

Deep Thermalization and Measurements of Quantum Resources

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Quantum resource theories provide a unified framework for characterizing useful quantum phenomena subject to physical constraints, but are notoriously hard to assess in experimental systems. In this Letter, we introduce a unified protocol for quantifying the resource-generating power of arbitrary quantum evolutions applicable to multiple resource theories. It is based on deep thermalization, which has recently gained attention for its role in the emergence of quantum state designs from partial projective measurements. Central to our approach is the use of projected ensembles, recently employed to probe deep thermalization, together with new twirling identities that allow us to directly infer the resource-generating powers of the underlying dynamics. These identities further reveal how resources build up and thermalize in generic quantum circuits. Finally, we show that quantum resources themselves undergo deep thermalization at the subsystem level, offering a complementary and another experimentally accessible route to infer the resource-generating powers. Our Letter connects deep thermalization to resource quantification, offering a new perspective on the essential role of various resources in generating state designs and on efficient estimation of quantum resources in experiment.

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Introduction—Quantum resource theories (QRTs) offer a unified framework for characterizing and manipulating operationally useful quantum features. Important examples are the QRTs of entanglement [1–4], nonstabilizerness [5–7], and coherence [8,9], which play central roles in achieving universal quantum computation [10–16], and asymmetry [17–19], which serves as a key resource in the context of quantum reference frames [17]. Each QRT specifies free states and operations easily accessible under specific experimental constraints, while the remainder is, conversely, identified as costly, i.e., resourceful [see Fig. 1(a)]. For example, in the QRT of entanglement, separable states and local operations and classical communication are considered free, while entangled states and nonseparable operations carry the resource. The ability of a unitary evolution to build up the desired resource when starting from a typical free state is called its “resource-generating power (RGP).” It remains an important question

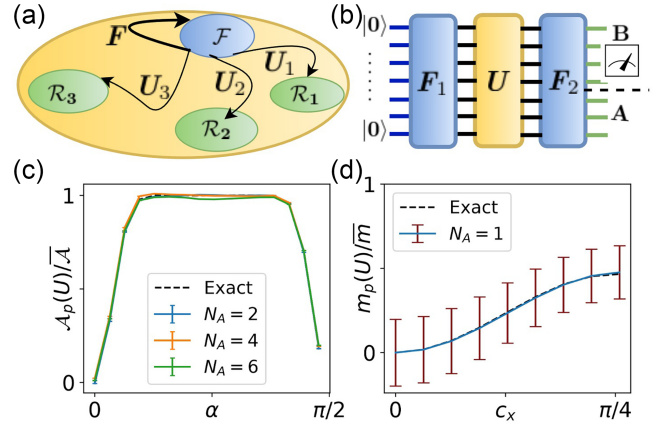


FIG. 1. (a) Abstract representation of free and nonfree elements: $\mathcal{F}$ denotes the set of free states/operators; U_i are nonfree operations mapping $\mathcal{F}$ outside itself, while the free operation F preserves $\mathcal{F}$. (b) Protocol for estimating the resource content of a unitary of interest U : prepare a random free state $|\psi\rangle$ (e.g., using a free unitary F_1 on a fiducial state), apply U followed by a random free operation F_2 , then perform measurements on a subsystem B and postprocess the outcomes. (c) Illustration of the use of the protocol to estimate the Z_2 -AGP ($A_p(U)/\bar{A}$) for $U = u^{\otimes N}$ with $u = \exp\{-i\alpha\sigma_x\}$, with $N = 10$ and subsystem measurements on $N_B = 4, 6, 8$ qubits. (d) Estimation of nonstabilizing power $m_p(U)/\bar{m}$ for two-qubit unitary $U = \exp\{-ic_x\sigma_x \otimes \sigma_x/2\}$ with projective measurements on $N_B = 1$ qubit.

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whether and how fast a unitary, when embedded in a generic quantum circuit, reaches the RGP of a typical, i.e., Haar-random unitary. A comprehensive understanding of this process—in analogy to equilibration in many-body systems [20–23] dubbed “thermalization of quantum resources”—remains an outstanding question. A closely related—and perhaps even more pressing—challenge concerns their experimental quantification [24,25].

