energy $F = 0.092746$, whose magnitude differs only by 0.8%.

From the TCSA plot [Fig. 2(a)] and our field identification (Table I), we can obtain operator flow from $M(2,7)$ to $M(2,5)$ [61]. Spin 0 is $\phi^2 \to i\phi$, $i\phi \to I$, $I \to i\Box\phi \sim \phi^2$, $\Box\phi^2 \to i\Box^2\phi$, $i\Box\phi \sim \phi^4 \to T\bar{T} \sim i\phi^3$. Spin 1 is $\partial\phi^2 \to i\partial\phi$, $i\partial\phi \to i\partial\Box\phi$. We expect in 3D the operator flow from $i\phi^5$ to $i\phi^3$ to be $\phi^k \to \phi^k$ in accordance with Fig. 1.

*Discussion*—We have presented evidence that the GL description of $M(2,7)$ is provided by the action (1). The upper critical dimension of this $i\phi^5$ universality class is $10/3$, so the 3D theory is rather weakly coupled, and this is supported by Fig. 1. While the scaling dimensions of operators appear to be close to the free dimensions, finding their precise values is an interesting problem. It would also be very interesting to calculate the scaling dimensions numerically in a lattice model, perhaps following the non-Hermitian $\mathcal{PT}$-symmetric Hamiltonian approach of [25], which used the Blume-Capel spin chain in an imaginary magnetic field. We hope that our estimates in $d = 3$ can be compared with numerical quantum criticality in the corresponding model on regularized spheres.

Let us note that $M(2,2n+1)$ minimal models have $\mathcal{PT}$ symmetry implied by OPE structure and no $\mathbb{Z}_2$ symmetry [62]. This also holds for scalar theories with odd potentials

$i\phi^{2n-1}$, where $\phi \to -\phi$ is not a symmetry but $\phi \to -\phi$, $i \to -i$ is a symmetry. Therefore, it is tempting to conjecture that the general GL description of $M(2, 2n+1)$ minimal models is

$$S_{2,2n+1} = \int d^d x \left( \frac{1}{2}(\partial_\mu \phi)^2 + \frac{g\phi^{2n-1}}{(2n-1)!} \right), \qquad g \in i\mathbb{R}. \quad (26)$$

If this is correct, then semiclassical arguments predict the family of RG flows from $M(n+1, n+2)$, which is described by the field theory with $\phi^{2n}$ interaction, to $M(2, 2n+1)$. These $M(p, q) \to M(p', q')$ RG flows have an interesting feature that $p' + q' = p + q$. The GL descriptions (26) appear to predict correct OPEs. However, by shifting $\phi$ in (26) and tuning the lower monomial coefficients, semiclassical arguments also predict RG flows $M(2, 2n+1) \to M(n, n+1)$, which do not happen in 2D for $n > 2$. In particular, for $n > 3$ such RG flows contradict the $c_{\text{eff}}$ theorem [38]. Clearly, more work is needed to ascertain the range of validity of the GL descriptions we have conjectured.

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*Data availability*—The data that support the findings of this article are openly available [63].

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