Quantum Scaling in Energy Correlators beyond the Confinement Transition

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We study the QCD scaling behavior of the small-angle energy-energy correlator (EEC), focusing on the transition between its perturbative preconfinement and nonperturbative postconfinement regimes. Applying the light-ray operator product expansion (OPE), we develop a formalism that describes the scaling of the EEC with the input energy Q in the transition and the postconfinement region, where the latter quantum scaling is determined by the $J = 5$ DGLAP anomalous dimension. A key result of our Letter is a novel connection between the light-ray OPE and the dihadron fragmentation function (DFF), where we show that the nonperturbative OPE coefficients correspond to moments of the DFF. This finding establishes a new paradigm for studying hadronization. Our theoretical predictions are validated against Monte Carlo simulations for both e^+e^- and pp collisions, showing excellent agreement. The potential role of the quantum scaling in the precision determination of α_s is also discussed.

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Introduction—Since its discovery in the 1970s, quantum chromodynamics (QCD) has been the established theory of the strong interactions, governing a rich array of phenomena from the confinement of quarks and gluons to the emergence of jets and the formation of quark-gluon plasma [1]. Despite five decades of progress, the complex, nonperturbative nature of QCD continues to pose significant challenges, necessitating the development of novel theoretical and experimental tools.

A key phenomenon in QCD that remains a mystery is the process of hadronization [2], a term describing the real-time dynamical transition of quarks and gluons into color-neutral hadrons, see Fig. 1(a). There are two popular approaches to describe this process: phenomenological models implemented in parton showers, such as string and cluster hadronization [3,4]; and factorization theorems, which formally separate perturbative and nonperturbative dynamics via universal fragmentation functions [5]. To progress beyond the current paradigm, new experimental probes sensitive to the formation of hadrons are needed, along with theoretical advances that can interpret them.

Recently, it has been revealed from CMS Open Data analysis [7] and direct experimental measurements [8–13], that the energy-energy correlator (EEC) [14] exhibits two scaling regimes when measured in jet substructure, which we call "preconfinement scaling" and the "postconfinement plateau," along with a smooth transition between them, as illustrated in Fig. 1(b). The preconfinement scaling regime has been understood in terms of perturbative quark/gluon splitting [15,16], along with systematic methods for incorporating nonperturbative corrections [17–20]. The postconfinement plateau is a new feature that has not been fully understood and has inspired many recent studies [21–25].

The EEC is a correlation function of the energy operator $\mathcal{E}(\boldsymbol{n})$ in a state created from e^+e^- annihilation at center-of-mass energy Q, denoted as $\langle \mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)\rangle_Q$. The energy operator is the simplest example of a light-ray operator [26,27], and it can be expressed as an integral of the local energy-momentum tensor [26–28],

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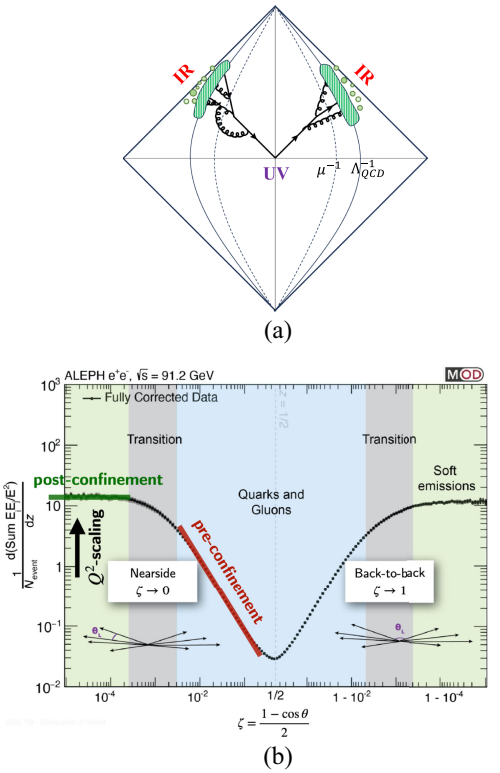