In this Letter, we build on recent advances in “deep thermalization” [26,27] to engineer a framework that enables (i) the experimental characterization of quantum resources and (ii) provides significant insights into their thermalization in generic quantum circuits [28]. Deep thermalization provides a refined description of statistical relaxation in many-body systems [26,27,29–48], extending beyond the Eigenstate thermalization hypothesis [49–54]. Rather than local observables, deep thermalization concerns *projected state ensembles* obtained by postselecting measurement outcomes on a larger subsystem in a generic nonintegrable quantum evolution. These ensembles deeply thermalize, meaning that they relax to universal distributions, such as state designs—finite ensembles of pure states reproducing Haar-random statistics [27,55–57]—or their finite-temperature counterparts, known as Scrooge ensembles [27,36,58]. It has become a subject of intensive study to analyze these ensembles and their dependency on the initial state [35], the dynamics [59–63], or the measurement basis [64,65]. For generic evolutions, the projected ensembles typically approach state designs, but when both the initial state and dynamics are free in a QRT sense, they deviate significantly from the designs [43,64,66]. This stems from the fact that free elements of QRTs occupy highly structured subsets of the Hilbert space [67]. Therefore, the onset of design behavior is intuitively linked to the presence of quantum resources. In this Letter, we formalize this intuition by devising a protocol to quantify the RGP of arbitrary evolutions, rooted in the projected ensembles. By particularly focusing on linear entropic resource measures, which are experimentally more accessible than nonlinear ones [68], we construct an unbiased estimator for the RGP. We find that this estimator is highly reliable, as its signal-to-noise ratio over free elements remains stable with increasing system size.

Our protocol can be immediately implemented on near-term quantum devices [see Supplemental Material (SM) [71] for a discussion of experimental feasibility]: a free state is evolved under a circuit composed of the target unitary and random free unitaries, followed by projective measurements on a subsystem. The resulting measurement outcome probabilities provide direct access to the RGP of the target unitary. Complementing this, we show that the resource content of the projected states serves as a proxy for the full dynamics, thereby establishing an experimentally viable framework for probing resources. We focus on asymmetry and nonstabilizerness in the main text, with

technical details and the cases of entanglement and coherence presented in the SM [71].

Quantifying resources—Many quantum resources admit entropy-based quantifiers, including entanglement [85] and stabilizerness [86]. Let $\mathcal{R}^{(\alpha)}(|\psi\rangle)$ denote the α -Rényi entropy quantifying the resource of a state $|\psi\rangle$. The corresponding linear entropy is $\mathcal{R}_{\text{lin}}(|\psi\rangle) = 1 - e^{-\mathcal{R}^{(2)}(|\psi\rangle)}$. Often, the linear entropies can be evaluated considering a number of t (determined by the underlying QRT) copies of the state of interest, $\mathcal{R}_{\text{lin}}(|\psi\rangle) = 1 - \text{Tr}(\mathbf{W}(|\psi\rangle\langle\psi|)^{\otimes t})$. The operator $\mathbf{W}$ is chosen to commute with $F^{\otimes t}$ for all free unitaries F and to satisfy $0 \leq \langle\psi^{\otimes t}|\mathbf{W}|\psi^{\otimes t}\rangle \leq 1$, with the upper bound attained when $|\psi\rangle$ is free, such that $\mathcal{R}_{\text{lin}}$ vanishes for free states.

The RGP of a unitary U is then defined as the average linear entropy achieved when U is applied to pure free states. I.e., denoting the average by $\langle\cdot\rangle_{|\psi\rangle}$, $\mathcal{R}_p(U) = \langle\mathcal{R}_{\text{lin}}(U|\psi\rangle)\rangle_{|\psi\rangle}$. RGPs quantify the information-theoretic complexity of implementing a quantum evolution within a QRT framework. They also bridge QRTs with out-of-time-ordered correlators and information scrambling [86–89]. In addition to characterizing the resource content generated by near-term quantum devices, our results thus provide a scope on experimentally probing quantum chaos.

Main results—Here, we give a high-level summary of our main results, followed by examples for specific QRTs. At the heart of our results are the twirling identities, common to all QRTs considered [21–23,71]. They state that averaging the action of a unitary U over free unitaries F and free states $|\psi\rangle$ yields simply a linear interpolation between the t ’th moments of free ($\rho^{(t)}$) and Haar-random states ($\mathbf{\Pi}^{(t)}$): $\langle(FU|\psi\rangle\langle\psi|U^\dagger F^\dagger)^{\otimes t}\rangle_{F,|\psi\rangle} = (1 - \alpha)\rho^{(t)} + \alpha\mathbf{\Pi}^{(t)}$, with $\alpha \propto \mathcal{R}_p(U)$ [90]. Taking advantage of these identities, we obtain a clear measurement protocol for computing RGP in the given QRT: (i) prepare a random pure bipartite free state $|\psi\rangle_{\text{free}} \in \mathcal{H}_A \otimes \mathcal{H}_B$, (ii) apply the target unitary U , whose RGP is to be estimated, (iii) apply a random free operation F , and (iv) measure a subset of the final state (say B) in the computational basis, record the outcome probabilities $\{p_b\}$, and repeat the above steps several times [see Fig. 1(b)]. The following theorem establishes one of our main results: an unbiased estimator of the RGP from the probabilities p_b .

