Editors' Suggestion

Warm Inflation with the Standard Model

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We show for the first time that warm inflation is feasible with standard model (SM) gauge interactions alone. Our model consists of a minimal extension of the SM by a single scalar inflaton field with an axionlike coupling to gluons and a monomial potential. The effects of light fermions, which were previously argued to render warm inflation with the SM impossible, are alleviated by Hubble dilution of their chiral chemical potentials. Our model features only one adjustable combination of parameters and accommodates all inflationary observables. We briefly discuss implications for axion experiments, dark matter, and the strong *CP* problem.

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Introduction—Cosmic inflation [1–4], an early period of accelerated cosmic expansion, provides the leading explanation for the observed near-homogeneity and isotropy of our Universe. However, it remains unknown what mechanism drove inflation, and how it is connected to microphysical theories of particle physics. Many models can effectively be parametrized by a single scalar field ϕ, the potential of which, $V(\phi)$, dominated the cosmic energy budget. However, simple renormalizable potentials with a single parameter have long been ruled out by observations of the cosmic microwave background (CMB) [5,6]. A plethora of models with more complicated potentials exist [7], but they usually contain more parameters, making an observational discrimination impossible. Moreover, for many models, it is not clear how they can be realized microphysically and related to theories of particle physics, in particular, the standard model (SM).

Some of these issues can be addressed by *warm inflation* [8], a variant of the inflationary paradigm in which dissipative effects maintain the presence of a thermal bath during inflation; see, e.g., Ref. [9] for a review. This allows for accelerated expansion on steeper potentials, alleviating the long-standing difficulty of protecting the flatness of the inflaton potential from quantum corrections [10–14]. Moreover, thermal fluctuations provide an additional source of scalar perturbations in the metric, effectively suppressing the tensor-to-scalar ratio r and, hence, rescuing models that would be observationally excluded in their absence. Finally, non-Gaussianities with a unique bispectral shape [15,16] provide a pathway toward discovering evidence of inflation.

Arguably, the main challenge of warm inflation has been to embed it in a microphysical theory [17]. The fundamental problem is that, in perturbative computations, the inflaton dissipation coefficient and thermal correction to the effective potential originate from the imaginary and real parts of the same Feynman diagrams, making it difficult to enhance the former without introducing sizable corrections to the latter that spoil the flatness of the potential. This issue can be avoided if the dissipation arises from an axionlike coupling to the topological winding number in non-Abelian gauge theory, which gives rise to a new form of thermal friction [18] known as sphaleron heating. It has been shown that sphaleron heating can indeed support warm inflationary dynamics [19] within a consistent quantum field theory framework [20], but previous realizations of this idea generally require new (dark) non-Abelian gauge interactions that serve no other evident purpose.

Here, we demonstrate that warm inflation can be induced directly by an axionlike coupling between the inflaton and quantum chromodynamics (QCD), the strong force of the SM. QCD has not previously been considered as a viable candidate for the required non-Abelian force due to the suppression of sphaleron heating by axial chemical potentials μ_A that it generates for the quarks [18,21,22]. In the present proposal, this problem is alleviated by the Hubble dilution of μ_A [23], which has been neglected in earlier

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works but is essential during inflation. We construct the first model of warm inflation that is driven by known SM gauge interactions alone. Our proposal is compatible with all observations and contains only two unknown parameters, one in the quartic inflaton potential and the other one in the coupling to gluons that is potentially experimentally testable.

Warm inflation—We consider inflation to be warm if there is a thermal bath with a temperature T that exceeds the Hubble rate H during a period of accelerated expansion ($T > H$). To achieve this, the inflaton field must source particles at a rate that can compensate for their Hubble dilution, and their self-interactions must be fast enough to maintain thermal equilibrium. This microphysical process can be captured by an effective description in which a thermal friction coefficient [24] Υ extracts energy from the slow-rolling inflaton field ϕ. The homogeneous equations of motion of the inflaton expectation value and radiation density $\rho_R \equiv (\pi^2/30) g_* T^4$ read

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V' = 0, \tag{1}$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon \dot{\phi}^2. \tag{2}$$

Here, dots denote derivatives with respect to time, prime denotes derivative with respect to ϕ, $H \equiv \dot{a}/a$ is the Hubble expansion, with a the scale factor, and g_* corresponds to the number of relativistic degrees of freedom in the thermal bath. The cases $Q \equiv \Upsilon/(3H) > 1$ and $Q < 1$ are commonly referred to as the strong and weak regime of warm inflation, respectively. In both regimes, the thermal bath can contribute to sourcing cosmological perturbations, but only in the strong regime does thermal friction dominate the slow-roll dynamics.