FIG. 1. (a) Schematic illustration of quark and gluon hadronizing into color-neutral hadrons; (b) illustration for different scaling behavior of EEC. Part figure (b) is adapted from [6].

$$\mathcal{E}(\boldsymbol{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt T_{0\boldsymbol{n}}(t, \vec{r}). \tag{1}$$

Taking the limit, (1) can be viewed as an integral along future null infinity, at a location $\boldsymbol{n}$ on the celestial sphere. The action of $\mathcal{E}(\boldsymbol{n})$ on a free particle state in the direction $\boldsymbol{n}$ gives its energy $\mathcal{E}(\boldsymbol{n})|\boldsymbol{p}\rangle = \omega_{\boldsymbol{p}}\delta^{(2)}[\boldsymbol{n} - (\boldsymbol{p}/|\boldsymbol{p}|)]|\boldsymbol{p}\rangle$. The energy operator has obvious importance in perturbative QCD (pQCD), due to its infrared and collinear safety properties [29], and has also drawn significant interest from a theoretical point of view; see [30] for a recent review.

In this Letter, we present two complementary perspectives on the Q^2-scaling of the EEC in the postconfinement plateau, providing insight into the dynamical hadronization transition. Our first perspective is based on the light-ray operator product expansion (OPE) [31,32]. Originally formulated in conformal field theory, the light-ray OPE has recently also found applications in QCD [18,33,34]. It expands a product of energy operators in a series of light-ray operators, whose scaling dimensions and Lorentz spins govern the dependence of the EEC on Q^2 and the angle, respectively. We focus here on the Q^2-scaling in the postconfinement and dynamical transition regimes.

Our second, complementary, perspective provides a description in terms of fragmentation functions. We demonstrate how the Q^2-scaling in the post-confinement regime can be described by the dihadron fragmentation function (DFF) formalism, a framework widely applied in hadron physics [35–44]. By bridging these two views, we establish a novel connection between the light-ray OPE and the phenomenological DFF formalism, a link that merits further exploration. To conclude, we show that the predictions from our OPE framework are in agreement with PYTHIA simulations, validating our approach.

Light-ray OPE analysis—A light-ray operator $\mathcal{D}(\boldsymbol{n})$ associated to a position $\boldsymbol{n}$ on the celestial sphere is characterized by a Lorentz spin $J_L \in \mathbb{C}$, which is its weight under a boost in the $\boldsymbol{n}$ direction. We also assign $\mathcal{D}(\boldsymbol{n})$ a dimension Δ_L equal to *minus* its mass dimension [26,27]. In a nonconformal theory, Δ_L can depend on a renormalization scale via the running coupling, $\Delta_L[J_L, \alpha_s(\mu^2)]$ [45,46]. The energy operator $\mathcal{E}$ has $J_L = -3$ and $\Delta_L = -1$ (independent of scale).

The work [46] argues that pQCD possesses an intrinsic set of renormalized light-ray operators $\mathcal{D}_{J_L,i}(\boldsymbol{n}, \mu^2)$, labeled by J_L, a Regge trajectory index i, and depending on a renormalization scale μ. These light-ray operators can be visualized on a Chew-Frautschi plot, which displays $1 - \Delta_L$ vs $-1 - J_L$ for each Regge trajectory, see Fig. 2. The scaling dimension $-\Delta_{L,i}[J_L, \alpha_s(Q^2)]$ characterizes the growth of matrix elements $\langle \mathcal{D}_{J_L,i}(\boldsymbol{n}, \mu^2)\rangle_Q$ with Q.