Theorem 1—Let $\mathcal{R}_p(U)$ denote an appropriately defined RGP of a unitary U in a QRT admitting for a twirling identity. Define $\tilde{P}^{(t)} = \sum_b p_b^t / 2^{N_B}$, where $t \in \mathbb{Z}^+$ is determined by the QRT, and $\{p_b\}$ are the subsystem measurement outcome probabilities obtained from the above protocol. Then,

$$\langle\mathcal{R}_{\text{est}}(U)\rangle_{F,|\psi\rangle} = \frac{\mathcal{R}_p(U)}{\bar{\mathcal{R}}} = \frac{k_2 - \langle\tilde{P}^{(t)}\rangle_{F,|\psi\rangle}}{k_2 - k_1}. \quad (1)$$

Here, k_1 and k_2 are constants determined by the underlying QRT and the subsystem sizes, $\langle \cdot \rangle_{F,|\psi\rangle}$ denotes averaging over free unitaries F and states $|\psi\rangle$, and $\bar{\mathcal{R}} = \langle \mathcal{R}_p(U) \rangle_{U \in \mathcal{U}(2^N)}$ is the Haar-averaged RGP.

The proof proceeds by using the twirling identities to evaluate $\langle p'_b \rangle$, thereby relating $\langle \tilde{P}^{(t)} \rangle$ to the corresponding RGPs [see Eqs. (5) and (7), along with the subsequent discussion, and also SM [71]]. Equation (1) encapsulates the fact that, for computational-basis measurements, $\langle p'_b \rangle$ is independent of b , causing $\langle \tilde{P}^{(t)} \rangle$ to simply linearly interpolate between two regimes: $\langle \tilde{P}^{(t)} \rangle = k_2$ for free U , and $\approx k_1$ for Haar-random U . A key advantage of our approach is that measuring only a subsystem suffices, thereby expanding projected ensembles from conceptual relevance into a highly useful tool.

Importantly, the variance of $\mathcal{R}_{\text{est}}$ over free elements decays at least exponentially with the size of the measured subsystem N_B for all the QRTs considered. Remarkably, in the QRT of entanglement, the chaotic limit yields an even stronger suppression, exponentially in N , making the protocol particularly attractive (see SM [71]). Consequently, the signal-to-noise ratio does not deteriorate even for large system sizes.

Moreover, the resource content of the projected states is an effective proxy for the full $\mathcal{R}_p(U)$. Namely, we further show that within the above protocol, for an arbitrary computational basis state $|b\rangle \in \mathcal{H}_B$, the averaged resource content of the projected states follows to leading order

$$\langle \mathcal{R}_{\text{PS}} \rangle_{F,|\psi\rangle} \approx \frac{\Omega \bar{\mathcal{R}}_A k_1}{(1-\Omega)k_2 + \Omega k_1}, \quad \text{where } \Omega = \frac{\mathcal{R}_p(U)}{\bar{\mathcal{R}}}. \quad (2)$$

Here, $\bar{\mathcal{R}}_A$ denotes the average resource of Haar-random states in the Hilbert space $\mathcal{H}_A$.

The QRT-specific tools we develop to prove the above results also enable a key result on thermalization of quantum resources, namely the equilibration of resource measures to the Haar-ensemble averages in generic quantum circuits. From the perspective of QRTs, generic circuits comprise layers of free and non-free operations, as shown in Fig. 2(a). By randomizing free operations and fixing the nonfree operation to U , we show that $\mathcal{R}_p$ converges to $\bar{\mathcal{R}}$ exponentially, a behavior ubiquitous across the QRTs considered,

Theorem 2—Let $U^{(n)} = F_n U F_{n-1} U \cdots F_1 U$, where each F_j (for all $1 \leq j \leq n-1$) is drawn independently from the group of free unitaries in a suitable QRT. Let U be a fixed unitary with finite $\mathcal{R}_p(U)$. Then,

$$\langle \mathcal{R}_p(U^{(n)}) \rangle_{\bar{F}} = \bar{\mathcal{R}} \left[1 - \left(1 - \frac{\mathcal{R}_p(U)}{\bar{\mathcal{R}}} \right)^n \right], \quad (3)$$

where $\langle \cdot \rangle_{\bar{F}}$ denotes independent averaging over the F_j 's.

