

Edge Reconstruction in a Quantum Spin Hall Insulator

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We study interaction-driven edge reconstruction in a quantum spin Hall insulator described by the Bernevig-Hughes-Zhang model with Kanamori-Hubbard interactions using the real-space density matrix renormalization group method in both the grand-canonical and canonical ensembles. For a two-dimensional cylinder with a smooth edge, we identify discrete particle-number transitions that lead to a spin-polarized edge state stabilized by an emergent ferromagnetic exchange interaction. The reconstruction is orbital-selective, occurring predominantly in the s -orbital channel. Our results reveal a microscopic mechanism for emergent fluctuating moments at the edge that could compromise the topological protection of helical edge states by time-reversal symmetry.

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Introduction—The theoretical prediction of a two-dimensional quantum spin Hall (QSH) insulator protected by time-reversal symmetry was first made by Kane and Mele for graphenelike systems, introducing the notion of counterpropagating or *helical* edge modes that are immune to nonmagnetic backscattering [1,2]. Building on this, Bernevig, Hughes, and Zhang (BHZ) constructed a descriptive four-band model for HgTe/CdTe quantum well heterostructures, demonstrating that an inverted band structure gives rise to gapless helical edges and a quantized conductance of $G = 2e^2/h$ in the absence of magnetic fields [3]. Shortly thereafter, König *et al.* [4] provided the first transport evidence for the quantum spin Hall effect, observing a robust $2e^2/h$ plateau at low temperatures in HgTe/(Hg,Cd)Te devices of micrometer length scales. However, subsequent experiments in longer devices have reported deviations from perfect quantization, with conductance decaying over distances as small as a few microns [5–10] and, in some cases, reporting an unexpectedly robust plateau emerging at $1e^2/h$ [11]. Beyond the original platform, QSH behavior has been observed in InAs/GaSb heterostructures [12–15], monolayer WTe₂ [16–19], and bismuthene on SiC [20,21], and moiré materials [22,23].

A variety of explanations for the sensitivity of QSH conductance to experimental parameters have been put forward, including the dynamical polarization of nuclear spins coupled to helical states [24–28], incoherent electromagnetic noise [29], the presence of Rashba spin-orbit coupling [11,30–34] combined with interaction mediated effects [35,36], scattering from magnetic impurities [37–46], and scattering from nonmagnetic impurities or electron puddles [47–50].

A more recent scenario to explain transport anomalies is the reconstruction of the charge density near the boundary itself, i.e., additional localized low-energy modes or extra one-dimensional edge channels may emerge proximate to the helical edge due to electrostatics, confinement, or interactions. This physics has been extensively studied in integer and fractional quantum Hall systems [51–58]. Prior theoretical work on edge reconstruction in QSH systems, largely within mean-field frameworks [59–62], has demonstrated that once multiple channels exist, time-reversal symmetry only protects each Kramers pair individually but does not forbid scattering between different pairs. Moreover, enhanced edge density of states can even trigger local moment formation (ferromagnetism), breaking time-reversal symmetry outright. Taken together, these effects provide backscattering mechanisms absent in the single-channel case, giving rise to a mesoscopic length scale that limits ballistic transport. However, a fully interacting real-space understanding of the effects of edge reconstruction is still lacking. Moreover, while the density matrix renormalization group (DMRG) method [63,64] has recently been established as a powerful tool for characterizing topological

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phases in idealized BHZ models [65], its application to mesoscopic systems with reconstructed edges remains largely unexplored.

In this Letter, we move beyond the limitations of previous mean-field and effective model approaches by applying DMRG to a fully interacting, multiorbital BHZ model. This constitutes a controlled microscopic treatment of realistic mesoscopic boundaries that preserves global time-reversal symmetry. Our exact many-body analysis reveals an orbital-selective edge reconstruction and isolates an emergent ferromagnetic exchange interaction that stabilizes magnetic moments on the edge. This is in stark contrast to mean-field approaches that are biased toward magnetic symmetry breaking states on the edge even in the absence of edge reconstruction. By providing a fully interacting, real-space mechanism for the generation of fluctuating magnetic moments at the boundary, our results establish a microscopic link between the topological properties of the bulk and the mesoscopic backscattering mechanisms that compromise ballistic transport in experimental devices [5–7,11].