Furthermore, [46] argues that any measurement of hadrons at infinity can be *matched* in the regime $Q \gg \Lambda_{\text{QCD}}$ onto a linear combination of pQCD detectors $\mathcal{D}_{J_L,i}(\boldsymbol{n}, \mu^2)$. From this perspective, to determine the scaling of $\langle \mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)\rangle_Q$ with Q, we should match $\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)$ [viewed as an operator in the effective field theory (EFT) of hadrons] onto a linear combination of QCD detectors. This matching is nonperturbative. However, by Lorentz invariance, it is diagonal in boost weight: hadronic operators with definite J_L match onto QCD light-ray operators with the same J_L.

Thus, we first decompose $\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)$ into hadronic operators with definite boost weight J_L. This is accomplished with the light-ray OPE in the EFT of hadrons,

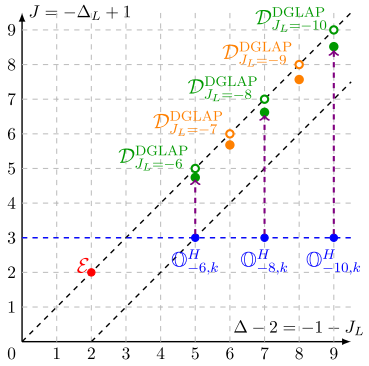


FIG. 2. Chew-Frautschi plot illustrating the light-ray OPE for the EEC in the free hadron region (blue) and detector matching (purple arrows). The detector positions in the free theory (open green circles) shift to their QCD-corrected locations at the scale μ (solid green points). Interaction between hadrons may induce odd J_L (orange) contributions.

$$\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)=\sum_{J_L,k}A^{(k)}_{J_L}C_{J_L}(\boldsymbol{n}_1,\boldsymbol{n}_2,\partial_{\boldsymbol{n}_2})\mathbb{O}^H_{J_L,k}(\boldsymbol{n}_2). \quad (2)$$

Here, each hadron operator $\mathbb{O}^H_{J_L,k}$ has definite boost weight J_L, and a label k representing different hadron operators. The differential operator

$$C_{J_L}(\boldsymbol{n}_1,\boldsymbol{n}_2,\partial_{\boldsymbol{n}_2})=|\boldsymbol{n}_{12}|^{-6-J_L}[1+O(|\boldsymbol{n}_{12}|)], \quad (3)$$

where $\boldsymbol{n}_{12}=\boldsymbol{n}_1-\boldsymbol{n}_2$, is entirely determined by Lorentz symmetry and sums over celestial descendants. (The operators $\mathbb{O}^H_{J_L,k}$ can also have nontrivial transverse spin [32,33], but we suppress this for brevity.) We emphasize that the operators $\mathbb{O}^H_{J_L,k}$ and OPE coefficients $A^{(k)}_{J_L}$ can be defined and computed entirely in the EFT of hadrons.

In the postconfinement regime, we can approximate hadrons as nearly free massless particles. In this approximation, the OPE (2) consists of a contact term proportional to the energy-squared operator $\mathcal{E}^{(2)}$, and a sum of double-twist light-ray operators $[\mathcal{E}\mathcal{E}]_{n,j}$ with boost weights $J_L=-6-2n-j$, transverse spin j, and classical dimension $\Delta_L=2\Delta_{L,\mathcal{E}}=-2$. The precise form of these operators and their OPE coefficients will not be important for us —only their quantum numbers. For simplicity, we will focus on spherically symmetric states, so that nonzero transverse spin operators do not contribute [32,33].

In high-energy scattering $Q\gg\Lambda_{\mathrm{QCD}}$, we can match the $\mathbb{O}^H_{J_L,k}$ onto perturbative detectors [46]. As illustrated in Fig. 2, the twist-2 DGLAP detectors $\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}=(\mathcal{D}^{\mathrm{DGLAP}}_{J_L,q},\mathcal{D}^{\mathrm{DGLAP}}_{J_L,g})^T$ have the largest $-\Delta_L$ for a fixed J_L and hence give the dominant contribution in the matching

$$\mathbb{O}^H_{J_L,k}\approx\vec{B}^{(k)}_{J_L}(\mu^2;\Lambda^2_{\mathrm{QCD}})\cdot[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R(\mu^2), \quad (4)$$