Interestingly, the same twirling identities that underlie Theorem 1 also lead to Theorem 2. Equation (3) tells us that

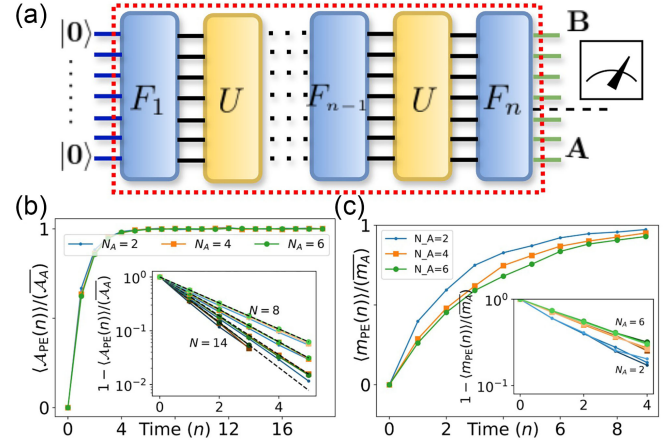


FIG. 2. (a) Illustration of repeated free and non-free operations on a fixed free state, followed by projective measurements on a subset of qubits. (b) Deep thermalization of the $\mathbb{Z}_2$ asymmetry for the above circuit with $N = 14$ and subsystem sizes $N_A = 2$ to 6 . The close agreement between the curves shows that subsystem asymmetry provides a reliable probe of the AGP. Inset: relaxation dynamics for $U = u^{\otimes N}$ with $u = e^{-i\pi\sigma_x/24}$ and $N = 8, 10, 12, 14$ (light to dark). For every N , the relaxation rates corresponding to different N_A nearly coincide. (c) Deep thermalization of nonstabilizerness under repeated interspersions of Clifford and non-Clifford operations for the unitary $U = \mathbb{I} \otimes \exp -i\pi\sigma_x \otimes \sigma_x/8 \otimes \mathbb{I}$, for $N = 12$ and different N_A . $\langle m(n)_{\text{PE}} \rangle / \langle m_A \rangle$ approaches $\langle m_p(U^{(n)}) \rangle_{\bar{C}}$ as $N_A \rightarrow N$. Inset: data for $N = 8, 10, 12$ (light to dark). In both (b) and (c), data is averaged over $\sim 10^3$ random circuit instances.

the thermalization rate is governed entirely by $\mathcal{R}_p(U)$, implying that, with random free operations, even arbitrarily small RGP suffice to generate any targeted amount of resource. We use Theorem 2, together with Eq. (2), to also probe the deep thermalization of quantum resources in circuits composed of free and resourceful operations (see Fig. 2; we define the deep thermalization of resources as the equilibration of resource measures in projected ensembles of states that undergo generic evolutions). In the following, we rigorously demonstrate the above results for the QRTs of $\mathbb{Z}_2$ -asymmetry and nonstabilizerness, to provide further intuition as well as specific numerical tests.

Main results for $\mathbb{Z}_2$ asymmetry—In the QRT of $\mathbb{Z}_2$ asymmetry, free states are given by eigenstates of the symmetry generator $\mathcal{G} = \sigma_z^{\otimes N}$. We define free unitaries as resource nonincreasing operations rather than merely those covariant with the symmetry group, thereby also including unitaries that anticommute with $\mathcal{G}$ (see SM [71] for a construction of random unitaries with symmetry via polar decomposition). The linear $\mathbb{Z}_2$ -asymmetry entropy of a pure state $|\psi\rangle \in \mathcal{H}^{2^N}$ can be computed using [71]

$$\mathcal{A}_{\text{lin}}(|\psi\rangle) = 1 - \text{Tr}[(\mathbf{Z}_0^{\otimes 2} + \mathbf{Z}_1^{\otimes 2})(|\psi\rangle\langle\psi|^{\otimes 2})], \quad (4)$$

where $\mathbf{Z}_s = (\mathbb{I} + (-1)^s \sigma_z^{\otimes N})/2$ is the projector onto the symmetry sector with charge $s \in \{0, 1\}$. $\mathcal{A}_{\text{lin}}$ vanishes iff

$|\psi\rangle$ is an eigenstate of $\mathcal{G}$ and remains invariant under the action of free unitaries, providing a faithful measure of Z_2 asymmetry. Correspondingly, the Z_2 -asymmetry generating power (Z_2 -AGP) of a unitary U is $\mathcal{A}_p(U) = \langle \mathcal{A}_{\text{lin}}(U|\psi) \rangle_{|\psi\rangle}$. Using elementary algebra, we can prove the following twirling identity [71]:

$$\langle (FU)^{\otimes 2} \rho_f (FU)^{\dagger \otimes 2} \rangle_F = \left[1 - \frac{\mathcal{A}_p(U)}{\bar{\mathcal{A}}} \right] \rho_f + \frac{\mathcal{A}_p(U)}{\bar{\mathcal{A}}} \mathbf{\Pi}^{(2)}. \quad (5)$$

Here, $\bar{\mathcal{A}} = 2^{N-1}/(2^N + 1)$ is the Haar-averaged AGP [71], ρ_f denotes the second moment of the ensemble of free states, and $\mathbf{\Pi}^{(2)} = (\mathbb{I} + \text{SWAP})/(2^N(2^N + 1))$ with SWAP acting nontrivially on $\mathcal{H}^{2^N} \otimes \mathcal{H}^{2^N}$ [91]. As the above equation highlights, while U imprints the resource onto free states, the random F spreads it across the system, yielding a convex combination of free (ρ_f) and Haar-randomized ($\mathbf{\Pi}^{(2)}$) parts, each weighted by terms proportional to $\mathcal{A}_p(U)$.