Model—We study an interacting real-space BHZ model on a two-dimensional cylinder with periodic boundary conditions along y and mixed boundaries along x : open on the left edge and with smooth potential confinement on the right. The Hamiltonian is given by

$$H = H_0 + H_{\text{int}} + \sum_{\mathbf{r}} V(r_x) n_{\mathbf{r}}, \quad (1)$$

where H_0 is the real-space noninteracting BHZ model, H_{int} is the multiorbital Kanamori-Hubbard interaction [66,67], and $V(r_x)$ captures the geometric details of the boundary with $n_{\mathbf{r}}$ the total electron density at site $\mathbf{r}$, including orbital and spin degrees of freedom. This explicitly breaks the mirror (left-right) symmetry but does not impact the global time-reversal symmetry.

The noninteracting Hamiltonian is [65,68–70]

$$H_0 = -m \sum_{\mathbf{r}, \alpha} n_{\mathbf{r}, \alpha} (\tau^z)_{\alpha\alpha} + B \sum_{\substack{\mathbf{r}, \alpha, \sigma \\ i=x, y}} c_{\mathbf{r}, \alpha, \sigma}^\dagger (\tau^z)_{\alpha\alpha} c_{\mathbf{r}+\hat{\mathbf{e}}_i, \alpha, \sigma} \\ + \frac{A}{2} \sum_{\substack{\mathbf{r}, \sigma \\ i=x, y}} \sum_{\substack{\alpha, \beta \\ \alpha \neq \beta}} c_{\mathbf{r}, \alpha, \sigma}^\dagger (D_\sigma^i)_{\alpha\beta} c_{\mathbf{r}+\hat{\mathbf{e}}_i, \beta, \sigma} + \text{H.c.}, \quad (2)$$

where the first term describes the on-site energy with $0 < |m/2B| < 2$ corresponding to the topological phase and $|m/2B| > 2$ the trivial phase [68]. The second term is the nearest neighbor hopping, and the third captures spin-orbit coupling with mixing matrices $(D_\sigma^x) = (-1)^\sigma (i\tau^x)$ and $(D_\sigma^y) = (-i\tau^y)$. Here, $\mathbf{r} = (r_x, r_y)$ represents the unit cell vector with components r_x and r_y along the lattice vectors $a\hat{\mathbf{e}}_x$ and $a\hat{\mathbf{e}}_y$, respectively, $\alpha, \beta = s, p$ denotes the orbital indices, and $\sigma = \uparrow, \downarrow$ represents the z -axis spin projection of an electron. The τ^i are Pauli matrices

in orbital space. The factor $(-1)^\sigma = -1(1)$ for $\sigma = \uparrow(\downarrow)$, respectively.

The multiorbital Hubbard interaction is [65–67]

$$H_{\text{int}} = U \sum_{\mathbf{r}, \alpha} n_{\mathbf{r}, \alpha, \uparrow} n_{\mathbf{r}, \alpha, \downarrow} + \left(U' - \frac{J_H}{2} \right) \sum_{\mathbf{r}} n_{\mathbf{r}, s} n_{\mathbf{r}, p} \\ - 2J_H \sum_{\mathbf{r}} \mathbf{S}_{\mathbf{r}, s} \cdot \mathbf{S}_{\mathbf{r}, p} + J_H \sum_{\mathbf{r}} (P_{\mathbf{r}, s}^\dagger P_{\mathbf{r}, p} + \text{H.c.}), \quad (3)$$

where $U' = U - 2J_H$, $\mathbf{S}_{\mathbf{r}, \alpha}$ is the spin operator, and $P_{\mathbf{r}, \alpha} = c_{\mathbf{r}, \alpha, \uparrow} c_{\mathbf{r}, \alpha, \downarrow}$ the pair-annihilation operator. Throughout this Letter, we measure lengths in units of the lattice spacing a and use the gap size m as the unit of energy, working in a fixed parameter regime: $A = 0.3, B = 0.5, U = 1.0$, and $J_H = 0.25$. This corresponds to the paramagnetic topological insulator phase [65].

