

Equivariant Localization for Higher Derivative Supergravity

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Conformal supergravity provides an effective off-shell formalism to study higher derivative actions. We show that the $D = 4$, $\mathcal{N} = 2$ theory admits equivariantly closed forms. These may be used to compute closed-form expressions for supersymmetric observables in a general class of supergravity theories with higher derivative couplings, without any need to solve equations of motion. We discuss applications to holography, presenting results for on-shell actions that are conjecturally valid to all orders in the perturbative $1/N$ expansion.

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Introduction—Equivariant localization is a powerful new technique in supergravity [1]. It allows one to compute various supersymmetric observables, including on-shell actions, black hole entropies and central charges, without solving equations of motion. The structure seems to be universal, applying to supergravity theories coupled to arbitrary matter in general dimensions. However, so far the theory has only been developed for two-derivative theories.

On the other hand, higher derivative corrections to supergravity play an important role in our understanding of quantum gravity. This includes the computation of all-order corrections to the Bekenstein-Hawking-Wald entropy of supersymmetric asymptotically flat black holes, matching to a microstate counting, as reviewed in [2,3]. More recently, higher derivative corrections to gauged supergravity have led to new tests of holography [4–6], with various conjectures for generalizations of these results to all derivatives and coupling to matter appearing [5,7–10].

An obstacle in extending any higher derivative program is that, in general, constructing explicit supersymmetric solutions is intractable. The Lagrangians are complicated, and solving the resulting equations of motion is a formidable task even for solutions with a high degree of supersymmetry/symmetry. Here we show that this obstacle can be bypassed by extending the localization techniques of [1] to conformal supergravity. This allows us to analyze a broad class of all-order higher derivative couplings with vector multiplet matter in $D = 4$, $\mathcal{N} = 2$ off-shell supergravity.

The approach contains three main ingredients. First, a conformal Killing vector field ξ that can be constructed as a bilinear in the Killing spinor parametrizing the preserved supersymmetry. Second, a set of polyforms built from Killing spinor bilinears and supergravity fields that are annihilated by the differential operator $d_\xi \equiv d - \xi \lrcorner$. Third, localization theorems [11,12] and supersymmetry allow one to evaluate integrals of these differential forms purely in terms of the fixed point set where $\xi = 0$. These integrals may then be used to compute physical observables of the supersymmetric configuration, including the action, in terms of global topological data.

Our results allow us to prove various aspects of the holographic conjectures [5,7–10] and also extend them, thereby providing new tools in supersymmetric holography. They also apply to ungauged supergravity, with applications to asymptotically flat solutions. Analogous equivariant structures in conformal supergravity in other dimensions should also exist, with a variety of applications. Additional results for the $D = 4$ case appear in [13] and a localization result for $D = 5$ AdS black holes appears in [14].

Conformal supergravity—Higher derivative terms in supergravity are most easily constructed by starting with conformal supergravity, which realizes supersymmetry off shell. We consider $D = 4$, $\mathcal{N} = 2$ conformal supergravity in Euclidean signature [15] (see Refs. [16,17] for reviews in Lorentzian signature). The theory is obtained by gauging the superconformal group whose bosonic group consists of the product of the Euclidean conformal group, $SO(5,1)$, and the R-symmetry group $SU(2) \times SO(1,1)$. Various multiplets may be constructed, with a proliferation of auxiliary fields required to close the superconformal algebra. For our purposes only a few key combinations of fields are important.

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The basic multiplet of the theory is the Weyl multiplet, which is associated with gauging the superconformal algebra. The bosonic fields are

$$\text{Weyl: } \{e_\mu^a, b_\mu, \mathcal{V}_{\mu j}^i, A_\mu, \omega_\mu^{ab}, f_\mu^a, T_{ab}, D\}. \quad (1)$$

There are gauge fields associated with general coordinate transformations, namely the orthonormal frame e_μ^a , dilatations, b_μ , and the $SU(2) \times SO(1, 1)$ R-symmetry, $\mathcal{V}_{\mu j}^i, A_\mu$. Here $\mu, \nu = 1, \dots, 4$ and $a, b = 1, \dots, 4$ are spacetime and frame indices, while $i, j = 1, 2$ are $SU(2)$ R-symmetry indices. The multiplet includes composite gauge fields ω_μ^{ab} , f_μ^a and auxiliary fields T_{ab}, D , as well as fermions, which we suppress. The antisymmetric tensor T_{ab} will feature prominently. The bosons are all real, except for $\mathcal{V}_{\mu j}^i$ which is anti-Hermitian. Since we are ultimately interested in Poincaré supergravity, we partially gauge fix by setting $b_\mu = 0$, as is common.