where μ is the factorization scale, $[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R$ are renormalized detectors, and the classical behavior of matching coefficients $\vec{B}^{(k)}_{J_L}$ is $\Lambda^{4+J_L}_{\mathrm{QCD}}$ by dimensional analysis. Combining (2) and (4), we obtain the leading light-ray OPE for $\langle\mathcal{E}\mathcal{E}\rangle_Q$ in the regime $\zeta<\Lambda^2_{\mathrm{QCD}}/Q^2$,

$$\langle\mathcal{E}\mathcal{E}\rangle_Q\approx\sum_{J_L}f_{J_L}(\zeta)\vec{R}_{J_L}(\mu^2;\Lambda^2_{\mathrm{QCD}})\cdot\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R(\mu^2)\rangle_Q. \quad (5)$$

Here, $f_{J_L}(\zeta)=\zeta^{(-6-J_L/2)}[1+O(\zeta)]$ is a celestial block, obtained by acting with the differential operator $C_{J_L}(\boldsymbol{n}_1,\boldsymbol{n}_2,\partial_{\boldsymbol{n}_2})$ on a light-ray one-point function [31]. $\vec{R}_{J_L}=\sum_k A^{(k)}_{J_L}\vec{B}^{(k)}_{J_L}$ is a combined OPE coefficient. The matrix element $\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R(\mu^2)\rangle_Q$ is angle-independent in a spherically symmetric state.

Let us make the free hadron approximation, so that the J_L appearing in the sum are those of the double-twist operators $[\mathcal{E}\mathcal{E}]_{n,0}: J_L=-6-2n$. Furthermore, we approximate the celestial block by its leading term $f_{J_L}(\zeta)\approx\zeta^{(-6-J_L/2)}=\zeta^n$ in the small angle limit. Making these substitutions, and defining $\mathrm{EEC}(\zeta,Q)=\langle\mathcal{E}\mathcal{E}\rangle_Q/Q^2$, we can write

$$\mathrm{EEC}(\zeta,Q)\approx Q^2\sum_{n\geq0}(\zeta Q^2)^n\vec{R}_{-6-2n}(\mu^2;\Lambda^2_{\mathrm{QCD}})\cdot\left(\frac{\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L=-6-2n}]_R(\mu^2)\rangle_Q}{Q^{4+2n}}\right). \quad (6)$$

The quantity in parentheses has vanishing classical dimension, and hence the leading term ($n=0$) in the OPE analysis predicts the classical Q^2 scaling for the height of the collinear plateau in $\mathrm{EEC}(\zeta,Q)$ [10,14,21,22].

This analysis can be immediately generalized to energy correlators with higher (possibly complex) energy weighting. We use $\mathcal{E}^{(\alpha)}$ to denote a detector measuring hadrons with weighting E^α, where $\mathrm{Re}(\alpha)>0$. The generalization of (6) to $\mathrm{E}^\alpha\mathrm{E}^\beta\mathrm{C}=\langle\mathcal{E}^{(\alpha)}\mathcal{E}^{(\beta)}\rangle_Q/Q^{\alpha+\beta}$ is

$$\mathrm{E}^\alpha\mathrm{E}^\beta\mathrm{C}(\zeta,Q)=Q^2\sum_{n\geq0}(\zeta Q^2)^n\vec{R}^{\alpha,\beta}_n(\mu^2;\Lambda^2_{\mathrm{QCD}})\cdot\frac{\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{-\alpha-\beta-4-2n}]_R(\mu^2)\rangle_Q}{Q^{2+\alpha+\beta+2n}}, \quad (7)$$

where the special case $\vec{R}^{1,1}_n$ reduces to $\vec{R}_{-6-2n}$ in (6).