To model a smooth boundary at one edge, as depicted in Fig. 1, we use an electrostatic potential $V(r_x)$ that increases linearly from the right helical edge over a width w , combined with an ionic potential at positions up to and including the right helical edge,

$$V(r_x) = \begin{cases} -\mu_0 + \varepsilon_h \delta_{r_x, L_x - w} & 0 \leq r_x \leq L_x - w \\ v_0(r_x - L_x + w) & L_x - w < r_x \leq L_x - 1 \end{cases},$$

where $\mu_0 = (3U - 5J_H)/2$ ensures charge neutrality in the bulk, v_0 is the slope of the confinement potential, and L_x is the total number of lattice sites along the cylinder. The additional additive term ε_h , acting at the right helical edge, prevents the accumulation of extra charge due to a softening of the confinement potential. In all our calculations, the value of ε_h is chosen such that the left and right helical edges are similar (e.g., no splitting of the Dirac point in Fig. 2, same particle number; see Supplemental Material for more details [71]).

Mean-field results—To complement our DMRG study, we performed mean-field calculations on two-dimensional cylinders, details are provided in Supplemental Material [71]. We evaluate the orbital and spin-resolved spectral

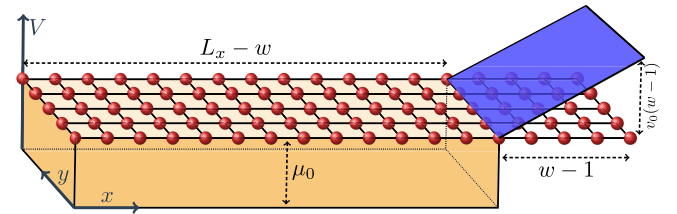


FIG. 1. Potential profile illustrating the combined ionic and confining potentials on a 2D lattice. The confining potential linearly increases in the extended edge region over a width w with slope v_0 from the left helical edge at site $r_x = L_x - w$. The ionic potential (orange) is uniformly set as the negative of the atomic half-filling chemical potential μ_0 . We take periodic boundary conditions in the y direction.

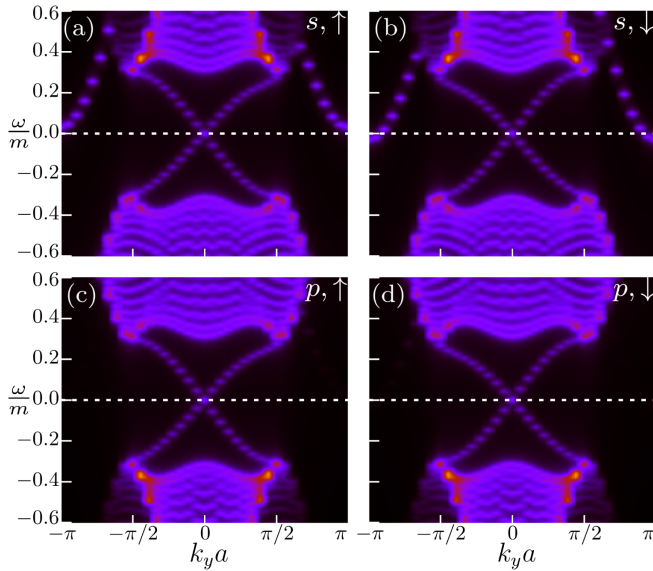


FIG. 2. Orbital and spin-resolved single-particle spectral function from self-consistent mean-field calculations on a 36×36 cylinder with confining width $w = 6$, confining potential slope $v_0 = 1.90$, and $\varepsilon_h = 0.3$. Panels (a) and (b) show the energy bands associated with the orbital- s channel for spin- $\uparrow$ and spin- $\downarrow$ particles, respectively. Panels (c) and (d) display the corresponding bands for the orbital- p channel. The plot demonstrates the orbital-selective nature of edge reconstruction.

function for momentum k_y around the cylinder. Our results in Fig. 2 reveal that at the onset of edge reconstruction for $v_0 = 1.9$, signaled by the appearance of a Fermi pocket at the Brillouin zone edge, a spin splitting emerges exclusively in the s -orbital bands, while the p -orbital bands remain mostly unaffected. Although spin splitting associated with edge reconstruction has been previously reported [59,60], its orbital character, specifically its confinement to the s channel, was missed due to a lack of orbital information. This orbital selectivity follows directly from the on-site energy term in Eq. (2), which assigns on-site energies $-m$ and $+m$ to s -orbital and p -orbital occupations, respectively, such that one can obtain reconstruction for orbital p by changing the crystal-field term $m \rightarrow -m$; see Supplemental Material [71].