Following the procedure in Fig. 1(b), Eq. (5) can be reformulated into an experimentally implementable protocol for measuring $\mathcal{A}_p(U)$. Given a random pure free state $|\psi\rangle \in \mathcal{H}_{AB}$ and a random free operation F , the probability of projecting on a fixed state $|b\rangle \in \mathcal{H}_B$ is $p_b = \text{Tr}(\langle b|FU|\psi\rangle\langle\psi|U^\dagger F^\dagger|b\rangle)$. By sampling over $|\psi\rangle$ and F , and averaging via Eq. (5), we obtain $\mathcal{A}_p(U)/\bar{\mathcal{A}} = (k_2 - \langle p_b^2 \rangle_{F,|\psi\rangle})/(k_2 - k_1)$, where $k_1 = \text{Tr}(\langle b^{\otimes 2} | \mathbf{\Pi}^{(2)} | b^{\otimes 2} \rangle)$, and $k_2 = \text{Tr}(\langle b^{\otimes 2} | \rho_f | b^{\otimes 2} \rangle)$. This validates the applicability of Theorem 1 for the QRT of Z_2 asymmetry. For computational-basis measurements, the constants are $k_{l \in \{1,2\}} = 2^{N_A}(2^{N_A} + l)/(2^N(2^N + l))$ for $0 < N_A < N$ [71]. Therefore, Born probabilities corresponding to subsystem measurements enable a direct evaluation of the AGP. Interestingly, these probabilities emerge naturally within the projected ensemble framework [27].

Remarkably, in the limit of large N , the variance of the estimator obtained from this protocol decays exponentially with N_B . Specifically, the variance ranges between $\sim 8/(2^{N_B} - 1)$ to $\sim 2/(2^{N_B} - 1)$ as U interpolates between a free and a Haar random unitary. For details, see SM [71]. Figure 1(c) shows the estimation of the Z_2 -AGP for an $N = 10$ qubit system using projective measurements on $N_B = 4, 6, 8$ qubits.

Similar to the measurement outcome probabilities, the projected states in this protocol also encode information about $\mathcal{A}_p(U)$. The projected state on $\mathcal{H}_A$ obtained upon projecting $FU|\psi\rangle$ onto $|b\rangle \in \mathcal{H}_B$ is given by $|\phi_b\rangle = \langle b|FU|\psi\rangle/\sqrt{p_b}$. By writing $|\tilde{\phi}_b\rangle = \langle b|FU|\psi\rangle$, the asymmetry in the projected state can be computed using $\mathcal{A}_{\text{PS}} = \text{Tr}[\mathbf{Z}_A^{(2)\perp}(|\tilde{\phi}_b\rangle\langle\tilde{\phi}_b|)^{\otimes 2}]/p_b^2$. Then, averaging over free states yields to zeroth order $\langle \mathcal{A}_{\text{PS}} \rangle_{F,|\psi\rangle} \approx (\mathcal{A}_p(U)\bar{\mathcal{A}}_A k_1)/(\bar{\mathcal{A}}\langle p_b^2 \rangle_{F,|\psi\rangle})$ (see SM [71]). By incorporating $\langle p_b^2 \rangle_{F,|\psi\rangle}$ obtained via the protocol, we recover

Eq. (2) for the Z_2 -AGP. When N_A is comparable to N , it follows that $k_1/k_2 \approx 1 - 2^{-N_A} + 2^{-N}$, leading to $\langle \mathcal{A}_{\text{PS}} \rangle/\bar{\mathcal{A}}_A \approx \mathcal{A}_p(U)/\bar{\mathcal{A}}$. This indicates that the normalized asymmetry of the projected states, when averaged over the free states and unitaries, behaves very similarly to the normalized AGP of U .