Following [18], we can further study the quantum scaling violation based on (7). To see the quantum scaling behavior, we strip off the classical Q dependence

$$F_{\alpha,\beta}(\zeta Q^2,Q^2)\equiv\frac{1}{Q^2}\mathrm{E}^\alpha\mathrm{E}^\beta\mathrm{C}(\zeta,Q). \quad (8)$$

The scaling behavior is controlled by the renormalization group (RG) equation for $[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R$

$$\frac{d}{d\ln\mu^2}[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R=\hat{\gamma}_{-J_L-1}\cdot[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R, \quad (9)$$

where $\hat{\gamma}_J=(\alpha_s/4\pi)\hat{\gamma}^{(0)}_J+\mathcal{O}(\alpha_s^2)$ is the timelike DGLAP anomalous dimension matrix [16]. It coincides with the spacelike anomalous dimension at leading order [47], following from the reciprocity relation [45,48,49]. As a consequence, we conclude that spin $J=5$ controls the quantum scaling of the postconfinement plateau. We note that in the preconfinement regime the reciprocity relation also works for the EEC in a fixed-coupling approximation [16,31,50].

By solving the RG equation, we introduce an evolution operator $U_{J_L}(\mu_1^2,\mu_2^2)$ to resum large logarithms related to the scaling violation

$$F_{\alpha,\beta}(\zeta Q^2,Q^2)=\sum_{n\geq0}(\zeta Q^2)^n\vec{R}^{\alpha,\beta}_n(\mu_J^2,\Lambda^2_{\mathrm{QCD}})\cdot U_{J_L}(\mu_J^2,\mu_H^2)\cdot\frac{\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R(\mu_H^2)\rangle_Q}{Q^{-J_L-2}}\bigg|_{J_L=-\alpha-\beta-4-2n}, \quad (10)$$

where the hard and jet scales are chosen to be $\mu_H\sim\mathcal{O}(Q)$, $\mu_J\sim\mathcal{O}(\Lambda_{\mathrm{QCD}})$. This is main prediction for the EEC in the

postconfinement and transition regions from the light-ray OPE perspective. Below, we will compare Eq. (10) with a Monte Carlo simulation at leading logarithmic accuracy. Under such an approximation, the evolution kernel $U_{J_L}(\mu_1^2,\mu_2^2)$ is

$$U_{J_L}^{\mathrm{LL}}(\mu_1^2,\mu_2^2)=\left[\frac{\alpha_s(\mu_2^2)}{\alpha_s(\mu_1^2)}\right]^{\hat{\gamma}^{(0)}_{-J_L-1}/\beta_0}, \tag{11}$$

where $\beta_0=\frac{11}{3}C_A-\frac{4}{3}n_fT_F$ is the one-loop beta function constant in QCD.

To summarize, the spectrum of operators $\mathbb{O}^H_{J_L,k}$ in the hadron light-ray OPE (2) determines the pQCD detector spectrum appearing in (5), whose coefficient exhibits scale evolution determined by the corresponding anomalous dimension. These statements continue to hold in the presence of hadron interactions, which can modify the spectrum of J_L's appearing. See examples in [51].

Inspired by the expansion (5) in the $\zeta Q^2\ll\Lambda^2_{\mathrm{QCD}}$ limit and extensive studies in $\zeta Q^2\gg\Lambda^2_{\mathrm{QCD}}$ limit [16–18,34,58], we conjecture a unified formula for the EEC in the collinear limit

$$\mathrm{EEC}(\zeta,Q)=\sum_i\int_\Gamma\frac{d\Delta_L}{2\pi i}f_{\Delta_L-\tau_i}(\zeta)\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^{\Delta_L+2}\times C_{\Delta_L,i}(\mu^2/\Lambda^2_{\mathrm{QCD}})\frac{\langle\mathcal{D}_{\Delta_L,i}(\mu^2)\rangle_Q}{Q^{-\Delta_L}}, \tag{12}$$

where the summation is over the ith Regge trajectory of light-ray operators labeled with twist $\tau=\Delta_L-J_L$. $\Gamma=c+i\mathbb{R}$ is a contour along the imaginary direction (see Fig. 3). Different contour deformations can reproduce known expansion structures in both preconfinement and postconfinement regions. We provide more details in [51].