DMRG results—We focus on quasi one-dimensional cylinders of size $L_x \times 3$, working in both the canonical and grand-canonical ensembles to investigate the microscopic nature of edge reconstruction. We first perform grand-canonical DMRG calculations [75,76] to identify the edge reconstruction regime by investigating changes in the particle number as the slope of the confining potential, v_0 , is decreased. Once the value of v_0 for which additional particles are energetically favorable on the reconstructed edge is known, we carried out high-precision canonical DMRG calculation [77–79] across adjacent particle sectors to determine the global ground states and analyze the spin and orbital character in real space. We considered up to

2000 DMRG states in the grand-canonical and up to 13000 states in the canonical ensemble calculations, imposing a maximum truncation error of 10^{-5} . The results presented here correspond to 12×3 cylinders with confining width $w = 4$ and $\varepsilon_h = 0.4$. This system size is sufficient to decouple the edges while maintaining a well-defined bulk. Finite size effects are explored in Supplemental Material [71].

Figure 3 describes the change in the average total number of particles $\langle N \rangle = \langle \sum_{\mathbf{r}} n_{\mathbf{r}} \rangle$ as a function of the slope of the confining potential v_0 , computed within the grand-canonical ensemble. As v_0 decreases from infinity, the system transitions from the hard-wall regime to the reconstructed regime. Initially, the system is stable at half-filling forming a broad plateau in $\langle N \rangle = N_0$, which for our 12×3 system with confining width $w = 4$ corresponds to $N_0 = (9 \cdot 3 \cdot 2 \cdot 2)/2 = 54$ particles (two spins, two orbitals per site). As v_0 is reduced, we observe a sharp jump in particle number, signaling the onset of edge reconstruction, with the additional charge now localized in the confined region (as demonstrated below in Fig. 5). This first jump corresponds to the $N_0 + 1$ particle sector. Further reduction of the confining potential induces another discrete jump in $\langle N \rangle$, yielding the $N_0 + 2$ sector, where two particles predominantly accumulate in the reconstructed region.

Having identified three distinct particle-number plateaus associated with edge reconstruction, we next turn to a canonical DMRG analysis to determine ground-state properties for N_0 , $N_0 + 1$, and $N_0 + 2$ corresponding to $v_0 = 1.53$, 1.40 , and 1.33 , respectively. In Fig. 4 we show the ground-state energy for different values of the fixed spin quantum number S_{tot}^z . For N_0 particles, the minimum occurs at $S_{\text{tot}}^z = 0$, expected for a half-filling. For $N_0 + 1$ particles, the lowest energy states are doubly degenerate between the $S_{\text{tot}}^z = \pm \frac{1}{2}$ sectors, reflecting the presence of an unpaired

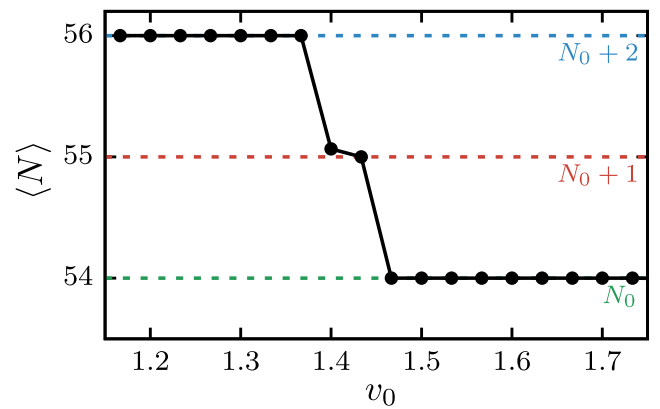


FIG. 3. Total number of particles $\langle N \rangle$ as a function of the slope of confining potential v_0 for a 12×3 cylinder with confining width $w = 4$, computed using grand-canonical DMRG. At large values of v_0 , the particle number stays on a plateau where the bulk is at half-filling. Reduction of v_0 leads to particle accumulation in the confined region, signaling the onset of edge reconstruction.