The Friedman equation governs the inflationary dynamics, $H^2 = (V(\phi) + \rho_R + \frac{1}{2}\dot{\phi}^2)/(3M_{\rm pl}^2) \approx V(\phi)/(3M_{\rm pl}^2)$, where $M_{\rm pl} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. A sustained period of accelerated expansion requires $V(\phi) \gg \rho_R, \dot{\phi}^2$, which can be assured by the usual slow-roll conditions, $\epsilon_V, \eta_V \ll 1$, if the slow-roll parameters are modified to be [26]

$$\epsilon_V \equiv \frac{M_{\rm pl}^2}{2(1+Q)}\left(\frac{V'}{V}\right)^2, \qquad \eta_V \equiv \frac{M_{\rm pl}^2}{(1+Q)}\frac{V''}{V}. \tag{3}$$

From Eq. (3), it is immediately apparent that in the strong regime of warm inflation the slow-roll conditions are more easily satisfied than in its cold counterpart, implying that a given number of e-folds,

$$N_* = \int H dt = \int_{\phi_{\rm end}}^{\phi_*} \frac{1}{M_{\rm pl}^2}\frac{V}{V'}(1+Q) d\phi, \tag{4}$$

can be achieved with a smaller field value ϕ_* or steeper potential (in the sense of larger V'/V). The slow-roll conditions suppress the first two terms in Eqs. (1) and (2), such that

$$\frac{\pi^2}{30} g_* T^4 \approx \frac{\Upsilon}{4H}\left(\frac{V'}{(3H+\Upsilon)}\right)^2 \tag{5}$$

defines the approximately constant steady-state temperature of the thermal bath T.

Sphaleron heating with the SM—Our model is defined by the extension of the SM [27]

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{\alpha_s}{8\pi}\frac{\phi}{f} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \tag{6}$$

where $G^a_{\mu\nu}$ is the QCD field strength tensor, $\tilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$ its dual, and $\alpha_s \equiv g^2/4\pi$ with g the QCD gauge coupling. The following considerations, however, hold for a general Yang-Mills SU(N_c) gauge group. We first neglect the presence of quarks and include their impact further below. For T larger than the confinement temperature, the equation of motion for the expectation value of ϕ reads [18]

$$\ddot{\phi} + 3H\dot{\phi} + V' = -\frac{\alpha_s}{8\pi f}\langle G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\rangle. \tag{7}$$

The thermal average of $G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ induces a thermal friction coefficient $\Upsilon_{\rm sph}$ due to real-time sphaleron processes that lead to biased transitions between vacua with different Chern-Simons number $N_{\rm CS}$, driven by the inflaton velocity $\dot{\phi}$. The friction coefficient $\Upsilon_{\rm sph}$ is related to the SU(N_c) sphaleron rate $\Gamma_{\rm sph} \equiv \lim_{t\to\infty}(N_{\rm CS}(t) - N_{\rm CS}(0))^2/(Vt)$ (also called the Chern-Simons diffusion rate) via the fluctuation-dissipation theorem [16,18–20,28]:

$$\frac{\alpha_s}{8\pi f}\langle G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\rangle \equiv \Upsilon_{\rm sph}\dot{\phi} = \frac{\Gamma_{\rm sph}}{2Tf}\frac{\dot{\phi}}{f}. \tag{8}$$

By comparing Eqs. (7) and (8) to Eq. (1), we can identify $\Upsilon = \Upsilon_{\rm sph}$. Using the estimate $\Gamma_{\rm sph} \approx \alpha_s^5 N_c^5 T^4$ [29], one finds [28]

$$\Upsilon_{\rm sph} \simeq (\alpha_s N_c)^5 \frac{T^3}{2f^2}, \tag{9}$$

where we neglected an additional prefactor of the order of 1 with a weak dependence on model parameters and dynamical quantities (see Refs. [20,29–31]).