Relation to the dihadron fragmentation—The nonperturbative nature of the postconfinement EEC allows the light-ray OPE analysis to be framed in terms of dihadron fragmentation, a topic that has received extensive attention in the literature [35–44]. Specifically, we consider the scenario where in the high energy limit $Q^2\gg\Lambda^2_{\mathrm{QCD}}$, the dominant contribution to the postconfinement EEC comes from a single parton fragmenting into a dihadron pair. In particular, the invariant mass of the hadron pair is related to their angular separation by $m^2/4=z_1z_2Q^2\zeta$, where z_i is the energy fraction carried by each hadron. In this picture, the standard EEC factorization form for small ζ values [16] remains valid, but the perturbative EEC jet function, see Ref. [16] or [51], is replaced by the nonperturbative mass-dependent DFF, see, e.g., Ref. [44]. The all order factorization reads

$$\mathrm{EEC}(\zeta,Q)=\int_0^1 dx x^2\vec{H}\left(x,\frac{Q^2}{\mu^2}\right)\int_0^\infty dm^2\delta\left(\zeta-\frac{m^2}{4x^2\xi_1\xi_2Q^2}\right)\cdot\sum_{(a,b)}\int d\xi_1d\xi_2\frac{\xi_1\xi_2}{4}\vec{D}_{h_ah_b}(\xi_1,\xi_2;m^2;\mu^2;\Lambda^2_{\mathrm{QCD}}), \tag{13}$$

where $\vec{H}=(2H_q,H_g)$ [59] is the hard function that produces the fragmenting parton with momentum fraction x. $\vec{D}_{h_ah_b}=(D_{h_ah_b/q},D_{h_ah_b/g})$ is the mass-dependent DFF. Note that this Eq. (13) reduces to the perturbative factorization in [16] when h_a and h_b comes from independent fragmentation (details in [51]).

In the postconfinement regime, we assume $\vec{D}_{h_ah_b}$ have analytic expansion around $0<m^2\ll\Lambda^2_{\mathrm{QCD}}$. Taylor expansion on m^2 gives

$$\frac{\mathrm{EEC}(\zeta,Q)}{Q^2}=\sum_{n\geq0}(4Q^2\zeta)^n\vec{\mathcal{H}}_{4+2n}\left(\frac{Q^2}{\mu^2}\right)\cdot\vec{\mathcal{J}}^{(n)}_{1,1}(\mu^2;\Lambda^2_{\mathrm{QCD}}), \tag{14}$$

where $\vec{\mathcal{H}}_{J-1}$ is the moment of the hard function $\vec{\mathcal{H}}_{J-1}(Q^2/\mu^2)\equiv\int_0^1 dx x^{J-1}\vec{H}(x;Q^2/\mu^2)$, and the nonperturbative jet function is given by

$$\vec{\mathcal{J}}^{(n)}_{\alpha,\beta}(\mu^2;\Lambda^2_{\mathrm{QCD}})=\frac{1}{n!}\sum_{(a,b)}\int_0^1 d\xi_1d\xi_2\xi_1^{1+\alpha+n}\xi_2^{1+\beta+n}\partial^n_{m^2}\vec{D}_{h_ah_b}(\xi_1,\xi_2;m^2;\mu^2;\Lambda^2_{\mathrm{QCD}})\Big|_{m^2=0}. \tag{15}$$

Critically, Eq. (14) is identical in form to the light-ray OPE in Eq. (6). This allows us to identify the nonperturbative OPE coefficients $\vec{R}_{-6-2n}$ with the moment of the nth derivative of the DFF

$$\vec{R}_{-6-2n}(\mu^2;\Lambda^2_{\mathrm{QCD}})=4^n\vec{\mathcal{J}}^{(n)}_{1,1}(\mu^2;\Lambda^2_{\mathrm{QCD}}), \tag{16}$$

and the detectors $[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L}]_R$ with the moments of the hard function $\vec{H}$ [34],

$$\langle[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L=-6-2n}]_R\rangle=Q^{4+2n}\vec{\mathcal{H}}_{4+2n}(Q^2/\mu^2). \tag{17}$$

Equations (16) and (17) established a dictionary between the light-ray OPE formalism and the DFF factorization theorem.