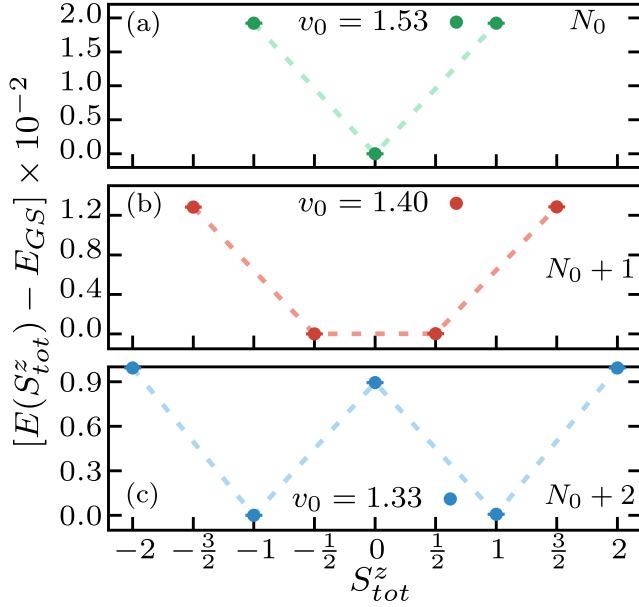


FIG. 4. Canonical DMRG ground-state energies as a function of total spin S_{tot}^z on a 12×3 cylinder with confining width $w = 4$. Panels (a)–(c) correspond to the N_0 , $N_0 + 1$, and $N_0 + 2$ particle sectors, with confining potentials $v_0 = 1.53$, 1.40 , and 1.33 , respectively. The ground state lies in the $S_{\text{tot}}^z = 0, \pm\frac{1}{2}$, and ± 1 sectors for N_0 , $N_0 + 1$, and $N_0 + 2$. In the $N_0 + 2$ case, the system favors a degenerate spin-polarized configuration, indicating an energetic preference for ferromagnetic alignment of spins at the reconstructed edge.

spin localized at the reconstructed edge. Surprisingly, for $N_0 + 2$ electrons, the ground states lie in the $S_{\text{tot}}^z = \pm 1$ sectors, with the $S_{\text{tot}}^z = 0$ state appearing at higher energy, indicating a spin-polarized edge with nonzero magnetic moment. This energetic preference supports an emergent ferromagnetic alignment of the two added edge-localized particles, consistent with an exchange-driven edge reconstruction as described below.

To gain further insight into the spatial and orbital character of these added electrons, we analyze the resulting real-space charge and magnetic properties of the helical and reconstructed states. Figure 5 shows the expectation value of the spatial distribution of added charge through the difference in orbital resolved y -averaged charge densities, $\langle n_{r_x, \alpha} \rangle = \frac{1}{L_y} \sum_{r_y} \langle n_{r, \alpha} \rangle$, between the ground states of different particle-number sectors. Panel (a) confirms that when adding a single electron to the system, it is localized in the confinement region $r_x > L_x - w$. With the addition of a second electron, the $N_0 + 2$ ground state now demonstrates some leakage of density onto the helical edge. In both cases, the excess density appears almost entirely in the orbital- s channel, with negligible contributions to the orbital- p channel, confirming the orbital-selective nature of edge reconstruction, consistent with our mean-field spectral analysis in Fig. 2.

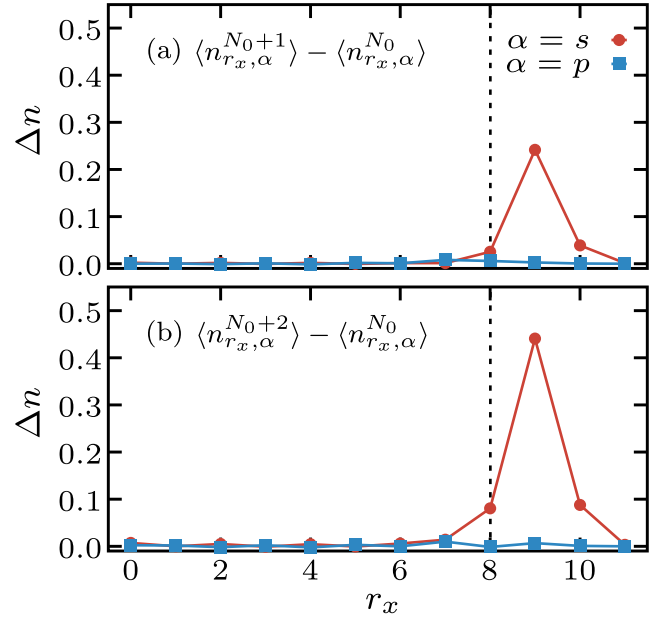


FIG. 5. Real-space charge density differences depicting how the reconstructed states are localized in the confinement region. Panel (a) shows the difference between the density profiles of the $N_0 + 1$ and N_0 sectors. (b) shows the difference between the $N_0 + 2$ and N_0 sectors. We average over the degenerate ground states for the cases of $N_0 + 1$ and $N_0 + 2$ electrons and along the periodic y direction. The vertical dashed lines depict the location of the right-helical edge.