We now illustrate Theorem 2 as a further consequence of the twirling identity in Eq. (5). The scenario we consider consists of interlacing free and nonfree operations [see Fig. 2(a)]. The free operations F_j at different time steps $j = 1 \dots n$ are random and independent, while the nonfree operations are arbitrary but taken here as nonrandom and identical at each step. The averaged AGP of $U^{(n)} = F_n U F_{n-1} U \dots F_1 U$ is

$$\langle \mathcal{A}_p(U^{(n)}) \rangle_{\bar{F}} = \bar{\mathcal{A}} \left[1 - \left(1 - \frac{\mathcal{A}_p(U)}{\bar{\mathcal{A}}} \right)^n \right]. \quad (6)$$

The proof follows from iteratively applying the twirling channel in Eq (5), see SM [71]. This result corresponds to Theorem 2 for the Z_2 AGP. Notably, $\bar{\mathcal{A}}$ is the only nontrivial fixed point of Eq. (6). Moreover, the above equation suggests that when free and nonfree operations are applied in conjunction, the AGP typically converges exponentially to its Haar-averaged value with rate $\lambda = -\ln[1 - \mathcal{A}_p(U)/\bar{\mathcal{A}}]$.

Main results for nonstabilizerness—Nonstabilizerness constitutes a key resource for achieving a quantum advantage, since stabilizer circuits are classically simulable [5–7]. In this QRT, the stabilizer states and Clifford group constitute the free states and unitaries. The nonstabilizerness of a pure state can be computed using the linear stabilizer entropy $m_{\text{lin}}(|\psi\rangle) = 1 - 2^N \text{Tr}[Q(|\psi\rangle\langle\psi|)^{\otimes 4}]$, where $Q = \sum_{i=1}^{2^{2N}} P_i^{\otimes 4}/2^{2N}$ is the projector onto the stabilizer space and P_i s are N -qubit Pauli strings [7,23,86,92–103]. m_{lin} is a fourth degree state polynomial. The nonstabilizing power of a unitary U is $m_p(U) = 1 - 2^N \text{Tr}[QU^{\otimes 4}\rho_s U^{\dagger \otimes 4}]$, where $\rho_s := \langle (|\psi\rangle\langle\psi|)^{\otimes 4} \rangle_{|\psi\rangle \in \text{STAB}(N)}$ is the fourth moment of the stabilizer states [23,86,104–107]. Analogous to Eq. (5), algebraic manipulations lead to the twirling identity (see SM [71])

$$\langle (CU)^{\otimes 4} \rho_s (CU)^{\dagger \otimes 4} \rangle_C = \left[1 - \frac{m_p(U)}{\bar{m}} \right] \rho_s + \frac{m_p(U)}{\bar{m}} \mathbf{\Pi}^{(4)}, \quad (7)$$

where $\mathbf{\Pi}^{(4)} = \mathbf{\Pi}^{(4)}/\text{Tr}(\mathbf{\Pi}^{(4)})$, with $\mathbf{\Pi}^{(4)} = (1/4!) \sum_{\pi \in S_4} \pi$, and π denotes the permutation operator acting on four copies of the Hilbert space $\mathcal{H}^{2^N}$. Similar to the Z_2 asymmetry, the above equation can be used to devise an experimental protocol to compute $m_p(U)$. Specifically, for a random stabilizer state $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ and a random Clifford unitary C , the probability to project onto a fixed computational-basis state $|b\rangle \in \mathcal{H}_B$ is related to $m_p(U)$ via Eq. (7) as

$m_p(U)/\bar{m} = (k_2 - \langle p_b^4 \rangle_{C,|\psi\rangle})/(k_2 - k_1)$, where $k_1 = \text{Tr}(\langle b^{\otimes 4} | \Pi^{(4)} | b^{\otimes 4} \rangle)$ and $k_2 = \text{Tr}(\langle b^{\otimes 4} | \rho_s | b^{\otimes 4} \rangle)$. Explicit expressions for k_1 and k_2 are given in the SM [71]. This result confirms the applicability of Theorem 1 to the QRT of nonstabilizerness. As one can show analytically [71], when N is large and U is a Haar-random unitary, the variance of the estimator decays like $\sim 72/(2^{N_B} - 1)$. We anticipate, and corroborate numerically, a similar scaling for the estimator's variance for more general unitaries [71].

The nonstabilizerness of the projected states can be probed using $m_{\text{PS}} \equiv 1 - 2^{N_A} \text{Tr}[Q_A(|\phi_b\rangle\langle\phi_b|)^{\otimes 4}]$, where $|\phi_b\rangle = \langle b|CU|\psi\rangle/\sqrt{p_b}$, and $Q_A = \sum_{j=0}^{2^{N_A}} (P_j)^{\otimes 4}/2^{2N_A}$ with P_j s being Pauli strings over N_A qubits. The averaged nonstabilizerness of the projected states, up to leading order becomes $\langle m_{\text{PS}} \rangle \approx (m_p(U)k_1\bar{m}_A)/(\bar{m}\langle p_b^4 \rangle)$ [71], where $\langle \cdot \rangle$ indicates averaging over the free elements C and $|\psi\rangle$. This is in accordance with Eq. (2). When N_A is comparable to N , we again have $k_1 \approx k_2$, leading to $\langle m_{\text{PS}} \rangle/\bar{m}_A \approx m_p(U)/\bar{m}$. This approximation becomes exact in the no-measurement limit, i.e., $N_A \rightarrow N$. Hence, this result offers another way of characterizing $m_p(U)$. Finally, we remind that the twirling identity in Eq. (7) also allows one to probe the nonstabilizing power of the circuits consisting of alternating free and resourceful unitaries. Let $U^{(n)} = C_n U C_{n-1} U \cdots C_1 U$, where each C_j ($\forall 1 \leq j \leq n-1$) is an independently drawn Clifford unitary and U is fixed. Then, Eq. (7) implies [23]