Next, we consider chiral and antichiral multiplets, $\Phi_\pm$, with the $\pm$ sign correlated with chirality. For brevity, we refer to both of these as “chiral multiplets.” The bosonic fields are

$$\text{Chiral } \Phi_\pm: \{A_\pm, B_\pm^{ij}, F_{ab}^\mp, C_\pm\}. \quad (2)$$

Here $A_\pm, C_\pm$ are real scalar fields, $F_{ab}^\mp$ are real anti-self-dual/self-dual two-forms, $F_{ab}^\mp = \mp (1/2!) \epsilon_{ab}^{cd} F_{cd}^\mp$, and $B_\pm^{ij} = B_\pm^{(ij)}$ and satisfies the reality condition $(B_\pm^{ij})^* = \epsilon_{ik} \epsilon_{jl} B_\pm^{kl}$. All fields in the multiplet transform under Weyl transformations with a particular weight. For example, if $A_\pm$ has weight w then $C_\pm$ has weight $w + 2$, and the multiplet is then said to have weight w . The product of two chiral (or two antichiral) multiplets of weights w_1 and w_2 yields a chiral (or anti-chiral) multiplet of weight $w_1 + w_2$, with components given in [15].

The Weyl multiplet is itself related to the constrained sum of chiral and antichiral tensor multiplets. Furthermore, using them we can construct scalar chiral and antichiral Weyl-squared multiplets, $\mathbb{W}_\pm^2$, of weight $w = 2$, whose components are given in [15].

The bosonic fields of a single vector multiplet $\mathbb{V}$ are

$$\text{Vector } \mathbb{V}: \{X_\pm, W_\mu, Y^{ij}\}. \quad (3)$$

Here $X_\pm$ are two real scalar fields with weight $w = 1$, W_μ is a real gauge field and $Y^{ij} = Y^{(ij)}$ satisfies $(Y^{ij})^* = \epsilon_{ik} \epsilon_{jl} Y^{kl}$. For multiple vector multiplets we append an index I . We focus on the case of Abelian multiplets for simplicity and write $F(W) = dW$ for the curvature. A vector multiplet is a constrained combination of a chiral and an antichiral multiplet each with weight $w = 1$; expressions for the components are given in [15].

Finally, recall the T-log multiplet [18]. The construction starts with an antichiral/chiral multiplet $\Phi'_\mp$ of weight $w' \neq 0$ in superspace. One then constructs the logarithm and applies four superspace derivatives to obtain a chiral/antichiral T-log multiplet of weight $w = 2$, denoted $\mathbb{T}_\pm$.

Chiral and antichiral multiplets of weight $w = 2$ may be used to construct supersymmetric invariant actions, with Lagrangian 4-forms $\mathcal{L}_\pm$ of weight $w = 4$. The bosonic part is given by

$$I_\pm = \int \mathcal{L}_\pm = \int \frac{1}{2} \left[C_\pm + \frac{1}{16} A_\pm (T^\pm)^2 \right] \text{vol}_4, \quad (4)$$

where $(T^\pm)^2 \equiv T_{ab}^\pm T^{\pm ab}$, and vol_4 denotes the usual volume form. Each multiplet above forms an off-shell representation of the superconformal algebra, i.e., without needing to impose equations of motion from an action.

Equivariantly closed forms—We are interested in *supersymmetric configurations* of off-shell conformal supergravity, which are defined to be bosonic configurations such that the corresponding supersymmetry transformations vanish. These transformations are parametrized by symplectic Majorana spinors ϵ^i and η^i , corresponding to the $\mathcal{N} = 2$ Q and S supersymmetries, respectively [19]. We will only use linear combinations of the supersymmetry transformations in which the η^i have been eliminated and hence only the Q -supersymmetry parameters will appear in what follows, with chiral components $\epsilon_\pm^i \equiv \frac{1}{2} (1 \pm \gamma_5) \epsilon^i$, where γ_5 is the chirality operator.