The perturbative shift symmetry of the axionlike coupling in Eq. (6) protects the inflaton potential from thermal corrections [16,18–20,31,32] that would otherwise endanger the slow-roll dynamics [17]. However, since nonperturbative contributions to $V(\phi)$ from instantons vanish at temperatures much larger than the QCD confinement scale $\Lambda_{\rm QCD}$, where $\alpha_s < 1$ (see, e.g., Ref. [33]),

the coupling to QCD cannot generate a potential that drives inflation [34].

The relevant interaction with SM quarks ψ is given by

$$\mathcal{L}_{\rm QCD} \supset \sum_f \left(\frac{i}{2} \bar{\psi}_f \not{D} \psi_f + \text{H.c.} \right), \tag{10}$$

where f denotes the sum over SM flavors. The chiral anomaly of $\mathrm{SU}(N_c)$ leads to a nonconservation of the axial fermion current $j_A^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma^5 \psi_f$ [18,36]:

$$\partial_t j_A^0 = -N_f \left\langle \frac{\alpha_s G^a_{\mu\nu} \tilde{G}^{a\mu\nu}}{4\pi} \right\rangle, \tag{11}$$

where the axial charge density j_A^0 corresponds to the difference in number density between right-handed and left-handed Dirac fermions. The quarks backreact onto the biased sphaleron transitions such that Eq. (8) changes to [18,29,37–41]

$$\frac{\alpha_s}{8\pi f} \langle G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \rangle = \Upsilon_{\rm sph} \left(\dot{\phi} + \frac{6 f j_A^0}{N_c T^2} \right). \tag{12}$$

In particular, Eqs. (11) and (12) imply that, in the absence of a rolling axion, sphalerons relax the axial charge as $\partial_t j_A^0 = -6(N_f/N_c)(\Gamma_{\rm sph}/T^3) j_A^0$ [29,37–41].

We define a chemical potential μ_A via

$$\begin{aligned} j_A^0 &= 2 N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{e^{(p-\mu_A)/T} + 1} - \frac{1}{e^{(p+\mu_A)/T} + 1} \right) \\ &= \frac{N_c N_f \mu_A T^2}{3}, \end{aligned} \tag{13}$$

where N_f denote the number of flavors, and we have taken the $\mu_A \ll T$ limit in the second line [42]. Using Eqs. (12) and (7) and the anomaly relation (11), we obtain the coupled evolution of the inflaton and the axial chemical potential for a quasi-steady-state temperature in an expanding universe [18]:

$$\ddot{\phi} + 3H\dot{\phi} + V' = -\Upsilon_{\rm sph} \left(\dot{\phi} + 2 N_f f \mu_A \right), \tag{14}$$

$$\dot{\mu}_A + 3H\mu_A = -\frac{6 f \Upsilon_{\rm sph}}{N_c T^2} \left(\dot{\phi} + 2 N_f f \mu_A \right). \tag{15}$$

Importantly, here we also take into account Hubble dilution of the chemical potential in the second line. When Hubble friction is neglected, there is no solution where $\ddot{\phi}$ vanishes asymptotically [18]; hence, slow roll cannot be sustained [19,21]. In a quasistatic situation, the effect of H can be incorporated in Eq. (14) by absorbing the term $\Upsilon_{\rm sph} 2 N_f f \mu_A$ into an effective dissipation coefficient [23]:

$$\Upsilon_{\rm eff} = \Upsilon_{\rm sph} \Big/ \left(1 + \frac{\Upsilon_{\rm sph}}{3H} \frac{N_f}{N_c} \frac{12 f^2}{T^2} \right). \tag{16}$$

Sphaleron heating is fully restored if $H/\Upsilon_{\rm eff} \gg 4 f^2 N_f / (T^2 N_c)$; i.e., the chemical potential is diluted faster than it builds up due to sphaleron transitions. Achieving this is generically easier in the weak regime of warm inflation where $\Upsilon_{\rm sph} < 3H$. Using the Friedmann equation and Eq. (9), we get

$$\Upsilon_{\rm eff} = N_c^5 \alpha_s^5 \frac{T^3}{2 f^2} \Big/ \left(1 + \frac{2 N_f N_c^4 \alpha_s^5 T}{\sqrt{V(\phi)/(3 M_{\rm pl}^2)}} \right). \tag{17}$$

Note that Eq. (17) can yield different functional dependencies of $\Upsilon_{\rm eff}$ on ϕ and T if the second term dominates the denominator, depending on the shape of $V(\phi)$:

$$\Upsilon_{\rm eff} \propto \frac{T^2}{f^2} \frac{\sqrt{V(\phi)}}{M_{\rm pl}}. \tag{18}$$