$$\langle m_p(U^{(n)}) \rangle_{\bar{C}} = \bar{m} \left[1 - \left(1 - \frac{m_p(U)}{\bar{m}} \right)^n \right]. \quad (8)$$

As this equation indicates, Theorem 2 is generic, showing the exponential convergence of quantum resources to their Haar-averaged values when $m_p(U)$ is finite.

Deep thermalization in QRTs—Here, we probe deep thermalization in the QRTs of asymmetry and nonstabilizerness under the circuit dynamics shown in Fig. 2(a). Deep thermalization in QRTs can be understood in two equivalent ways: (i) through the thermalization of quantum resources in the projected ensembles, which reflect their subsystem behavior, and (ii) through the emergence of approximate state designs from these ensembles. Both perspectives are intimately related: the thermalization of resources is necessary for the projected ensembles to approximate t designs ($t = 2$ and 4 for asymmetry and nonstabilizerness, respectively). In the following, we discuss the former perspective, leaving the latter to the End Matter.

For the Z_2 asymmetry, we take the initial states to be random Z_2 symmetric. At discrete time $n = 0$, measurements in the computational basis ensure that the projected states on the complementary subsystem remain Z_2 symmetric, and therefore $\langle \mathcal{A}_{\text{PE}} \rangle_{|\psi\rangle, \bar{F}} = 0$, where $\mathcal{A}_{\text{PE}} = \sum_b p_b \mathcal{A}_{\text{PS}}(|\phi_b\rangle)$ denotes the resource content of the projected ensemble for a single circuit instance. As the states evolve under the circuit dynamics, their global

asymmetry grows and relaxes to the Haar-averaged value in accordance with Eq. (6). This indicates that the states become Haar-like at the level of their second moment. Since ensembles of two-design states yield projected ensembles that also approximate two designs [27], the asymmetry of the projected ensembles likewise grows and thermalizes to the Haar value, which we identify as deep thermalization of Z_2 asymmetry. Equation (2) further shows that the asymmetry of the projected states closely tracks the AGP of the full dynamics. In particular, for $N_A \sim N$, we have $\langle \mathcal{A}_{\text{PS}}(n) \rangle_{|\psi\rangle, \bar{F}}/\bar{\mathcal{A}}_A \approx \langle \mathcal{A}_p(U^{(n)}) \rangle_{\bar{F}}/\bar{\mathcal{A}}$. Consequently, the collective asymmetry of the projected ensemble can be expected to follow a similar exponential relaxation as in Eq. (6), as confirmed numerically in Fig. 2(b). The figure shows $\langle \mathcal{A}_{\text{PE}}(n) \rangle_{\bar{F}}/\bar{\mathcal{A}}_A$ for $N = 8, 10, 12, 14$, with $N_A = 2, 4, 6$, all converging to the normalized AGP $\langle \mathcal{A}_p(U^{(n)}) \rangle_{\bar{F}}/\bar{\mathcal{A}}$, indicating the deep thermalization of the asymmetry. The inset compares $\langle \mathcal{A}_{\text{PE}}(n) \rangle_{\bar{F}}$ with the AGP of the dynamics, $\langle \mathcal{A}(U^{(n)}) \rangle_{|\psi\rangle, \bar{F}}$, showing remarkable agreement even for small subsystems ($N_A = 2$), consistent with our analytical predictions.

We now turn to the behavior of nonstabilizerness in projected ensembles, given by $m_{\text{PE}} = \sum_b p_b m_{\text{PS}}(|\phi_b\rangle)$, under interlaced Clifford and non-Clifford dynamics. At $n = 0$, computational-basis measurements leave the projected states in the stabilizer set, yielding $m_{\text{PE}} = 0$. As the dynamics proceeds, the nonstabilizerness of the projected ensemble, averaged over initial stabilizer states and Clifford unitaries, grows and relaxes exponentially toward the Haar value $\bar{m}_A$ for all system and subsystem sizes considered [Fig. 2(c)]. For larger subsystems, $\langle m_{\text{PE}}(n) \rangle/\bar{m}_A$ closely tracks the nonstabilizing power of the full dynamics. The inset shows even faster relaxation of $\langle m_{\text{PE}}(n) \rangle$ than the global evolution. This numerical behavior is consistent with Eq. (8).