Given a supersymmetric configuration with spinor ϵ^i , we introduce the following real scalars, real 1-form and complex 2-form bilinears:

$$\begin{aligned} S_\pm &\equiv \bar{\epsilon}_{i\pm} \epsilon_\pm^i, & \xi^b &\equiv 2i \bar{\epsilon}_{i+} \gamma_{(1)} \epsilon_-^i, \\ \Upsilon_{\pm i}^j &\equiv \bar{\epsilon}_{i\pm} \gamma_{(2)} \epsilon_\pm^j. \end{aligned} \quad (5)$$

Here $\gamma_\mu = \gamma_\mu^\dagger$, $\gamma_{(r)} \equiv (1/r!) \gamma_{\mu_1 \dots \mu_r} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_r}$ and $\bar{\epsilon} \equiv \epsilon^\dagger$. The supersymmetry transformations imply that the vector ξ^μ , dual to the 1-form ξ^b , is a conformal Killing vector [20]. Furthermore, since $\|\xi^b\|^2 = 4S_+ S_-$ this vector has fixed points when either $S_+ = 0$ or $S_- = 0$, and at such a fixed point, the spinor ϵ^i will be of definite chirality ϵ_-^i or ϵ_+^i , respectively.

Following [1], we next construct polyforms Ψ out of the bilinears (5) and supergravity fields, which are closed under the equivariant exterior derivative $d_\xi \equiv d - \xi \lrcorner$, so $d_\xi \Psi = 0$. Starting from the Weyl multiplet and a *general* chiral multiplet $\Phi_\pm$ of weight $w = 2$, with components given in (2), we find that $\Psi^\pm = \Psi_4^\pm + \Psi_2^\pm + \Psi_0^\pm$ are equivariantly closed, where

$$\begin{aligned}
 \Psi_4^\pm &\equiv \frac{1}{2} \left[C_\pm + \frac{1}{16} A_\pm (T^\pm)^2 \right] \text{vol}_4, \\
 \Psi_2^\pm &\equiv 2S_\mp \left(F^\mp + \frac{1}{2} A_\pm T^\pm \right) - B_\pm^{ij} \varepsilon_{jk} \Upsilon_{\mp i}^k, \\
 \Psi_0^\pm &\equiv \pm 4A_\pm S_\mp^2.
 \end{aligned} \tag{6}$$

Note that $\Psi_4^\pm$ is the Lagrangian 4-form in (4); we emphasize that $d_\xi \Psi^\pm = 0$ follows from off-shell supersymmetry, without imposing equations of motion that follow from a Lagrangian.

Next, for each vector multiplet we can construct an additional equivariantly closed form

$$\Psi_{(F)} \equiv F(W) + 2(X_+ S_- - X_- S_+), \tag{7}$$

where $F(W)$ is the field strength. This is used to derive the key “gluing rules” and “UV/IR relations,” below.

To evaluate $\Psi_0^\pm$ in (6) at fixed points of ξ , below, we need further input from supersymmetry. Starting from a general chiral multiplet $\Phi'_\pm$ of arbitrary weight w' , we obtain the expression (A2) in the Appendix for the 2-form $d\xi^\flat$. At a fixed point where $\xi = 0$, the skew eigenvalues of $\frac{1}{2}d\xi^\flat$ define the weights b_i , $i = 1, 2$, of the action of the vector field ξ . The connected components of the fixed point set are either isolated points (“nuts”) or surfaces (“bolts”) [21]. At a nut, $b_1, b_2 \neq 0$ and we can locally write $\xi = \sum_{i=1}^2 b_i \partial_{\varphi_i}$, where ∂_{φ_i} rotate each of the $\mathbb{R}_i^2$ in the tangent space $\mathbb{R}^4 = \mathbb{R}_1^2 \oplus \mathbb{R}_2^2$ at the nut. At a bolt, which will be some fixed Riemann surface, either $b_1 = 0$ or $b_2 = 0$, with $b\partial_\varphi$ rotating the normal $\mathbb{R}^2$. A computation using (A2) reveals that at a $\mp$ chirality fixed point we have

$$\begin{aligned}
 (b_1 \mp b_2)^2 &= \frac{1}{16} S_\mp^2 A_\pm|_{\mathbb{W}_\pm^2}, \\
 (b_1 \pm b_2)^2 &= \frac{2}{w'} S_\mp^2 A_\pm|_{\mathbb{T}_\pm},
 \end{aligned} \tag{8}$$

where $A_\pm|_{\mathbb{W}_\pm^2} = (T^\mp)^2$ and $A_\pm|_{\mathbb{T}_\pm}$ given by (A1) are the lowest weight components of the Weyl-squared multiplet and the T-log multiplet, respectively. Equations (8) play a crucial role in localizing higher derivatives.