In addition to the gauge interactions in (10), quarks also have Yukawa couplings, $y_f H \overline{\psi_f} \psi_f$, which mediate chirality violation processes either by generating fermion masses [43] or through thermal scatterings [44,45]. Both make the SM flavors distinguishable. An equation of motion for each chemical potential μ_{A_f} can be obtained from (15) by omitting the factor N_f and adding a term $-\Gamma_{{\rm ch}f} \mu_A$ on the rhs, which can effectively be incorporated into $\Upsilon_{\rm eff}$ by expressing the factor N_f terms of a sum over flavors with the replacement $3H \to 3H + \Gamma_{{\rm ch}f}$. If all $\Gamma_{{\rm ch}f}$ are either much smaller or much larger than H and $\Upsilon_{\rm sph}$, one can simply continue using the unflavored equation (16) but set N_f to the number of SM flavors for which $\Gamma_{{\rm ch}f} \ll \Upsilon_{\rm sph}$, which for $\Gamma_{{\rm ch}f} \sim 10^{-2} y_f^2 T$ [44] amounts to $y_f^2 \lesssim 10^3 \alpha_s^5 N_c^4$. This condition is violated only for the top quark; hence, we use (16) with $N_f = 5$ [46]. We further set $N_c = 3$ and obtain α_s from running to the energy scale T with the known SM β functions [47,48].

Working model—We choose the quartic potential

$$V(\phi) = \lambda \phi^4 \tag{19}$$

to fully determine the Lagrangian (6) with two unknown model parameters f and λ. Matching the observed amplitude $A_s(k_*) \approx 2 \times 10^{-9}$ of scalar perturbations $\Delta_s^2(k) = A_s(k_*)(k/k_*)^{n_s(k_*)-1}$ at the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ [5] establishes a relation between them, effectively leaving us with one single free parameter. Practically, we use Eqs. (4) and (5), with $\Upsilon = \Upsilon_{\rm eff}$ and (19) to numerically solve for the temperature T and, subsequently, $\Upsilon_{\rm eff}$ as a function of ϕ, λ, and f. Varying λ scans along values of $Q = \Upsilon_{\rm eff}/(3H)$. With this and the approximation of instantaneous reheating,

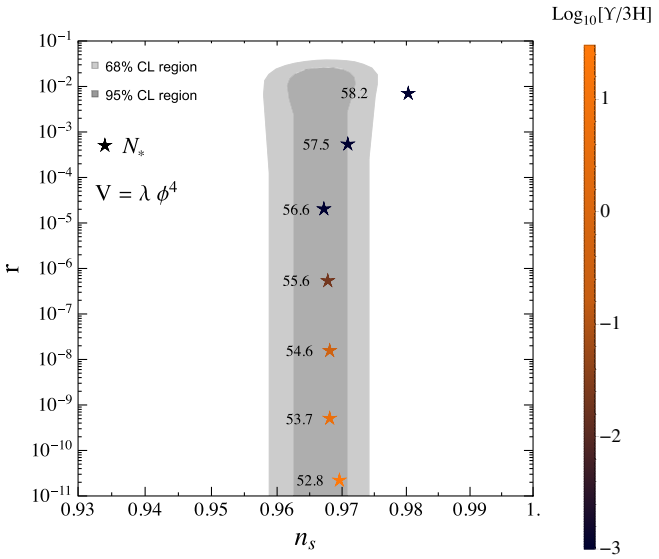


FIG. 1. The predictions of warm inflation with the SM for a quartic inflaton potential. The allowed 68% and 95% contour regions correspond to the constraints from the combined analysis of Planck [5] and BICEP/Keck [6] data. Proposed and upcoming CMB experiments will improve this bound by about a factor of 4 (AliCPT [51,52], Simons Observatory [53], and Simons Array [54]) to 20 (LiteBIRD [55] and CMB-S4 [56]).

we can compute N_*, A_s, n_s, and the tensor-to-scalar ratio $r \equiv \Delta_h^2/\Delta_s^2$ from the relations given in the Appendix. Since our model effectively has only one free parameter, we predict a line in the $n_s - r$ plane, as displayed in Fig. 1.