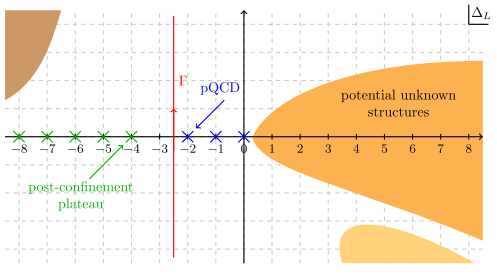


FIG. 3. Conjectured contour in Δ_L plane for light-ray OPE of EEC in the QCD high-energy collision. Little knowledge is known about complex structure in Δ_L.

As a consistency check, we compare their renormalization group evolution. The hard function $\vec{\mathcal{H}}_{4+2n}(\mu)$ for the single inclusive parton production obeys the same DGLAP evolution as $[\vec{\mathcal{D}}^{\mathrm{DGLAP}}_{J_L=-6-2n}]_R$ in Eq. (9), since the moment variable is given by $J-1=-J_L-2=4+2n$. By consistent condition $d\mathrm{EEC}/d\ln\mu^2=0$, the jet function must then evolve as

$$\frac{d\vec{\mathcal{J}}^{(n)}_{1,1}(\mu^2;\Lambda^2_{\mathrm{QCD}})}{d\ln\mu^2}=\vec{\mathcal{J}}^{(n)}_{1,1}(\mu^2;\Lambda^2_{\mathrm{QCD}})\cdot\hat{\gamma}_{5+2n}, \tag{18}$$

where $\hat{\gamma}_J$ is the DGLAP anomalous dimension matrix. This evolution correctly matches the known evolution of the DFF at nonzero angular separation [35]. This agreement in the scaling behavior from both the DFF and OPE perspectives validates the connection between these two independent approaches.

The relation is straightforwardly generalized to the energy correlators with higher energy weights in Eq. (7), which will relate to higher moments of the DFF, i.e., $\mathcal{J}^{(n)}_{\alpha,\beta}=\vec{R}^{\alpha,\beta}_n$.

Phenomenological studies—In this section, we validate our theoretical setup by comparing the leading logarithm (LL) prediction from Eq. (10) with Monte Carlo simulations from PYTHIA 8.2 [60]. We begin by studying global energy correlators in e^+e^- annihilation, using $e^+e^-\to\gamma/Z^*\to X(q\bar{q})$ and $e^+e^-\to h^*\to X(gg)$ to source the quark and gluon initiated energy correctors, respectively.

Figure 4 presents this comparison for the $F_{1,1}$ and $F_{2,2}$ correlators at the center-of-mass energies of $Q=200$, 500, and 1000 GeV. As detailed in the figure caption, our theoretical predictions are derived by fitting the light-ray OPE to the $Q=500$ GeV simulation data and then evolving the result to the other energies. Details on the initial condition fitting are given in the [51]. Across all scenarios, the predictions (solid lines) show excellent agreement with the Monte Carlo simulation (dots). At lower energies, we expect $1/Q$ correction and higher-loop effects are important, the analysis of which we leave to future work. It is interesting to note that the convergence of the light-ray OPE fitting breaks down around $Q^2\zeta\sim\mathcal{O}(50)$ GeV^2, setting the boundary between the post- and preconfinement regimes.