Localization—We can now evaluate the action (4) for a general off-shell supersymmetric configuration, for an arbitrary chiral or antichiral multiplet of weight $w = 2$. The key point is that the Lagrangian 4-form appearing in (4) is the top form of the equivariantly closed form given in (6). Thus, we can evaluate using the Berline-Vergne-Atiyah-Bott (BVAB) theorem [11,12],

$$\int_M \Psi_4^\pm = I_\pm^{\text{nuts}} + I_\pm^{\text{bolts}} + I_\pm^{\partial M}. \tag{9}$$

Here $I_\pm^{\text{nuts}}, I_\pm^{\text{bolts}}$ are the fixed point contributions, which we assume have no component on the boundary ∂M of the

four-manifold M , and $I_\pm^{\partial M}$ is a specific boundary contribution. To simplify formulae, we now assume the fixed point set does not have orbifold singularities [22].

We henceforth focus on a specific class of theories which, after gauge fixing, are associated with Poincaré supergravity theories, both gauged and ungauged, which are relevant for holography and supersymmetric black holes, as mentioned in the introductory section. Consider chiral multiplets that are constructed from $n_v + 1$ vector multiplets $\mathbb{V}^I$, $I = 0, \dots, n_v$, and the higher derivative chiral Weyl-squared multiplets $\mathbb{W}_\pm^2$ and T-log multiplets $\mathbb{T}_\pm$ constructed from $\log \Phi'_\mp$, where $\Phi'_\mp$ has weight w' . These may be combined into a chiral multiplet of weight $w = 2$ by introducing general real functions $\mathcal{F}^\pm(X_\pm^I, A_\pm|_{\mathbb{W}_\pm^2}, A_\pm|_{\mathbb{T}_\pm})$, satisfying

$$\begin{aligned}
 &\lambda^2 \mathcal{F}^\pm(X_\pm^I, A_\pm|_{\mathbb{W}_\pm^2}, A_\pm|_{\mathbb{T}_\pm}) \\
 &= \mathcal{F}^\pm(\lambda X_\pm^I, \lambda^2 A_\pm|_{\mathbb{W}_\pm^2}, \lambda^2 A_\pm|_{\mathbb{T}_\pm}).
 \end{aligned} \tag{10}$$

The bosonic component fields of this chiral multiplet can be obtained from [15], with lowest weight field given by $A_\pm|_{\mathcal{F}_\pm} = \mathcal{F}^\pm(X_\pm^I, A_\pm|_{\mathbb{W}_\pm^2}, A_\pm|_{\mathbb{T}_\pm})$.

The expressions (6) show that the action for a chiral multiplet only receives contributions from fixed points where the spinor e^i has $-$ chirality, while the action for an antichiral multiplet only receives contributions where the spinor e^i has $+$ chirality. We first consider the nut contributions, which can be written in the compact form

$$\begin{aligned}
 I_\pm^{\text{nuts}} &= \sum_{\text{nuts} \mp} \pm \frac{16\pi^2}{b_1 b_2} \\
 &\times \mathcal{F}^\pm \left(S_\mp X_\pm^I, 16(b_1 \mp b_2)^2, \frac{w'}{2} (b_1 \pm b_2)^2 \right),
 \end{aligned} \tag{11}$$

where we used (8). For the bolts, which are Riemann surfaces Σ_g , of genus g , we find

$$\begin{aligned}
 I_\pm^{\text{bolts}} &= \sum_{\text{bolts} \mp} 16\pi^2 \left[\mathfrak{p}_{\Sigma_g}^I \partial_{X_\pm^I} \mathcal{F}^\pm \mp \mathcal{F}^\pm \int_{\Sigma_g} c_1(N\Sigma_g) \right. \\
 &\quad \left. - 32 \frac{\partial \mathcal{F}^\pm}{\partial A_\pm|_{\mathbb{W}_\pm^2}} \left(\int_{\Sigma_g} c_1(T\Sigma_g) \mp c_1(N\Sigma_g) \right) \right. \\
 &\quad \left. + w' \frac{\partial \mathcal{F}^\pm}{\partial A_\pm|_{\mathbb{T}_\pm}} \left(\int_{\Sigma_g} c_1(T\Sigma_g) \pm c_1(N\Sigma_g) \right) \right].
 \end{aligned} \tag{12}$$