We find that our model is compatible with observations in both the strong regime and the weak regime, consistent with what was previously found for dark gauge groups [16]. For $Q \gg 1$, the denominator in Eq. (17) suppresses the efficiency of sphaleron heating ($\Upsilon_{\rm eff} \ll \Upsilon_{\rm sph}$), and $\Upsilon_{\rm eff}$ follows the scaling in Eq. (18). We do not compare to CMB observables in this regime, as we can no longer compute perturbations reliably with the formulas given in the Appendix. This suppression is, however, already alleviated for $Q \lesssim \mathcal{O}(10)$, i.e., within the strong regime. Column 7 in Table I shows that the deviation of (17) from $\Upsilon_{\rm sph}$ amounts to less than 10%. For smaller Q, the restoration becomes increasingly efficient, so that $\Upsilon_{\rm eff}$ further approaches $\Upsilon_{\rm sph}$ in the weak regime. Thermally produced cosmological perturbations become negligible for $Q < 10^{-4}$, practically recovering standard cold inflation. The predicted warm inflation temperatures lie below $T \lesssim 6 \times 10^{14}$ GeV, with temperatures decreasing for larger values of Q. The corresponding quartic couplings for which inflation is warm lie below $\lambda \lesssim 10^{-13}$. We provide a comprehensive list of all the relevant parameters discussed here in Table I.

In our model, the interaction of the inflaton ϕ with the SM is already fully specified. Therefore, the transition from inflation to radiation domination is calculable, and, in the strong regime, accelerated expansion directly evolves to a period dominated by a quark-gluon plasma. This is reflected in the fact that we can uniquely predict r as a function of n_s (see Fig. 1), with no additional uncertainty arising from unknown interactions of the inflaton with the SM. Moreover, the direct coupling of the inflaton to QCD can lead to observable signatures at low energies [57]. The values for the coupling $f \sim 10^{10}$–10^{13} GeV lie within reach of proposed searches for axionlike particles [62,63]. However, Hubble dilution alone suffices to ensure that $\phi \ll f$ at the temperature of the QCD phase transition, implying that ϕ oscillations around the minimum of the potential $V(\phi) \simeq \lambda\phi^4 + \Lambda_{\rm QCD}^4(1 - \cos(\phi/f - \theta))$ contribute only a small fraction to the dark matter density, where θ is the CP-violating angle in QCD. Since $\lambda\phi^4$ still gives an important contribution to the potential at low energies, satisfying the experimental bound on CP violation in the strong interaction requires $|\phi/f - \theta| \ll 10^{-10}$ at the minimum, i.e., the same amount of tuning as in QCD, and can potentially be achieved with a standard QCD axion [64–66].

Conclusion—We showed for the first time that warm inflation can be driven by sphaleron heating from SM gauge interactions alone. Our model comprises a minimal extension of the SM by a single scalar inflaton field ϕ with an axionlike coupling to gluons (6). A crucial point is that the suppressing effect which light fermions have on the efficiency of sphaleron heating is partially alleviated by the expansion of the Universe, leading to the effective friction coefficient in Eq. (16). Notably, this coefficient can have different functional dependencies on the inflaton field value for different effective potentials in Eq. (18). Additionally, it predicts a smooth transition from a warm inflationary period to a Universe dominated by a quark-gluon plasma. For a monomial quartic potential (19), the model contains only two parameters and is consistent with CMB observations as well as laboratory experiments. Its minimality in terms of both the potential during inflation and the clear connection to the SM make this scenario highly predictive and a target for tests with future CMB and large-scale structure surveys as well as laboratory experiments, offering a unique opportunity to probe the connection between cosmic inflation and particle physics.

Note added—Recently, CMB predictions of our model were further investigated in [67,68]. Both works confirm that our proposal is compatible with current observational bounds.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

Appendix—The spectrum of cosmic perturbations for warm inflation has not been derived within quantum field theory at the same level of rigor as for standard inflation, and the predictions presented in the literature exhibit minor discrepancies [16,22,67–73]. However, our main conclusions are not sensitive to these differences. We use the predictions for A_s from [16] (recently confirmed by [68]):

$$\Delta_s^2(\phi_*) = \frac{H^2(\phi_*)}{4\pi^2\dot{\phi}^2(\phi_*)}(H(\phi_*)^2 + H(\phi_*)T(\phi_*)F[Q(\phi_*)]), \tag{A1}$$

where [16,22]

$$F(Q_*) = 3.24\times10^{-4}Q_*^7 + 168Q_*\left(\frac{1}{3}\left(1+\frac{9Q_*^2}{25}\right)+\frac{2}{3}\tanh\frac{1}{30Q_*}\right). \tag{A2}$$

The resulting scalar index $n_s - 1 \equiv d\ln\Delta_s^2/dN$ cannot be written in closed form in all generality for the regime in which Q transitions from the weak to the strong regime, and we, thus, calculate it numerically.