The magnetic properties of the edge are shown in Fig. 6, which depicts the orbital and site resolved y -averaged magnetization $\langle S_{r_x, \alpha}^z \rangle$ for the ground states of the three particle sectors of interest. For N_0 particles (half-filling) the total and local magnetization are zero as seen in panel (a). As the confinement potential is softened and the system transition to the $N_0 + 1$ state in panel (b), the additional electron causes a local moment in the confinement region mostly in the s channel. However, we also observe an interaction induced increase in the p -channel moment directly on the right helical edge with a concomitant response on the left helical edge due to the restriction to $S_{\text{tot}}^z = +\frac{1}{2}$. Panel (c) for $N_0 + 2$ and $S_{\text{tot}}^z = +1$ shows a further enhancement of correlated local moments in the confinement region and helical edges, indicative of a coupling between magnetic moments on the reconstructed and helical edges. Thus, the observed ferromagnetic alignment of the two added edge-localized electrons provides real-space evidence of an exchange driven edge magnetism associated with edge reconstruction.

Emergent spin Hamiltonian—DMRG calculations supplemented by the exact diagonalization (ED) analysis for two particles on a single rung (see End Matter) allow us to infer an effective exchange Hamiltonian between the two spin- $\frac{1}{2}$ particles localized in the reconstructed region for $N_0 + 2$. From ED, we find a singlet-triplet splitting

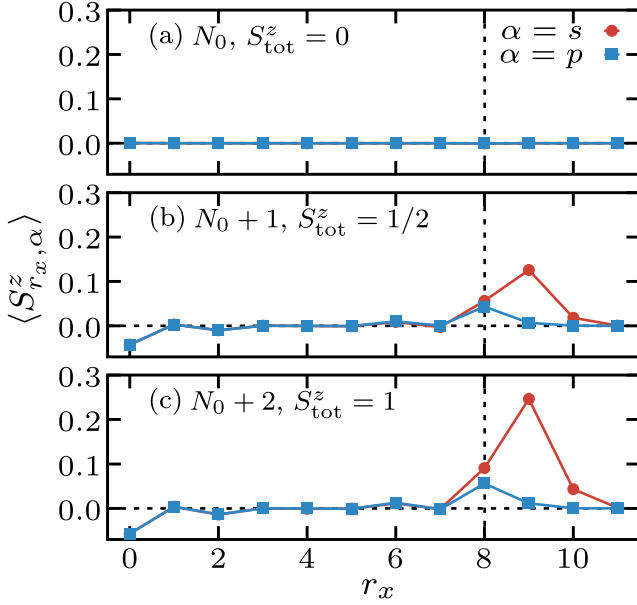


FIG. 6. Real-space orbital and site-resolved magnetization for the ground states of different particle-number sectors with fixed S_{tot}^z quantum numbers. Panel (a) shows magnetization for N_0 particles in the $S_{\text{tot}}^z = 0$ state, panel (b) for $N_0 + 1$ in the $S_{\text{tot}}^z = +\frac{1}{2}$ state, and panel (c) for $N_0 + 2$ in the $S_{\text{tot}}^z = +1$ state. The vertical dashed line depicts the right-helical edge. We observe a p -channel magnetic response at the location of the helical edge, indicating that there may be an interaction coupling between the reconstructed and helical edges.

$\Delta E = 0.2167$ that is considerably larger than the spin exchange anisotropy $\Delta E_a = 0.0089$ (due to the combination of Hund's coupling and spin-dependent hopping, shown in Fig. 4). Taken together this physics can be described by $H_{\text{exc}} = -J(\mathbf{S}_i \cdot \mathbf{S}_j) - \Delta S_i^z S_j^z$, where the first term describes the magnitude of a singlet-triplet splitting, and the second term describes the anisotropy between the components of the triplet state with $S^z = 0$ and $S^z = \pm 1$. This simple picture yields $J = \Delta E \simeq 0.2167$ and for our 12×3 cylinder, $\Delta = 2\Delta E_a \simeq 0.0178$, indicating a finite anisotropic ferromagnetic coupling that stabilizes a spin-polarized edge configuration. For a finite size scaling analysis of these couplings, see Supplemental Material [71].