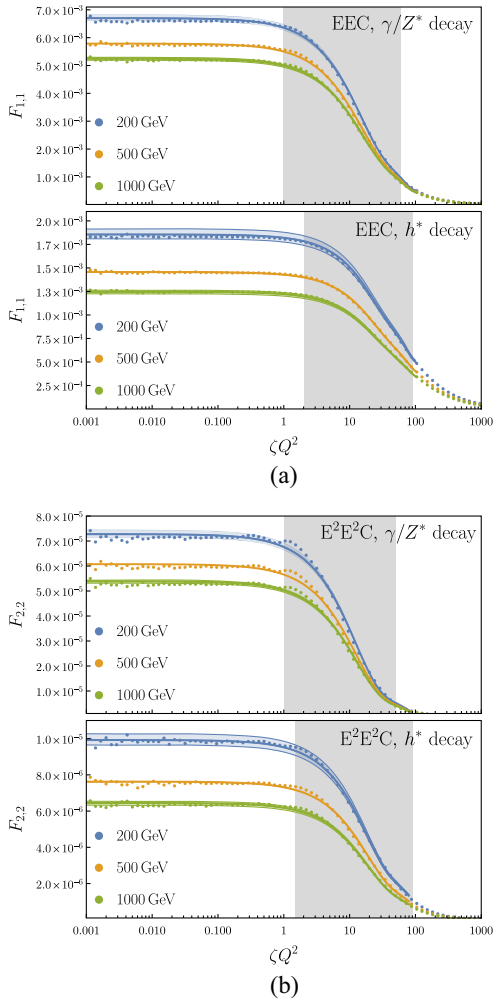


FIG. 4. Validation of the LL-evolved light-ray OPE against PYTHIA simulations in e^+e^- annihilation. (a) shows the $F_{1,1}$ correlator, and (b) shows the $F_{2,2}$ correlator for both quark-initiated (top) and gluon-initiated (bottom) jets. The solid lines are theoretical predictions obtained by fitting the OPE in Eq. (7) to the PYTHIA data (dots) at $Q=500$ GeV and then evolving the result to $Q=200$ and 1000 GeV with a scale choice of $\mu_H=Q/4$. The bands are scale uncertainties obtained by varying μ_H up and down by a factor of 2.

Our framework automatically extends to energy correlators measured inside jets at the LHC, as well as track-based measurements [61,62], which offer superior angular resolution [6], making them more suitable for investigating the postconfinement regime. Detailed studies on this, along with a discussion on its potential application to the precise determination of α_s, can be found in [51].

Conclusion and outlook—We developed a light-ray OPE approach to studying the Q^2-scaling behavior of the EEC at small angles, focusing on the postconfinement and dynamical transition regimes. We successfully described the Q^2-scaling observed in PYTHIA simulations for a range of energies and collision processes in the transition and the postconfinement plateau, where in the latter case, the quantum scaling is determined by the DGLAP anomalous dimension with $J=5$. We also discuss energy correlators with more general weights, $\mathrm{E}^\alpha\mathrm{E}^\beta\mathrm{C}$.

Furthermore, we established a direct connection between the light-ray OPE formalism and the mass-dependent dihadron fragmentation function framework, demonstrating that the OPE coefficients correspond to moments of (the derivative of) the DFF.

Our Letter opens up several avenues for future exploration. Perhaps the primary priority is to test the quantum scaling behavior predicted in this Letter experimentally. This would likely require track-based measurement to

reach high accuracy in the postconfinement regime, for which our predictions remain valid. We further propose to extract α_s from precise scaling measurements. Our findings also suggest that the nearside EEC provides a direct probe of the mass-dependent DFF. This offers a novel experimental method to measure these nonperturbative distribution functions, which are crucial for understanding the hadronization process. The established link between the phenomenological DFF formalism and the theoretical light-ray OPE creates a new paradigm for studying nonperturbative QCD.

Note added—Recently, we became aware of Ref. [63], where the scaling behavior in the near-side region was also studied using the transverse momentum dependent dihadron fragmentation formalism. Related work appears in the following Letters [64,65].

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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