Here $\mathcal{F}^\pm$ are evaluated at $\mathcal{F}^\pm[b^{-1}S_\mp X_\pm^I, 16, (w'/2)]$ where recall $\xi = b\partial_\varphi$ rotates the normal direction to the bolt Σ_g . $T\Sigma_g, N\Sigma_g$ denote tangent and normal bundles. Fixing an orientation these are complex line bundles, and c_1 denotes the first Chern class, in particular $\int_{\Sigma_g} c_1(T\Sigma_g) = 2 - 2g$. The magnetic flux $\mathfrak{p}_{\Sigma_g}^I$ in (12) is given by

$$\mathfrak{p}_{\Sigma_g}^I = \frac{1}{4\pi} \int_{\Sigma_g} F^I(W), \quad (13)$$

where recall $F^I(W)$ is the field strength of the Abelian gauge field in the vector multiplet $\mathbb{V}^I$.

The expressions (11) and (12) depend on chirality signs $\pm$, weights b_i , and fluxes/topological invariants, but also the scalars $S_{\mp} X_{\pm}^I$ evaluated at the fixed points. The former are global data, while the latter are not. However, the equivariantly closed forms $\Psi_{(F)}^I$ given in (7) may be used to systematically relate the scalars at different fixed points, and also to their values on the boundary ∂M , as explained in [23–25].

First consider a two-sphere $\Sigma = S^2$ [26] to which ξ is tangent. If ξ is not identically zero on Σ (so Σ is not a bolt), ξ will have fixed points at the north and south poles of the S^2 , which will be two isolated fixed points in the sum in (11). Then from BVAB we deduce

$$\mathfrak{p}_{S^2}^I = \frac{1}{2b_N} (\Psi_{(F)0|N}^I - \Psi_{(F)0|S}^I), \quad (14)$$

where $\Psi_{(F)0}^I = 2(X_+^I S_- - X_-^I S_+)$ is the zero-form part of $\Psi_{(F)}^I$ in (7), and we used the fact that the weights at the poles are related by $b_S = -b_N$. The spinor ϵ^i will have a fixed $\pm$ chirality at each pole and correspondingly $\Psi_{(F)0|\pm}^I = \mp 2X_{\mp}^I S_{\pm}$. Equation (14) then determines the scalars at one fixed point in terms of scalars at the other fixed point, together with a magnetic flux $\mathfrak{p}_{S^2}^I$. The topological formula $\chi(M) = 1 + b_2(M)$, valid for simply connected M and in particular for toric M , ensures there are $\chi(M) - 1$ gluing conditions for data at $\chi(M)$ nuts, leaving just one scalar (for each I) undetermined. Here $\chi(M)$, $b_2(M)$ are the Euler and second Betti number of M . These are called the “gluing rules” (the name is inspired by [27]).

Next consider a noncompact two-dimensional submanifold to which ξ is tangent. Assume this has topology $\mathbb{R}^2$ and intersects the boundary ∂M on an S^1 , where we define the holonomy $\Delta^I \equiv (4\pi)^{-1} \int_{S^1} W^I = (4\pi)^{-1} \int_{\mathbb{R}^2} F^I(W)$. The origin of the $\mathbb{R}^2$ necessarily lies on the fixed point set of ξ , and applying Stokes’ theorem and the BVAB theorem leads to the relation

$$\frac{1}{2b} \Psi_{(F)0|_{\text{fixed point}}}^I = \Delta^I + i \frac{2\pi}{b} \sigma^I, \quad (15)$$

where $\sigma^I \equiv -i(4\pi)^{-1} \Psi_{(F)0|_{S^1}}^I$ is the boundary value of $\Psi_{(F)0}^I$. These “UV/IR relations” (see also Ref. [28]), combined with the gluing rules, are crucial in the holographic context as they allow one to interpret the action in terms of boundary field theory data [23–25].