Discussion—Our results identify a microscopic route to edge magnetism in a quantum spin Hall system: sample boundary softening through a spin-independent electrostatic potential drives an interaction-induced, orbital-selective spectral reconstruction in which the s channel predominantly admits charges and exhibits spin splitting, while the p channel remains mostly incompressible. The orbital selectivity is tied to the crystal-field splitting and thus a p -orbital polarized edge reconstruction could be realized via strain tuning. This mechanism is tied to the topological band inversion and can be readily applied to systems with p - d [80,81] or d - f [82,83] orbital character.

Plateaus followed by jumps in the total particle number in the grand-canonical ensemble signal generic edge reconstruction with canonical calculations at $N_0, N_0 + 1, N_0 + 2$ linking the charge steps to real-space spin densities. The latter can be used to diagnose the presence of a finite ferromagnetic exchange coupling from the energy differences between different total spin sectors, explaining the observed ground-state spin polarization at the reconstructed edge even though the Hamiltonian preserves time-reversal symmetry. While we study a finite lattice model, from a Luttinger-liquid viewpoint a ferromagnetic tendency can arise when the spin stiffness (the coefficient of the quadratic gradient term) changes sign, with a higher-gradient term stabilizing the field theory and yielding unconventional dynamical scaling [84]. However, the Lieb-Mattis theorem [85] forbids true ferromagnetism for a single occupied subband with interactions shorter than the interparticle spacing, and in the opposite limit of long-range Coulomb forces a ferromagnetic state competes with Wigner crystallization [86]. We therefore expect that the mesoscopic devices in the current era of experiments should exhibit fluctuating edge moments rather than stable long-range ferromagnetism with implications for the protection of ballistic transport.

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Data availability—All data, code, and analysis scripts that support the findings of this study are available online [84].

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End Matter

Appendix A: Exact diagonalization of a three-site ring—Both our mean-field and DMRG calculations indicate that a spin splitting emerges when more than one particle occupies the confined region ($N_0 + 2$), with the added charge predominantly localized near the right-helical edge. Specifically, in the case of two additional particles, they become localized at the rung adjacent to the right-helical edge, where the confining potential is minimal (but nonzero). To isolate the origin of this mechanism and quantify the scale of the singlet-triplet gap, we consider an exactly solvable system in which two particles are confined entirely to a three-site ring with two orbitals per site. The reduced Hamiltonian along the y direction is $H_{\text{ring}} = H_s + H_p + H_{sp}$, where

$$H_\alpha = (\tau^z)_{\alpha\alpha} B \sum_{r_y, \sigma} (c_{r_y, \alpha, \sigma}^\dagger c_{r_y+1, \alpha, \sigma} + \text{H.c.}) - (\tau^z)_{\alpha\alpha} m \sum_{r_y} n_{r_y, \alpha} + U \sum_{r_y} n_{r_y, \alpha, \uparrow} n_{r_y, \alpha, \downarrow} \quad (\text{A1})$$

$$H_{sp} = \frac{A}{2} \sum_{r_y, \sigma} (c_{r_y, s, \sigma}^\dagger c_{r_y+1, p, \sigma} - c_{r_y, p, \sigma}^\dagger c_{r_y+1, s, \sigma} + \text{H.c.}) + \left(U' - \frac{J_H}{2} \right) \sum_{r_y} n_{r_y, s} n_{r_y, p} - 2J_H \sum_{r_y} \mathbf{S}_{r_y, s} \cdot \mathbf{S}_{r_y, p} + J_H \sum_{r_y} (P_{r_y, s}^\dagger P_{r_y, p} + \text{H.c.}). \quad (\text{A2})$$

We have omitted the confinement potential $v_0 \sum_{r_y, \alpha, \sigma} n_{r_y, \alpha, \sigma}$, which acts as a uniform chemical potential that can be absorbed as an energy offset.