The scalar-to-tensor ratio is

$$r = \frac{8\dot{\phi}_*^2}{M_{\rm pl}^2 H_* T_* F(Q_*)}. \tag{A3}$$

Equation (A2) was derived for $\Upsilon_{\rm sph}$ rather than $\Upsilon_{\rm eff}$, for which the temperature no longer scales as a simple power law. This is a subtle complication, as it has been shown that the scalar perturbations are quite sensitive to the scaling with temperature in the friction coefficient for $Q > 1$ [69,70]. However, in the absence of calculations of the scalar power spectrum given $\Upsilon_{\rm eff}$, we proceed with Eq. (A1) as the best available approximation, and column 7 in Table I confirms *a posteriori* that the error is small.

Finally, the number of inflationary e-foldings after the generation of CMB perturbations is (cf. [74])

$$N_* = \ln\left(\left(\frac{g_{*0}}{g_*}\right)^{1/3}\frac{T_0}{T_{RH}}\frac{H_*}{k_*}\right), \tag{A4}$$

to be matched with Eq. (4). Here, $g_{*0} = 3.94$ [75] is the effective number of degrees of freedom today, $T_0 \approx 2.7$ K corresponds to the present CMB temperature, H_* is the Hubble scale at CMB generation, and we take $k_* = 0.05\ {\rm Mpc}^{-1}$ as the pivot scale. Given that the inflaton interactions are fully specified in our model, reheating is, in principle, calculable. In Eq. (A4), we have taken the transition from inflation to radiation domination as instantaneous and believe this to be a good approximation due to the strong coupling $\sim\alpha T/f$ with SM degrees of freedom. Consequently, we set T_{RH} to the temperature of the thermal plasma when $\epsilon_V(\phi_{\rm end}) = 1$.

TABLE I. Parameters of interest for warm SM inflation. λ, the potential slope, is a free parameter and scans the parameter space along Q. ϕ_* and f are fixed by N_*, and $A_s \approx 2\times10^{-9}$. All other parameters are derived. The EFT cutoff is $\Lambda_{\rm cutoff} \equiv (\alpha_s/8\pi f)^{-1}$. We present values to three significant figures to facilitate reproducibility when using Eqs. (9) and (A2). However, when accounting for theoretical uncertainties, we estimate that only one significant figure can be considered as reliable for the derived quantities (two figures for n_s, theoretical uncertainty not shown in Fig. 1).

λ	$(\phi_*/M_{\rm pl})$	$f[10^{12}$ GeV]	$T[10^{12}$ GeV]	α_s	T/H	$(2N_f N_c^4\alpha_s^5 T/\sqrt{\lambda\phi_*^4/3M_{\rm pl}^2})$	$T/\Lambda_{\rm cutoff}$	N_*	Q	n_s	r
10^{-21}	5.02	0.146	8.75	0.0269	7820	8.99×10^{-2}	0.064	52.8	14.9	0.9696	2.19×10^{-11}
10^{-20}	6.19	0.264	19.1	0.0263	3551	3.63×10^{-2}	0.076	53.7	9.22	0.9681	5.02×10^{-10}
10^{-19}	8.22	0.518	44.1	0.0257	1474	1.34×10^{-2}	0.087	54.6	4.78	0.9681	1.54×10^{-8}
10^{-18}	11.1	1.05	101	0.0251	582	4.69×10^{-3}	0.097	55.6	2.19	0.9678	5.29×10^{-7}
10^{-17}	15.6	2.30	219	0.0246	202	1.46×10^{-3}	0.093	56.6	0.661	0.9672	2.01×10^{-5}
10^{-16}	20.0	4.95	329	0.0243	58.9	4.03×10^{-4}	0.064	57.5	0.0889	0.9709	5.34×10^{-4}
10^{-15}	21.4	8.71	381	0.0242	18.8	1.26×10^{-4}	0.042	58.2	0.0122	0.9803	6.96×10^{-3}