Now, to obtain on-shell results for Poincaré supergravity, either gauged or ungauged, one needs to add compensating

multiplets and carry out gauge fixing. For instance, for the theories of vector multiplets described here, one can use compensating hyper or tensor multiplets. It is possible to show that neither of them contributes to the localized actions (11) and (12) [13]. Boundary terms need to be carefully taken into account. These include those arising from the BVAB formula (9), and in any physical application one will also have Gibbons-Hawking-York-Myers boundary terms [29–31], as well as boundary terms to remove divergences, either by implementing holographic renormalization in the gauged case, or background subtraction in the ungauged case, and potentially also a Legendre transform. We will not analyze these issues further here. However, importantly, in the case of two-derivative on-shell $D = 4$, $\mathcal{N} = 2$ gauged supergravity, it has been shown that the combination of all these boundary terms precisely cancels [25]. Thus, for these two-derivative theories, the holographic on-shell action is exactly given by the fixed point result. Moreover, we have shown that at the two-derivative level, the results in this Letter are in precise alignment with those of [25] after gauge fixing. It is therefore plausible that our fixed point result for off-shell configurations given in (11) and (12) does indeed give the correct physical result more generally.

Our expression for the nut contribution (11) was insightfully conjectured in [7,10], based on extrapolation of known solutions, where gluing rules were also discussed. Here we have provided a general proof of (11) as well as obtained the bolt contribution (12). In addition, we have provided a precise prescription for computing the gluing rules in general and, furthermore, explained the important UV/IR dictionary.

ABJM theory—As an application to holography, we consider the gravity dual of ABJM theory [32]. Specifically, we consider $D = 4$ STU gauged supergravity with vector multiplets labeled by $I = 0, 1, 2, 3$. Solutions to the two-derivative theory uplift on $S^7/\mathbb{Z}_k$ with N units of 7-form flux to $D = 11$ supergravity [33], where they are dual to the $d = 3$, $U(N) \times U(N)$ ABJM theory with Chern-Simons level $k \in \mathbb{Z}$. The derivative expansion in gravity corresponds to a $1/N$ expansion in field theory, with k fixed. It is not known how to compute higher derivative corrections to the $D = 4$ prepotential directly from M theory in $D = 11$, but an all-derivative prepotential was conjectured in [8]. Absorbing the overall Newton constant into the prepotential, in our conventions the latter reads

$$\mathcal{F} = \frac{\sqrt{2kX^0X^1X^2X^3}}{48\pi} \times \left[N_k - \frac{1}{3k} \frac{\frac{1}{16} f_W^2(X^I) A|_W^2 + \frac{2}{9} f_T(X^I) A|_T^2}{16X^0X^1X^2X^3} \right]^{3/2}, \quad (16)$$

where $N_k \equiv N - (k/24)$ and

$$f_{\mathbb{W}^2}(X^I) \equiv -2 \sum_{I < J} X^I X^J,$$

$$f_{\mathbb{T}}(X^I) \equiv \sum_{I=0}^3 (X^I)^2 - \frac{1}{\sum_{I=0}^3 X^I} (X^0 + X^1 - X^2 - X^3) \\ \times (X^0 - X^1 + X^2 - X^3)(X^0 - X^1 - X^2 + X^3), \quad (17)$$

and in (13) we now take $\mathbf{p}_{\Sigma_g}^I \in \mathbb{Z}$, associated with smooth uplifted solutions. In the Euclidean theory $\mathcal{F}^\pm$ may in principle be specified independently, but Wick rotating the Lorentzian theory implies $\mathcal{F}^+ = \mathcal{F}^- = \mathcal{F} = \mathcal{F}(X^I, A|_{\mathbb{W}^2}, A|_{\mathbb{T}})$. The expression (16) was essentially deduced from field theory results on S^3 , up to a logarithmic term and the N^0 term in the all-order $1/N$ expansion. The veracity of the conjecture comes from consistency with a number of other known results [8,9]. In particular, combining (16) with the gluing rules for black hole solutions with topology $\mathbb{R}^2 \times S^2$ gives agreement with field theory results for the topologically twisted index (TTI) and superconformal index (SCI) of ABJM theory on $S^1 \times S^2$. Notice this is a form of consistency check of precision holography, rather than a fully independent computation of each side of the duality.