The resulting Hilbert space has dimension $|H_{\text{ring}}| = \binom{12}{2} = 66$, and we perform exact diagonalization for the same model parameters as in the main text. We show the low-lying spectrum in Table I resolved by total spin quantum number (S_{tot}). We find a threefold degenerate spin triplet as ground state for the fully interacting ring geometry

TABLE I. Energy eigenvalues for the lowest six eigenstates of the three-site ring and their total spin quantum numbers (S_{tot}). The second column uses the parameters from the main text. In the third column, we set the interaction to zero ($U = U' = J_H = 0$). We find that in the noninteracting case the ground state is sixfold degenerate, containing both singlet and triplet states at the same energy. Interactions lift this degeneracy and stabilize a spin triplet ground state. We measure energies in units of the gap m and lengths in units of the lattice spacing a .

S_{tot}	Energy	Energy, $U = J_H = 0$
1	−3.04242	−3.04467
1	−3.04242	−3.04467
1	−3.04242	−3.04467
0	−2.82576	−3.04467
0	−2.82576	−3.04467
0	−2.47194	−3.04467

TABLE II. Energy eigenvalues for the lowest six eigenstates of the three site ring with two particles oriented along x direction corresponding to their total spin- z quantum number (S_{tot}^z). The second column uses the parameters from the main text. In the third and fourth column, we set $J_H = 0$ and $A = 0$, respectively. We find that there is a third degenerate ground state with $S_{\text{tot}}^z = 0$ if either the Hund's coupling J_H or the orbital mixing A are set to zero.

S_{tot}^z	Energy	Energy, $J_H = 0$	Energy, $A = 0$
-1	-3.04242	-3.03734	-3.0
1	-3.04242	-3.03734	-3.0
0	-3.03881	-3.03734	-3.0
0	-2.82791	-2.82259	-2.78228
0	-2.82791	-2.82259	-2.78228
0	-2.47194	-2.46787	-2.42069

separated by a finite gap with the singlet. In the absence of interaction ($U = U' = J_H = 0$), the ground state becomes sixfold degenerate due to the degeneracy of the lowest single-particle momentum states, resulting in both singlet and triplet at the same energy. The interaction therefore lifts this degeneracy and stabilizes a triplet ground state, depicting the vital role of interactions in the formation of magnetic moments at the reconstructed edge.

When the spatial orientation of the ring is changed from y to x , the orbital-mixing structure ($A \neq 0$) of the non-interacting BHZ Hamiltonian breaks $SU(2)$ symmetry. While the triplet ground state persists for two interacting particles, the triplet manifold becomes internally split,

with the Hund's coupling favoring $S^z = \pm 1$ over $S^z = 0$ as the ground state, thereby selecting an easy-axis direction for the total spin. The corresponding result for a three-site ring oriented along x with two particles is shown in Table II. Thus, the interacting BHZ model at fractional filling exhibits fluctuating moments with anisotropic spin exchange.

Appendix B: Emergent spin Hamiltonian—The singlet-triplet splitting $\Delta E = 0.2167$ in Table I is considerably larger than the spin exchange anisotropy that we obtained from our canonical DMRG calculations $\Delta E_a = 0.0089$ (Fig. 4) for two additional particles in the reconstructed region.

Based on this combined ED and DMRG analysis, we suggest an effective emergent exchange Hamiltonian for the $N = 2$ reconstructed edge with form,

$$H_{\text{exc}} = -J(\mathbf{S}_i \cdot \mathbf{S}_j) - \Delta S_i^z S_j^z. \quad (\text{B1})$$

The first term describes the origin of the observed singlet-triplet splitting, whereas the second term describes the anisotropy between the $S^z = 0$ and $S^z = \pm 1$ states. For two particles, within this simple picture we obtain the following eigenvalues: $E_{S^z=\pm 1} = (-J - \Delta)/4$, $E_{S^z=0, \text{triplet}} = (-J + \Delta)/4$, and $E_{S^z=0, \text{singlet}} = (3J + \Delta)/4$. From our DMRG analysis we obtain $\Delta E_a = E_{S^z=0, \text{triplet}} - E_{S^z=\pm 1}$, which allows us to estimate $\Delta = 2\Delta E_a \simeq 0.0178$. The ED singlet-triplet splitting is described by the limit $\Delta \approx 0$, which gives $J = \Delta E \simeq 0.2167$.