Discussion—In this Letter, we have developed a protocol to quantify and characterize the resource-generating power of unitaries across a broad class of resource theories. The quantum resources the framework encompasses have implications for quantum computation, quantum foundations, and the classical simulability of quantum dynamics. A central outcome is an unbiased estimator whose variance over free elements decays at least exponentially with the size of the measured subsystem, enabling scalable experimental characterization of quantum resources. Beyond its practical utility, our framework reveals that free and resourceful operations together drive the thermalization of resource measures toward their Haar values, while deep thermalization provides a complementary subsystem-level proxy for the resource content of dynamics.

Being built on twirling identities, our framework naturally extends to quantifying the resource content of arbitrary quantum states. Although our protocol relies on random free operations, it remains an open question whether similar twirling identities emerge when using local free operations [24,25,70], which are often more scalable in

practice. In addition, developing efficient methods for sampling free unitaries constitutes an important direction for future research. Our main results hinge on twirling identities like Eqs. (5) and (7) (see also SM [71]), where averaging free unitaries yields a weighted sum of free and Haar moments. Any QRT admitting such a twirling identity will lead to the same conclusions. Investigating whether this framework extends to other QRTs, such as non-Gaussianity [16,108–113], or nonstabilizerness for qudits [114,115], is a compelling direction of future research.

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Data availability—All data supporting the findings of this study are publicly available in the Zenodo repository at [116].

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End Matter

Emergent state designs in QRTs—Complementary to the analysis in the main text regarding the thermalization of quantum resources in projected ensembles, we further characterize deep thermalization by studying how the projected ensembles of typical free generator states converge to state t designs [117]. For this purpose, we use the trace-distance measure between the moments, given by $\Delta^{(t)}(\tau) = \|\mathcal{M}_{\text{PE}}^{(t)}(n) - \Pi_A^{(t)}\|_1$. At $n = 0$, the first moment of the projected ensemble already coincides with the Haar moment ($\mathbb{I}A/2^{N_A}$), while higher moments remain symmetrized. Under the dynamics generated by $U^{(n)}$ in Fig. 2(a), these moments relax exponentially in time toward their Haar counterparts. Figures 3(a) and 3(b) show this relaxation for the second and fourth moments, respectively, for $N = 8, 10, 12$ with $N_A = 2$, where measurements on N_B qubits are performed in the computational basis and results are averaged over $\sim 10^2$ circuit realizations. The second-moment distance $\Delta^{(2)}(n)$ decays exponentially with time, and its long-time value decreases exponentially with increasing N_B . There are well-known situations where first nontrivial deviations from Haar statistics occur at higher moments (e.g., stabilizer states without projections form exact three-designs, but not four designs [118,119]). Therefore, we also study $\Delta^{(4)}(n)$, which again shows exponential decay, indicating rapid convergence of the projected

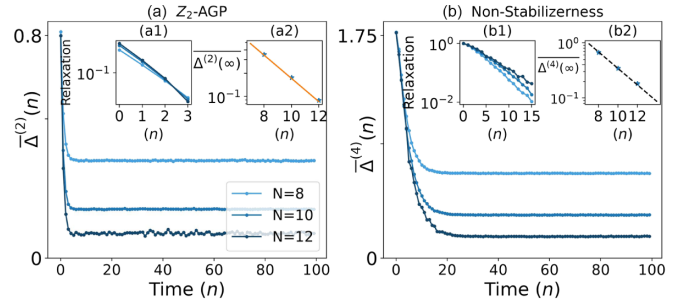


FIG. 3. Deep thermalization, quantified by the trace distance between the moments of the projected ensemble and that of the Haar ensemble, in the QRT of (a) $\mathbb{Z}_2$ asymmetry and (b) non-stabilizerness. In both cases, the initial free state evolves under repeated interspersings of random free and nonfree operations as depicted in Fig. 2(a), followed by measurements in the computational basis. The data are averaged over $\sim 10^2$ realizations of the circuit and the initial free state. The subsystem size is kept fixed at $N_A = 2$. Insets [(a1),(b1)]: The trace distance exhibits exponential relaxation toward its long-time average across all system sizes, consistent with global thermalization. We quantify this relaxation using $1 - \bar{\Delta}^{(t)}/\Delta^{(t)}(\infty)$, where $\Delta^{(t)}(\infty)$ denotes the long-time average and $t = 2, 4$ correspond to the $\mathbb{Z}_2$ -AGP and nonstabilizerness, respectively. (b1),(b2) The long-time average decays exponentially with increasing system size.

ensemble toward Haar-like statistics, with long-time values that vanish exponentially with system size.