Our bolt formula (12) allows us to present a new result for ABJM theory on a general class of supersymmetric 3-manifolds. The conformal Killing vector ξ is by assumption nowhere zero on the boundary ∂M . Provided all the orbits of ξ close, ∂M is then a Seifert-fibered 3-manifold [34]. This is the total space of a circle orbibundle over an orbifold Riemann surface $\Sigma_{g,d}$: the Riemann surface Σ_g has d orbifold points, locally $\mathbb{C}/\mathbb{Z}_{n_i}$, $i = 1, \dots, d$, with the circle orbibundle specified by an integer degree $p \in \mathbb{Z}$ and parameters $0 \leq m_i < n_i$. Such 3-manifolds are naturally the boundary of the associated complex line orbibundle, with ξ rotating the $\mathbb{R}^2 = \mathbb{C}$ fibers. We take this to be our bulk M filling the boundary Seifert manifold, which has a bolt $\Sigma_{g,d}$ at the origin of $\mathbb{R}^2 = \mathbb{C}$. Then we may simply substitute (16) into (12), suitably modified to include orbifold points, with

$$\int_{\Sigma_{g,d}} c_1(T\Sigma_{g,d}) = 2 - 2g - d + \sum_{i=1}^d \frac{1}{n_i}, \\ \int_{\Sigma_{g,d}} c_1(N\Sigma_{g,d}) = -p + \sum_{i=1}^d \frac{m_i}{n_i}. \quad (18)$$

$I_{\pm}^{\text{bolt}}$ then gives an all-order prediction for saddle points of ABJM theory on this general class of 3-manifolds, where the finite N partition function is known [36].

While this general large N expansion has not been computed in field theory, Ref. [37] studied the case that $\Sigma_{g,d}$ is smooth, so $d = 0$, but with general Chern number p (further setting $p = 0$ gives the TTI). The map from gravity

variables to field theory variables is the same as in the two-derivative theory, given in [25]

$$\text{--chirality: } b^{-1} S_- X_+^I = \Delta^I + i \frac{2\pi}{b} \sigma^I = 2[\Delta_{\text{there}}^I], \\ \mathbf{p}_{\Sigma_g}^I = 2 \left(\mathbf{n}^I + p D^I - \frac{1}{2} p \nu_R \right). \quad (19)$$

Here the first equation is the UV/IR relation (15), and also gives the relation to the field theory variables Δ_{there}^I in [37]. Note that the constraint $\sum_{I=0}^3 [\Delta_{\text{there}}^I] = 1$ is equivalent to $b = \frac{1}{2} \sum_{I=0}^3 S_- X_+^I$ [13]. We refer to [37] for definitions of all the quantities on the right-hand side of (19), but note that $\Delta_{\text{there}}^I = [\Delta_{\text{there}}^I] + D^I$ with D^I integers, and $\sum_{I=0}^3 \Delta_{\text{there}}^I = 2\nu_R$, where $\nu_R = +\frac{1}{2}$ for these negative chirality solutions. It is straightforward to check that I_+^{bolt} given by (12) matches the full $1/N$ expansion given in Equation (3.25) of [37] (up to the logarithmic term and N^0 term), and that the positive chirality branch I_-^{bolt} matches the second saddle point discussed in [37]. This is a remarkable agreement and provides nontrivial support for (16), as well as our general expectation that in evaluating the higher derivative actions using localization, the total boundary contributions will vanish.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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End Matter

Appendix—We record here that the lowest weight fields of the T-log multiplets, $\mathbb{T}_\pm$, expressed in terms of the components of $\Phi'_\mp$ are given by

$$A_\pm|_{\mathbb{T}_\pm} = \frac{C_\mp}{A_\mp} + \frac{1}{4A_\mp^2} \left[2F^{\pm ab} F_{ab}^\pm - \epsilon_{ij}\epsilon_{kl} B_{\mp}^{ik} B_{\mp}^{jl} \right], \quad (\text{A1})$$

where the components $A_\mp$ of $\Phi'_\mp$ are assumed to be non-vanishing.

A key equation in our analysis is an expression for the 2-form $d\xi^b$, which we can decompose into self-dual and anti-self-dual components, $d\xi^b = (d\xi^b)^+ + (d\xi^b)^-$. Using (anti) chiral multiplets $\Phi'_\pm$ of arbitrary weight w' we find

$$(d\xi^b)^\pm = \mp \frac{1}{2} S_\pm T^\pm \pm \frac{1}{w'A_\mp} \left[2S_\mp F^\pm + B_{\mp}^{ij}\epsilon_{jk}\Upsilon_{\mp i}^k \mp 2(DA_\mp \wedge \xi^b)^\pm \right]. \quad (\text{A2})$$

This follows from imposing supersymmetry for the Weyl multiplet and a chiral multiplet of weight w' , with components (2). The covariant derivative acting on A is $D_\mu A_\pm = (\partial_\mu \pm w'A_\mu)A_\pm$. Evaluating the norms of these 2-forms at a fixed point of chirality $\pm$ leads to (8).